

MATH 1620LR Test 1

Name _____

Banner ID _____

Section LR _____

Please do not separate these pages!

1. (32 pts) Consider the improper integral

$$I = \int_k^{\infty} \frac{1}{x^2 - x} dx$$

- (a) First evaluate the indefinite integral

$$\int \frac{1}{x^2 - x} dx.$$

Express your answer in terms of a single logarithm.

(b) Evaluate the improper integral I for $k = 3$

(c) For which values of k will the improper integral I converge, and for which will it diverge? Justify your answer.

2. (33 pts) Evaluate each integral:

(a)

$$\int \sqrt{x} \ln x \, dx$$

(b)

$$\int_0^{\frac{3\sqrt{2}}{2}} \frac{x^2}{(9-x^2)^{3/2}} \, dx$$

(c)

$$\int_0^{1/3} \tan^{-1}(3x) dx$$

(Careful: this is arctan, not tan)

3. (24 pts) Determine whether each integral converges or diverges. Justify your answers completely.

(a)

$$\int_0^{10} \frac{1}{t^6 + \sqrt{t}} dt$$

(b)

$$\int_1^{\infty} \frac{2x + 1}{\sqrt{4x^3 - 1}} dx$$

4. (21 pts) Suppose $f(x)$ and $g(x)$ are continuous for all $-\infty < x < \infty$. Suppose $\int_1^\infty f(x) dx$ converges, and that both $\int_{-\infty}^1 f(x) dx$ and $\int_{-\infty}^1 g(x) dx$ diverge. For each integral below, state whether it is convergent, divergent, or if the information above is insufficient to determine whether it converges or diverges. No justification necessary.

(a)

$$\int_{-\infty}^{\infty} f(x) dx$$

(b)

$$\int_1^{1,000} g(x) dx$$

(c)

$$\int_{-1,000}^{\infty} f(x) dx$$

(d)

$$\int_{-\infty}^1 (f(x) + g(x)) dx$$

(e)

$$\int_1^{\infty} (f(x) - 1) dx$$

(f)

$$\int_{-\infty}^{-1,000} g(x) dx$$

(g)

$$\int_{-\infty}^1 f(x)g(x) dx$$

Any extra work goes here

