Questions for recitation 24 March 2021

1. Exercises 11.4: #29-32, 40-43, 45-46.

   Exercises 11.3: #22, 34, 41-44

2. Determine if each of the series below converges or diverges. If possible, for each convergent series, determine the sum of the series. Be sure to fully motivate your answers.

   (a) $\sum_{n=3}^{\infty}(-1)^n(\cos[1])^n$

   (b) $\sum_{n=0}^{\infty}\frac{2^n+5}{3^n}$

   (c) $\sum_{n=3}^{\infty}\frac{1}{(n+2)\ln(n+2)}$

   (d) $\sum_{n=0}^{\infty}\frac{n^3-n^{2/3}+18}{\sqrt{n^7-n^5+2}-1}$

   (e) $\sum_{n=3}^{\infty}\frac{\ln n}{\ln(\ln n)}$

   Solution:

   (a) This is geometric, and converges to $\frac{-\cos[1]^3}{1+\cos[1]}$.

   (b) This is the sum of 2 geometric series, and converges to $\frac{1}{1-\frac{2}{3}}+\frac{5}{1-\frac{1}{3}}$.

   (c) This is a positive, decreasing, and continuous function of n, so it has the same behavior as $\lim_{t\to\infty}\int_3^t \frac{1}{(x+2)\ln(x+2)}\,dx = \lim_{t\to\infty}\ln(\ln(x+2))|_3^t \to \infty$.

   (d) Diverges by the limit comparison test to $b_n = \frac{1}{\sqrt{n}}$, as $\frac{a_n}{b_n} \to 1$ and $\sum b_n \to \infty$ by a p-test.

   (e) Diverges by the divergence test

3. Find the sum of the series

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+\frac{1}{16}+\frac{1}{18}+\cdots$$

   where the terms are the reciprocals of the positive integers whose only prime factors are 2s and 3s.

Solution: We can arrive at this series by multiplying the series containing all of the reciprocals of the positive integers whose only prime factor is 2 to the series containing only the 3s, or:

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots = \left(\sum_{i=0}^{\infty} \frac{1}{2^i} \right) \left(\sum_{j=0}^{\infty} \frac{1}{3^j} \right) = \left(\frac{1}{1 - \frac{1}{2}} \right) \left(\frac{1}{1 - \frac{1}{3}} \right) = 3$$