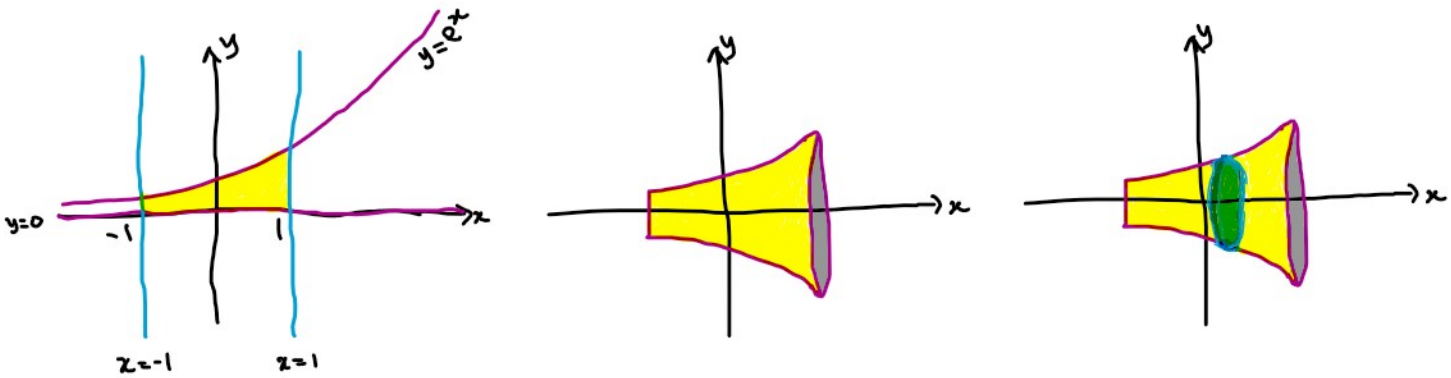
(4) $y = e^x$, $y = 0$, $x = -1$, $x = 1$; about the x-axis



Area of disk $A(x) = \pi r^2$

$$= \pi y^2$$

$$= \pi (e^x)^2$$

$$= \pi e^{2x}$$

So,

$$\text{Volume of solid} = \int_{-1}^{1} A(x)\,dx$$

$$= \int_{-1}^{1} \pi e^{2x}\,dx$$

$$= \frac{\pi e^{2x}}{2}\Bigg|_{-1}^{1}$$

$$= \frac{\pi}{2}\left(e^{2(1)} - e^{2(-1)}\right)$$

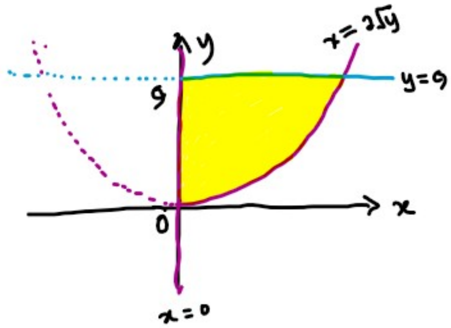
$$= \frac{\pi}{2}\left(e^{2} - e^{-2}\right)$$

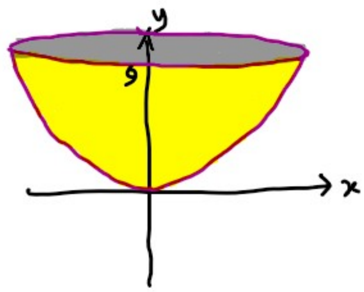
(5) $x = 2\sqrt{y}$, $x = 0$, $y = 9$; about the y-axis

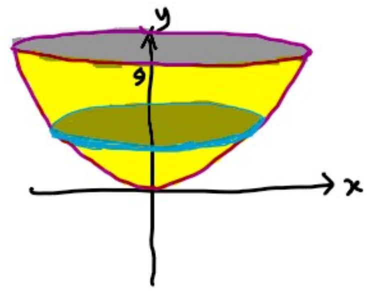
$y = \frac{x^2}{4}$, $x \geq 0$ (only for the sake of drawing)

(5) $x = 2\sqrt{y}$, $x = 0$, $y = 9$; about the y-axis

NB: $x = 2\sqrt{y} \Rightarrow y = \frac{x^2}{4}, x \geq 0$ (only for the sake of drawing)







Area of disk $A(y) = \pi r^2$

$= \pi x^2$

$= \pi (2\sqrt{y})^2$

$= 4\pi y$

So,

Volume of solid $= \int_0^9 A(y)\,dy$

$= \int_0^9 4\pi y\,dy$

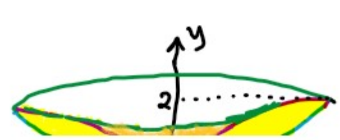
$= \left.\frac{4\pi y^2}{2}\right|_0^9$

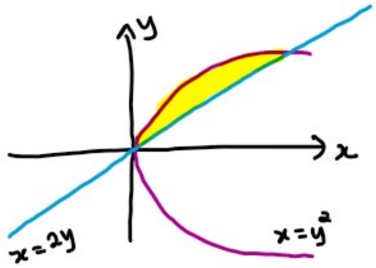
$= 2\pi (9^2 - 0^2)$

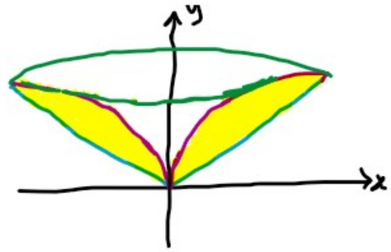
$= 2\pi (81)$

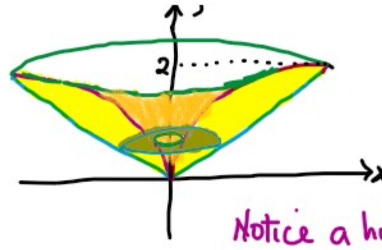
$= \underline{\underline{162\pi}}$

(9) $y^2 = x$, $x = 2y$; about the y-axis.









Notice a hollow in the disk

Area of hollow disk (a washer $\quad A(y) = \pi r_o^2 - \pi r_i^2$

(r_o and r_i represent outer & inner radii) respectively

$$= \pi x_o^2 - \pi x_i^2$$

$$= \pi (2y)^2 - \pi (y^2)^2$$

$$= \pi (4y^2 - y^4)$$

But the curve and the line intersect when

$$x = y^2 = 2y$$

$$\Rightarrow \quad y(y-2) = 0$$

$$\Rightarrow \quad y = 0, \ y = 2$$

So,

Volume of solid $= \int_0^2 A(y)\,dy$

$$= \int_0^2 \pi (4y^2 - y^4)\,dy$$

$$= \pi \left(\frac{4y^3}{3} - \frac{y^5}{5} \right) \Big|_0^2$$

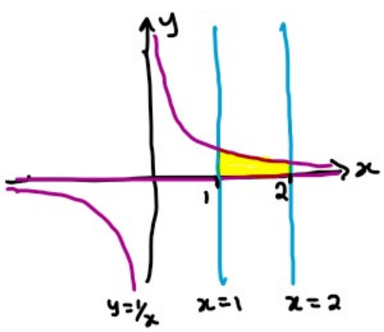
$$= \pi \left(\frac{4 \cdot 2^3}{3} - \frac{2^5}{5} \right) - \pi \left(\frac{4 \cdot 0^3}{3} - \frac{0^5}{5} \right)$$

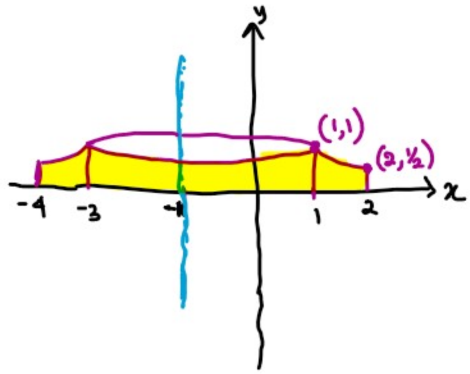
$$= \pi \left(\frac{4 \cdot 8}{3} - \frac{32}{5} \right)$$

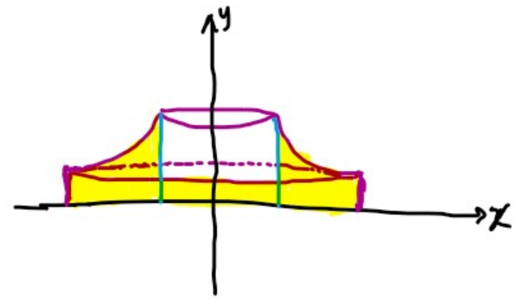
$$= \pi \left(\frac{32}{3} - \frac{32}{5} \right)$$

$$= \frac{64\pi}{15}$$

(16) $\underbrace{xy = 1}_{y=\frac{1}{x}}$, $y = 0$, $x = 1$, $x = 2$; about $x = -1$



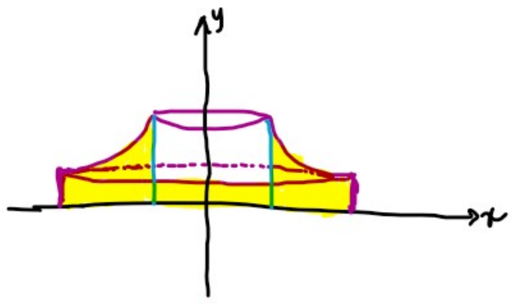




a

Area of hollow disk (washer) in a

$$A_a(y) = \pi(r_o^2 - r_i^2)$$
$$= \pi(3^2 - 2^2)$$
$$= 5\pi$$



b

Area of hollow disk (washer) in b

$$A_b(y) = \pi(r_o^2 - r_i^2)$$
$$= \pi(x_o^2 - x_i^2)$$
$$= \pi\left[\left(\tfrac{1}{y} - (-1)\right)^2 - \left(1 - (-1)\right)^2\right]$$
$$= \pi\left[\left(\tfrac{1}{y} + 1\right)^2 - 2^2\right] = \pi\left(\frac{1}{y^2} + \frac{2}{y} + 1 - 4\right) = \pi\left(\frac{1}{y^2} + \frac{2}{y} - 3\right)$$

Hence,

$$\text{Volume of solid} = \int_0^{1/2} A_a(y)\,dy + \int_{1/2}^{1} A_b(y)\,dy$$
$$= \int_0^{1/2} 5\pi\,dy + \int_{1/2}^{1} \pi\left(\frac{1}{y^2} + \frac{2}{y} - 3\right)dy$$
$$= 5\pi y\Big|_0^{1/2} + \pi\left(-\frac{1}{y} + 2\ln y - 3y\right)\Big|_{1/2}^{1}$$

$$\ldots \; \pi\left(-1 \; + 2\ln\tfrac{1}{2} - 3(\tfrac{1}{2})\right)$$

$$= 5\pi y\Big|_0 \quad \cdots \quad \Big|_{1/2}$$

$$= 5\pi\left(\tfrac{1}{2}-0\right) + \pi\left(-\tfrac{1}{1} + 2\ln 1 - 3(1)\right) - \pi\left(-\tfrac{1}{1/2} + 2\ln\tfrac{1}{2} - 3(\tfrac{1}{2})\right)$$

$$= \frac{5\pi}{2} + \pi(-4) - \pi\left(-\tfrac{7}{2} - 2\ln 2\right)$$

$$= \frac{5\pi}{2} - 4\pi + \frac{7}{2}\pi + 2\pi\ln 2$$

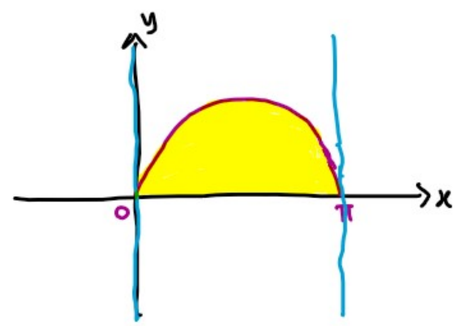
$$= 2\pi + 2\pi\ln 2$$

$$= 2\pi(1+\ln 2)$$

(39) $\pi\int_0^{\pi} \sin x\,dx = \int_0^{\pi} \pi\left(\sqrt{\sin x}\right)^2 dx$

$$= \int_0^{\pi} \pi y^2\,dx \quad \text{where } y = \sqrt{\sin x}$$

$$= \int_0^{\pi} A(x)\,dx$$

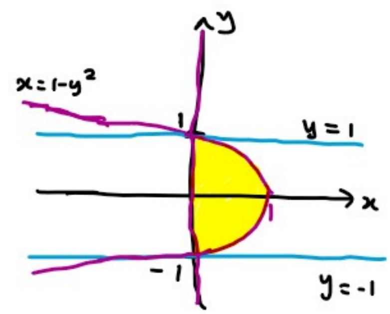


So
The region is the rotation of the area bounded by $y = \sqrt{\sin x}$, $y=0$, $x=0$ and $x=\pi$ about the x-axis.

(40) $\pi\int_{-1}^{1} (1-y^2)^2\,dy = \int_{-1}^{1} \pi(1-y^2)^2\,dy$

$$= \int_{-1}^{1} \pi x^2\,dy \quad \text{where } x = 1-y^2$$

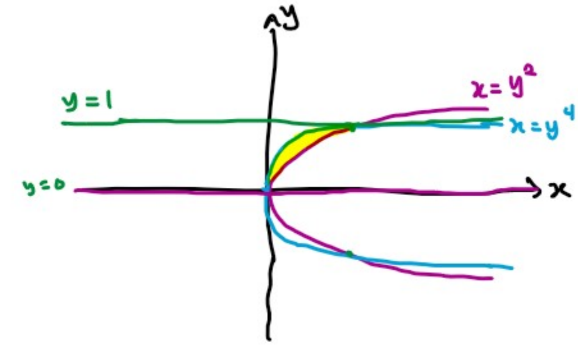
$$= \int_{-1}^{1} A(y)\,dy$$



So, the region is the rotation of the area bounded by $x = 1-y^2$

So, the region is the rotation of the area bounded by $x = 1 - y^2$ $x = 0$, $y = -1$ and $y = 1$ about the y-axis.

(41) $\pi \int_0^1 (y^4 - y^8)\, dy = \int_0^1 \pi \left((y^2)^2 - (y^4)^2 \right) dy$

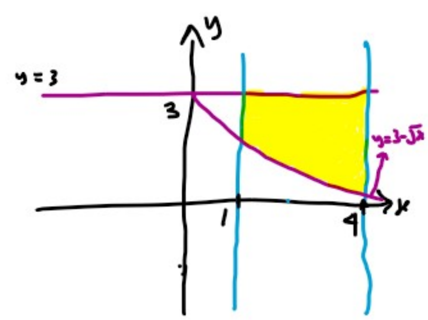


$= \int_0^1 \pi \left(x_o^2 - x_i^2 \right) dy$

$= \int_0^1 A(y)\, dy$

where $x_o = y^2$ and $x_i = y^4$ are the outer & inner radii respectively.

So the region is the rotation of the area bounded by $x = y^2$, $x = y^4$, $y = 1$, and $y = 0$ about the y-axis.



(42) $\pi \int_1^4 \left[3^2 - (3 - \sqrt{x})^2 \right] dx = \int_1^4 \pi \left[3^2 - (3 - \sqrt{x})^2 \right] dx$

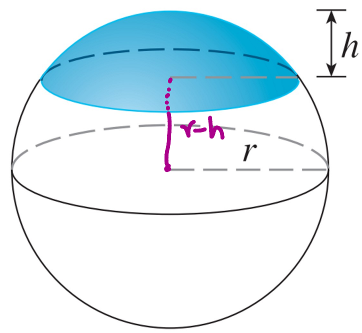
$= \int_1^4 \pi \left[y_o^2 - y_i^2 \right] dx$

where $y_o = 3$, $y_i = 3 - \sqrt{x}$

$= \int_1^4 A(x)\, dx$

So the region is the rotation of the area bounded by $y = 3$, $y = 3 - \sqrt{x}$ $x = 1$, $x = 4$ about x-axis.

49. A cap of a sphere with radius r and height h



Area of a disk $= \pi x^2$

$= \pi(r^2 - y^2)$ since $x^2 + y^2 = r^2$

So,

$$\text{Volume of solid} = \int_{r-h}^{r} A(y)\,dy$$

$$= \int_{r-h}^{r} \pi(r^2 - y^2)\,dy$$

$$= \pi\left(r^2 y - \frac{y^3}{3}\right)\Bigg|_{r-h}^{r}$$

$$= \pi\left(r^2 \cdot r - \frac{r^3}{3}\right) - \pi\left(r^2(r-h) - \frac{(r-h)^3}{3}\right)$$

$$= \pi\left(r^3 - \frac{r^3}{3}\right) - \pi\left(r^3 - r^2 h - \frac{(r-h)(r^2 - 2rh + h^2)}{3}\right)$$

$$= \pi\left(r^3 - \frac{r^3}{3}\right) - \pi\left(r^3 - r^2 h - \frac{r^3 - 2r^2 h + rh^2 - r^2 h + 2rh^2 - h^3}{3}\right)$$

$$= \pi\left(r^3 - \frac{r^3}{3}\right) - \pi\left(\frac{3r^3 - 3r^2 h - (r^3 - 3r^2 h + 3rh^2 - h^3)}{3}\right)$$

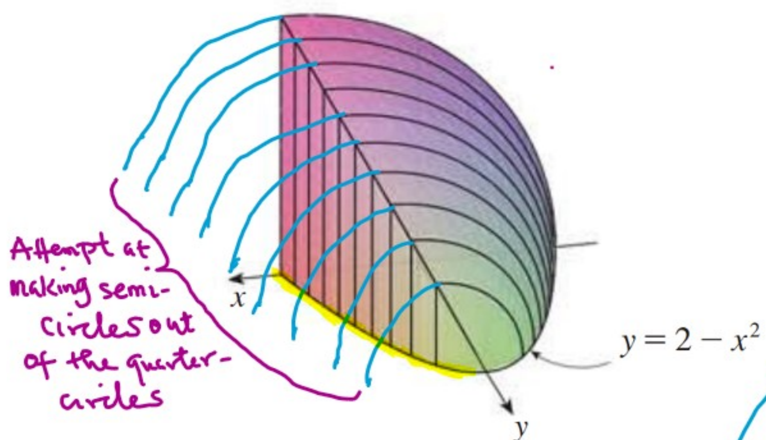
$$= \pi\left(\frac{3r^3 - r^3}{3}\right) - \pi\left(\frac{3r^3 - 3r^2 h - r^3 + 3r^2 h - 3rh^2 + h^3}{3}\right)$$

$$= \pi\left(\frac{2r^3}{3}\right) - \pi\left(\frac{2r^3 - 3rh^2 + h^3}{3}\right)$$

$$= \frac{\pi}{3}\left(2r^3 - 2r^3 + 3rh^2 - h^3\right)$$

$$= \frac{\pi}{3}\left(3rh^3 - h^3\right)$$

60. The base of S is the region enclosed by $y = 2 - x^2$ and the x-axis. Cross-sections perpendicular to the y-axis are quarter-circles.



$y = 2 - x^2$

$\Rightarrow x^2 = 2 - y$

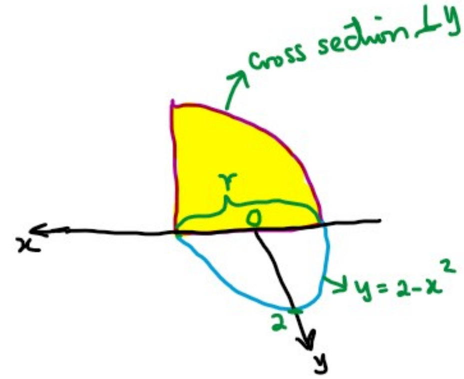
$\Rightarrow x = \pm\sqrt{2-y}$

So radius of each cross section perpendicular to y-axis

$r = \sqrt{2-y} - \left(-\sqrt{2-y}\right)$

$= 2\sqrt{2-y}$

Notice that the center of each of the quarter-circles is on the highlighted part of the base curve $y = 2 - x^2$



Area of a quarter-circle $A(y) = \frac{\pi}{4} r^2$

$= \frac{\pi}{4}\left(2\sqrt{2-y}\right)^2$

$= \frac{4\pi}{4}(2-y)$

$= \pi(2-y)$

Hence,

Volume of $\int^{2} A(y)\,dy$

Volume of solid $= \int_0^2 A(y)\,dy$

$$= \int_0^2 \pi(2-y)\,dy$$

$$= \pi\left(2y - \frac{y^2}{2}\right)\Big|_0^2$$

$$= \pi\left(2(2) - \frac{2^2}{2}\right) - \pi\left(2(0) - \frac{0^2}{2}\right)$$

$$= \pi(4-2)$$

$$= 2\pi$$