1. Find $\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{e^{2x}-1} dx$

Take $u = e^{2x}-1$. Then $\frac{du}{dx} = 2e^{2x}$

$$\Rightarrow \quad \frac{du}{2e^{2x}} = dx$$

Thus,

$$\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{e^{2x}-1} dx = \int_{\ln 2}^{\ln 3} \frac{e^{2x}}{u} \cdot \frac{du}{2e^{2x}}$$

$$= \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{1}{u} du$$

$$= \frac{1}{2} \ln u \Big|_{x=\ln 2}^{x=\ln 3}$$

$$= \frac{1}{2} \ln\left(e^{2x}-1\right) \Big|_{\ln 2}^{\ln 3}$$

$$= \frac{1}{2} \ln\left(e^{2\ln 3}-1\right) - \frac{1}{2}\ln\left(e^{2\ln 2}-1\right)$$

$$= \frac{1}{2} \ln\left(e^{\ln 9}-1\right) - \frac{1}{2}\ln\left(e^{\ln 4}-1\right)$$

$$= \frac{1}{2} \ln(9-1) - \frac{1}{2}\ln(4-1)$$

$$= \frac{1}{2}\ln 8 - \frac{1}{2}\ln 3$$

$$= \frac{1}{2}\ln\left(\frac{8}{3}\right)$$

2. Find $\int_{1/2}^{1} 8x^{-2}(1+\frac{1}{x})^{-3}dx$

Take $u = 1+\frac{1}{x}$. Then $\frac{du}{dx} = -x^{-2}$

$$\Rightarrow \quad \frac{du}{-x^{-2}} = dx$$

Thus,

$$\int_{1/2}^{1} 8x^{-2}\left(1+\frac{1}{x}\right)^{-3}dx = \int_{1/2}^{1} 8x^{-2} \cdot u^{-3} \cdot \frac{du}{-x^{-2}}$$

$$= \int_{1/2}^{1} 8u^{-3}du$$

$$= 8 \cdot \frac{u^{-3+1}}{-3+1}\Bigg|_{x=1/2}^{x=1}$$

$$= -\frac{8}{2}\left(1+\frac{1}{x}\right)^{-2}\Bigg|_{1/2}^{1}$$

$$= -4\left(\left(1+\frac{1}{1}\right)^{-2} - \left(1+\frac{1}{1/2}\right)^{-2}\right)$$

$$= -4\left(2^{-2} - 3^{-2}\right)$$

$$= 4\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= 4\left(\frac{9-4}{36}\right)$$

$$= \frac{20}{36}$$

3. Sketch the curve $y = f(x)$, where f satisfies all the following:

(a) f is continuous everywhere

(b) $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$, and $f'(1) = 0$

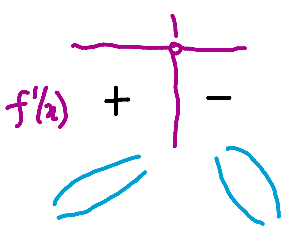
(c) $f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$, $f''(x) < 0$ on $(-2, 2)$

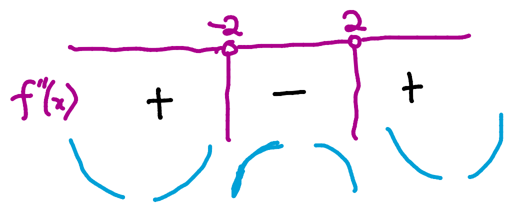
(d) $\lim_{x\to-\infty} f(x) = -\infty$

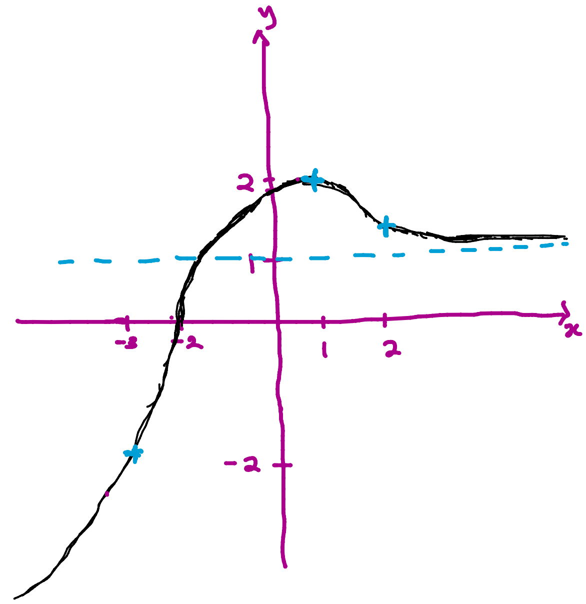
(e) $\lim_{x\to\infty} f(x) = 1$

(f) $f(-3) = -2$

(g) $f(1) = 2$







4. Let $g(x) = \int_{\sqrt{x}}^{1} \cos(t^2)dt$ be defined on $(0, 2\pi)$. Find the intervals of increase and decrease of g.

Since $\cos(t^2)$ is continuous,

$$g(x) = \int_{\sqrt{x}}^{1} \cos(t^2)dt \Rightarrow g'(x) = \cos(t^2)\Big|_{t=1} \cdot (1)' - \cos(t^2)\Big|_{t=\sqrt{x}} \cdot (\sqrt{x})'$$

$$= \cos(1)\cdot 0 - \cos\left((\sqrt{x})^2\right) \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$= -\frac{\cos x}{2\sqrt{x}}$$

(Recall that $F(x) = \int_{g(x)}^{h(x)} f(t)dt \Rightarrow F'(x) = f(h(x))h'(x) - f(g(x))g'(x)$ provided f is continuous)

Next, we compute the critical numbers of f by finding x in $(0, 2\pi)$ such that

$$g'(x) = \begin{cases} 0 \\ DNE \\ \pm\infty \end{cases}$$

Since 0 is not in $(0, 2\pi)$, it follows that $g'(x) = 0$ is the only possibility.

Thus,

$$g'(x) = 0 \Rightarrow -\frac{\cos x}{2\sqrt{x}} = 0$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = \pi/2, \frac{3\pi}{2}$$

		$\pi/2$		$\frac{3\pi}{2}$	
$-\cos x$	$-$		$+$		$-$
$\sqrt{x}$	$+$		$+$		$+$
$g'(x)$	$-$		$+$		$-$
g	$\searrow$		$\nearrow$		$\searrow$

Hence, g is decreasing on $(0, \pi/2), (\frac{3\pi}{2}, 2\pi)$ and increasing on $(\pi/2, \frac{3\pi}{2})$.

5. If $f(x) = 10 + \int_9^{x^2} \sin(\frac{\pi\sqrt{t}}{2})dt$, what is $f(3)$ and $f'(3)$?

$$f(x) = 10 + \int_9^{x^2} \sin\left(\frac{\pi\sqrt{t}}{2}\right) dt$$

$$\rightarrow f(3) = 10 + \int_9^{3^2} \sin\left(\frac{\pi\sqrt{t}}{2}\right)dt$$

$$= 10 + \int_9^{9} \sin\left(\frac{\pi\sqrt{t}}{2}\right)dt$$

$$= 10 + 0$$

$$= \underline{\underline{10}}$$

Also, by continuity of $\sin\left(\frac{\pi\sqrt{t}}{2}\right)$,

$$f'(x) = \frac{d}{dx}(10) + \frac{d}{dx}\int_9^{x^2} \sin\left(\frac{\pi\sqrt{t}}{2}\right) dt$$

$$= 0 + \sin\left(\frac{\pi\sqrt{t}}{2}\right)\Bigg|_{t=x^2} \cdot (x^2)' - \sin\left(\frac{\pi\sqrt{t}}{2}\right)\Bigg|_{t=9} \cdot (9)'$$

$$= \sin\left(\frac{\pi\sqrt{x^2}}{2}\right) \cdot 2x - \sin\left(\frac{\pi\sqrt{9}}{2}\right) \cdot 0$$

$$= 2x \sin\left(\frac{\pi}{2}x\right)$$

So,

$$f'(3) = 2(3)\sin\left(\frac{3\pi}{2}\right)$$

$$= -6$$

6. A terabyte of data contains 2^{40} bytes. Use linearization to approximate 2^{40} without a calculator. (Hint: Use $f(x) = x^4$, then $2^{40} = f(2^{10}) = f(1024)$. With $a = 1000$, $f(1024) \approx L(1024) = f(a) + f'(a)(1024 - a) = 1000^4 + 4 \cdot 1000^3 \cdot (1024 - 1000) = 1,096,000,000,000$. In actuality, $2^{40} = 1,099,511,627,776$.)

Using $f(x) = x^4$, we have that $f(2^{10}) = 2^{40}$.

But $2^{10} = 1024$. Thus

$$L(1024) = f(1000) + f'(1000)(1024 - 1000)$$

$$= 1000^4 + 4(1000^3)(24)$$

$= 1000\,000\,000\,000\,000 + 96\,000\,000\,000\,000$

$= 1096\,000\,000\,000\,000$

7. Express $\int_1^4 (x^2 - 4x + 2)dx$ as a limit of Reimann sums, and then evaluate the limit. (Hint: Recall that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.)

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$ and so $x_i = 1 + \frac{3}{n} i$

Thus,

$$\int_1^4 (x^2 - 4x + 2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i), \quad f(x) = x^2 - 4x + 2$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} f\left(1 + \frac{3}{n} i\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\left(1 + \frac{3}{n} i\right)^2 - 4\left(1 + \frac{3}{n} i\right) + 2 \right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(1 + \frac{6}{n} i + \frac{9}{n^2} i^2 - 4 - \frac{12}{n} i + 2\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\frac{9}{n^2} i^2 - \frac{6}{n} i - 1\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\frac{9}{n^2} \sum_{i=1}^{n} i^2 - \frac{6}{n} \sum_{i=1}^{n} i - \sum_{i=1}^{n} 1 \right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{6}{n}\cdot\frac{n(n+1)}{2} - n\right)$$

$$= \lim_{n\to\infty}\left(\frac{9(n+1)(2n+1)}{2n^2} - \frac{9(n+1)}{n} - 3\right)$$

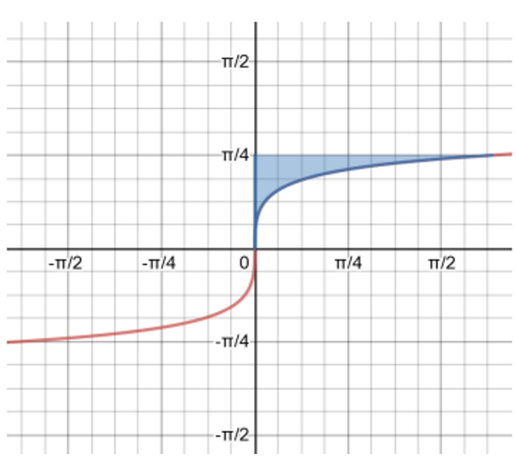
$$= \lim_{n\to\infty}\left(\frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 9\left(1+\frac{1}{n}\right) - 3\right)$$

$$= \frac{9}{2}(1+0)(2+0) - 9(1+0) - 3$$

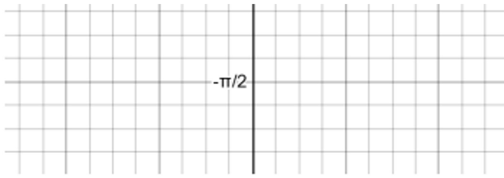
$$= 9 - 9 - 3$$

$$= -3$$

8. $\int_a^b f(x)dx$ can be interpreted as the net area contained between f and the x-axis for $a \le x \le b$. How would you interpret $\int_c^d f(y)dy$? Use this interpretation to determine the area of the region below where the curve is given by $x = \sec^2 y \tan^3 y$.



$\int_c^d f(y)dy$ can be interpreted as the area under the curve $x = f(y)$ from c to d.



$$\text{Area of region} = \int_0^{\pi/4} \sec^2 y \tan^3 y \, dy$$

$$u = \tan y \Rightarrow \frac{du}{dy} = \sec^2 y \Rightarrow \frac{du}{\sec^2 y} = dy$$

Thus,

$$\text{Area of region} = \int_0^{\pi/4} \sec^2 y \cdot u^3 \cdot \frac{du}{\sec^2 y}$$

$$= \int_0^{\pi/4} u^3 \, du$$

$$= \frac{u^4}{4} \Bigg|_{y=0}^{y=\pi/4}$$

$$= \frac{(\tan y)^4}{4} \Bigg|_0^{\pi/4}$$

$$= \frac{1}{4}\left(\tan^4 \frac{\pi}{4} - \tan^4 0\right)$$

$$= \frac{1}{4}\left(1^4 - 0^4\right)$$

$$= 1/4$$