Section 12.6: 4, 11, 21, 25, 29, 32, 34

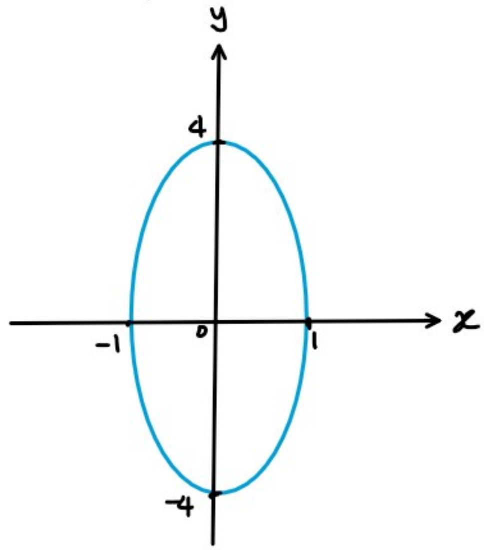
3–8 Describe and sketch the surface.

3. $x^2 + z^2 = 1$

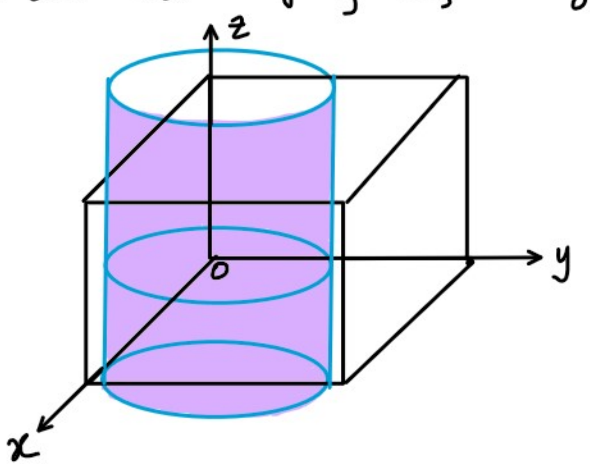
4. $4x^2 + y^2 = 4$

(a) In standard form,

$$4x^2 + y^2 = 4 \Rightarrow x^2 + \frac{y^2}{4} = 1,$$ an ellipse



Stacking the ellipses over the range of z, we get

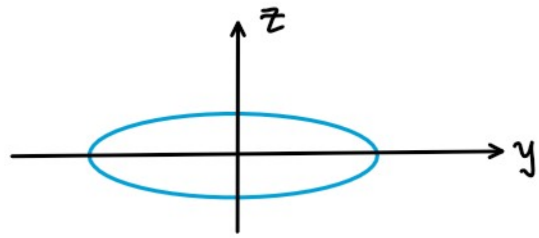


which is an elliptic cylinder.

11–20 Use traces to sketch and identify the surface.

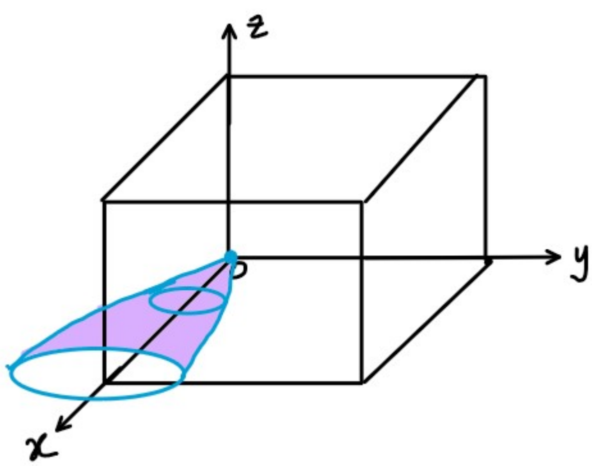
11. $x = y^2 + 4z^2$ **12.** $4x^2 + 9y^2 + 9z^2 = 36$

For any fixed $x \geq 0$, the trace on yz plane is an ellipse given as



with axes lengths proportional to x. e.g., at $x=0$, the ellipse is a point and it expands out as x increases.
NB: $x < 0$ does not make sense.

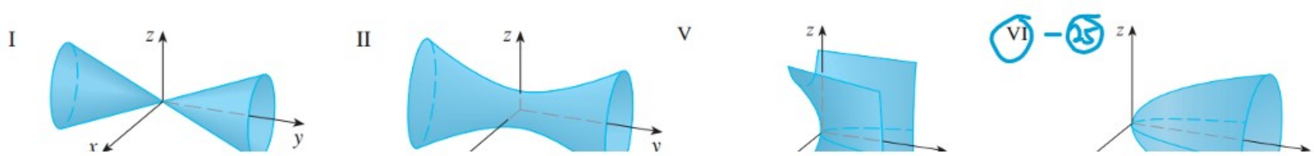
Stacking these ellipses along the x-axis, we obtain

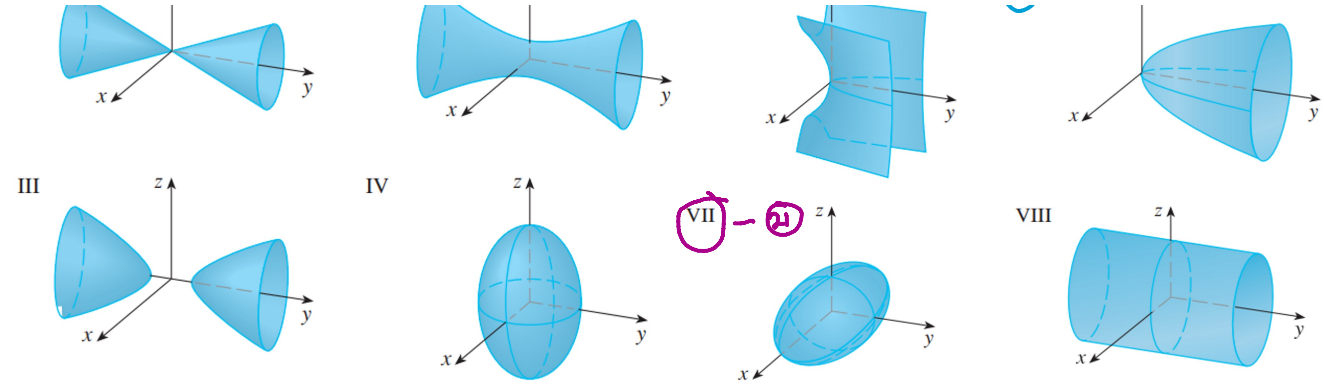


which is an elliptic paraboloid.

21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choice.

21. $x^2 + 4y^2 + 9z^2 = 1$ **22.** $9x^2 + 4y^2 + z^2 = 1$





21. $x^2 + 4y^2 + 9z^2 = 1$

For $-1 \le x \le 1$, $4y^2 + 9z^2 = 1 - x^2$ is a family of ellipses that vanish at both bounds ($x = -1$ and $x = 1$).

For $-\frac{1}{2} \le y \le \frac{1}{2}$, $x^2 + 9z^2 = 1 - 4y^2$ is a family of ellipses that vanish at both bounds (at $x = -\frac{1}{2}$ and at $x = \frac{1}{2}$).

For $-\frac{1}{3} \le z \le \frac{1}{3}$, $x^2 + 4y^2 = 1 - 9z^2$ is a family of ellipses that vanish at both bounds (at $x = -\frac{1}{3}$ and at $x = \frac{1}{3}$)

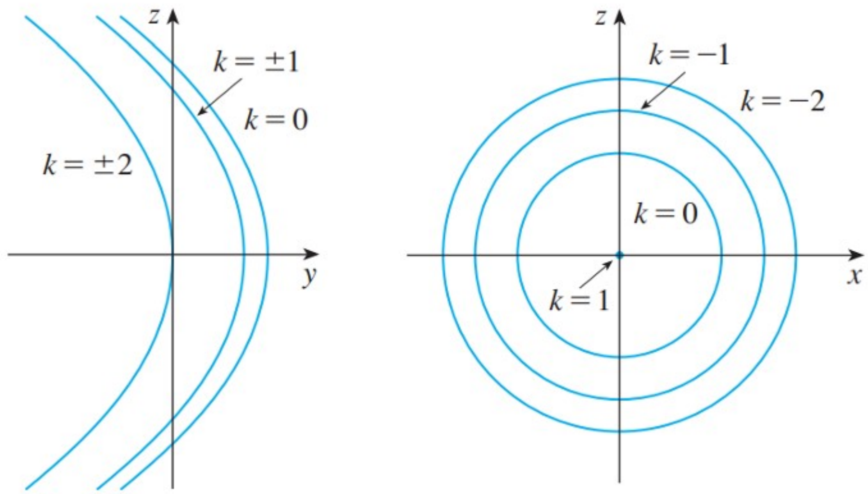
Putting everything together, we obtain an ellipsoid with major axis as the x-axis as shown in (VII).

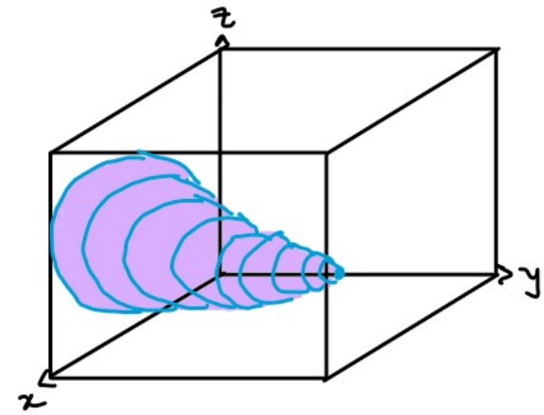
25. $y = 2x^2 + z^2$ **26.** $y^2 = x^2 + 2z^2$

Following similar arguments as in Q 11, we see $y = 2x^2 + z^2$ is an ellipse with z as the major axis for a fixed y. Stacking these ellipses along the y-axis gives (VI)

29–30 Sketch and identify a quadric surface that could have the traces shown.

29. Traces in $x = k$ Traces in $y = k$





Circular paraboloid

31–38 Reduce the equation to one of the standard forms, classify the surface, and sketch it.

31. $y^2 = x^2 + \frac{1}{9}z^2$

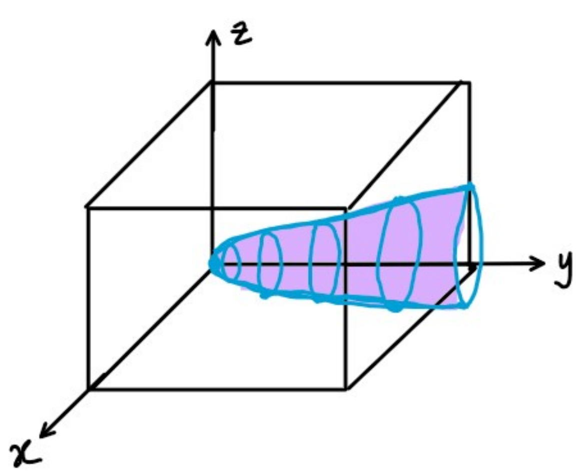
32. $4x^2 - y + 2z^2 = 0$

33. $x^2 + 2y - 2z^2 = 0$

34. $y^2 = x^2 + 4z^2 + 4$

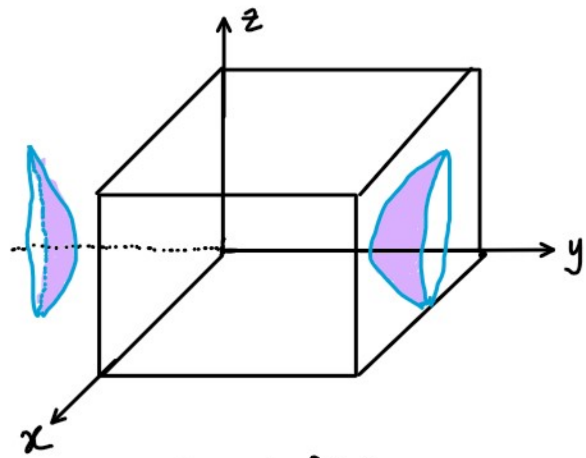
32 $4x^2 - y + 2z^2 = 0 \Rightarrow y = 4x^2 + 2z^2$

$\Rightarrow \frac{y}{1} = \frac{x^2}{(1/2)^2} + \frac{z^2}{(1/\sqrt{2})^2}$, equation of an elliptic paraboloid.



34 $y^2 = x^2 + 4z^2 + 4 \Rightarrow y^2 - x^2 - 4z^2 = 4$

$$\Rightarrow \frac{y^2}{2^2} - \frac{x^2}{2^2} - \frac{z^2}{1^2} = 1$$, equation of a hyperboloid of two sheets.



Hyperboloid of two sheets