Section 13.2: 5, 10, 17, 21, 26

3–8

(a) Sketch the plane curve with the given vector equation.

(b) Find $\mathbf{r}'(t)$.

(c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.

4. $\mathbf{r}(t) = \langle t^2, t^3 \rangle, \quad t = 1$

(a) $r(1) = \langle t^2, t^3 \rangle$

Since $r = \langle x, y \rangle$, we have

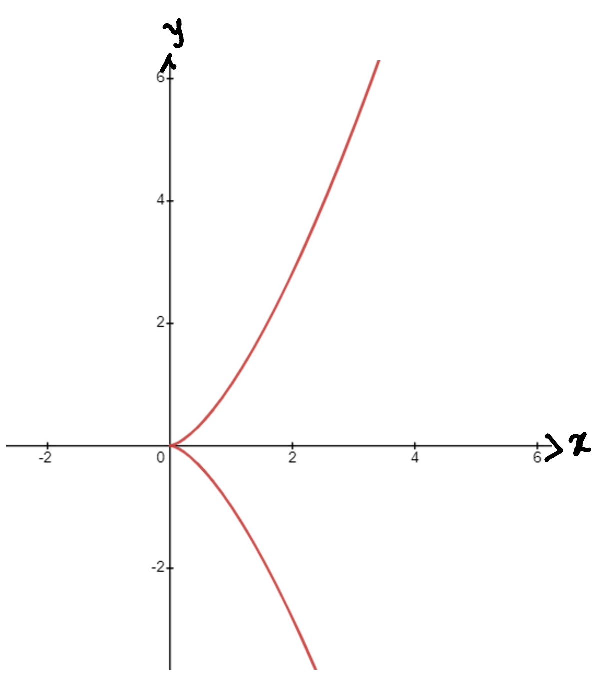
$$\langle x, y \rangle = \langle t^2, t^3 \rangle$$

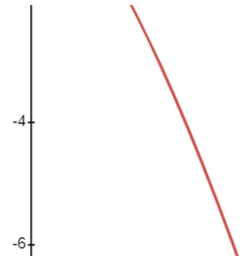
$$\Rightarrow x = t^2, \; y = t^3 \Rightarrow t = y^{1/3}$$

$$\Rightarrow x = t^2 \Big|_{t = y^{1/3}}$$

$$= y^{2/3}$$

So, the plane curve is the graph of $x = y^{2/3}$



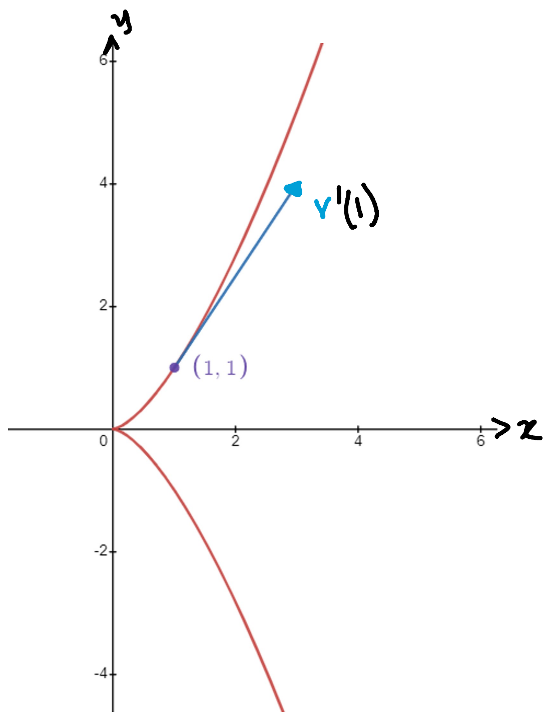


(b) By definition,

$r'(t) = \langle (t^2)', (t^3)' \rangle$

$= \langle 2t, 3t \rangle.$

(c) $r'(1) = \langle 2t, 3t \rangle \Big|_{t=1} = \langle 2, 3 \rangle.$



9–16 Find the derivative of the vector function.

9. $\mathbf{r}(t) = \left\langle \sqrt{t-2}, 3, 1/t^2 \right\rangle$

10. $\mathbf{r}(t) = \langle e^{-t}, t - t^3, \ln t \rangle$

10 $r(t) = \langle e^{-t}, t - t^3, \ln t \rangle$

By definition,

$$r'(t) = \langle (e^{-t})', (t-t^3)', (\ln t)' \rangle$$

$$= \langle -e^{-t}, 1 - 3t^2, \tfrac{1}{t} \rangle.$$

17–20 Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

17. $\mathbf{r}(t) = \left\langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2 \right\rangle, \quad t = 2$

$$r'(t) = \langle 2t - t, 3, t^2 + t \rangle$$

$$\Rightarrow r'(2) = \langle 2t - t, 3, t^2 + t \rangle \Big|_{t=2} = \langle 2, 3, 6 \rangle.$$

$$\Rightarrow |r'(2)| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$$

Hence, the unit tangent vector at $t = 2$ is

$$T(2) = \frac{r'(2)}{|r'(2)|}$$

$$= \frac{1}{7} \langle 2, 3, 6 \rangle.$$

21. If $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

$$r'(t) = \langle 1, 2t, 3t^2 \rangle \Longrightarrow r'(1) = \langle 1, 2, 3 \rangle \Longrightarrow |r'(1)| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

So

$$T(1) = \frac{r'(1)}{|r'(1)|} = \frac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle.$$

Also,

$$r''(t) = \langle 0, 2, 6t\rangle$$

$$\Rightarrow r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$$= (12t^2 - 6t^2)i - (6t - 0)j + (2-0)k$$

$$= 6t^2 i - 6tj + 2k$$

$$= \langle 6t^2, -6t, 2\rangle.$$

23–26 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

26. $x = \sqrt{t^2+3}$, $y = \ln(t^2+3)$, $z = t$; $(2, \ln 4, 1)$

$$x'(t) = \frac{t}{\sqrt{t^2+3}}, \quad y'(t) = \frac{2t}{t^2+3}, \quad z'(t) = 1.$$

Vector equation of the curve:

$$r(t) = \langle \sqrt{t^2+3}, \ln(t^2+3), t\rangle.$$

$$\Rightarrow r'(t) = \left\langle \frac{t}{\sqrt{t^2+3}}, \frac{2t}{t^2+3}, 1\right\rangle$$

With $z = t$ then at $(2, \ln 4, 1)$, we have $t = 1$.

Thus, a direction vector of the tangent line is

$$v = r'(1) = \left\langle \frac{1}{\sqrt{1+3}}, \frac{2}{1+3}, 1\right\rangle = \left\langle \frac{1}{2}, \frac{1}{2}, 1\right\rangle$$

So equation of the line is

$$r = r_0 + t v$$

$$= \langle 2, \ln 4, 1 \rangle + t \langle \tfrac{1}{2}, \tfrac{1}{2}, 1 \rangle$$

$$= \langle 2 + \tfrac{t}{2}, \ln 4 + \tfrac{t}{2}, 1 + t \rangle$$

$\Rightarrow x = 2 + \frac{t}{2}, \; y = \ln 4 + \frac{t}{2}, \; z = 1 + t$ are the parametric equations.