Section 15.2: 1, 15, 24, 56

1–6 Evaluate the iterated integral.

1. $\int_1^5 \int_0^x (8x - 2y)\, dy\, dx$

$$\int_1^5 \underbrace{\int_0^x (8x-2y)\,dy}\,dx = \int_1^5 \left[8xy - y^2\right]_{y=0}^{y=x} dx$$

$$= \int_1^5 \left[\left(8x(x) - (x)^2\right) - \left(8x(0) - 0^2\right)\right] dx$$

$$= \int_1^5 \left(8x^2 - x^2\right) dx$$

$$= \int_1^5 7x^2\, dx$$

$$= \frac{7}{3}x^3 \Big|_1^5$$

$$= \frac{7}{3}\left(5^3 - 1^3\right)$$

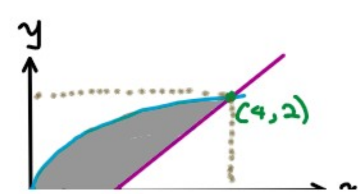
$$= \frac{7}{3}(125-1)$$

$$= \frac{7(124)}{3}$$

$$= \frac{868}{3}.$$

15–16 Set up iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why it's easier.

15. $\iint_D y\, dA$, D is bounded by $y = x - 2$, $x = y^2$



The curves intersect when

$$y + 2 = x = y^2$$

The curves intersect when

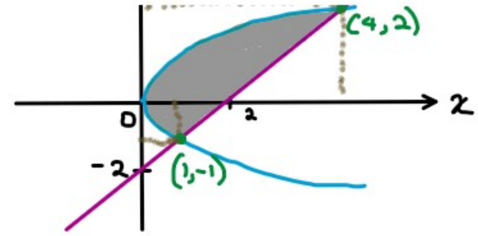
$$y+2 = x = y^2$$

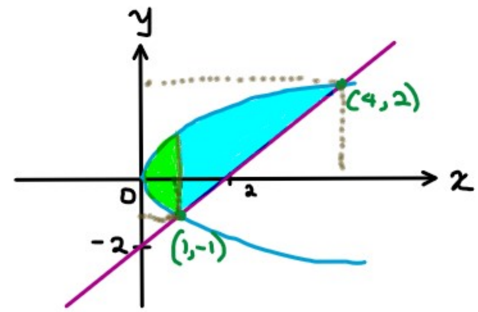
$$\Rightarrow \quad y^2 - y - 2 = 0$$

$$\Rightarrow \quad (y-2)(y+1) = 0$$

$$\Rightarrow \quad y = -1, 2$$

$y=-1 \Rightarrow x = 1$ and $y = 2 \Rightarrow x = 4$





Type II region:

$$\iint_D y\, dA = \int_{-1}^{2}\int_{y^2}^{y+2} y\, dx\, dy$$

Type I region: $x = y^2 \Rightarrow y = \pm\sqrt{x}$, so split region into two according to $y = \sqrt{x}$ and $y = -\sqrt{x}$ as shown.

$$\iint_D y\, dA = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y\, dy\, dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y\, dy\, dx$$

The resulting iterated integrals from type I and type II are not difficult to evaluate, but that of type II is simpler since it involves only one integral. Thus,

$$\iint_D y\, dA = \int_{-1}^{2}\int_{y^2}^{y+2} y\, dx\, dy$$

$$= \int_{-1}^{2} \Big[xy\Big]_{x=y^2}^{x=y+2} dy$$

$$= \int_{-1}^{2} \left[(y+2)y - y^2(y)\right] dy$$

$$= \int_{-1}^{2} \left(y^2 + 2y - y^3\right) dy$$

$$= \left[\frac{y^3}{3} + y^2 - \frac{y^4}{4}\right]_{-1}^{2}$$

$$= \left(\frac{2^3}{3} + 2^2 - \frac{2^4}{4}\right) - \left(\frac{(-1)^3}{3} + (-1)^2 - \frac{(-1)^4}{4}\right)$$

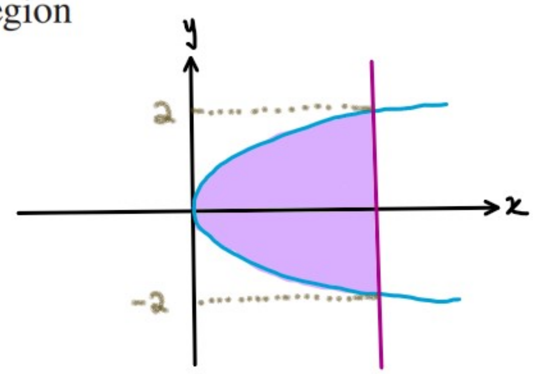
$$= \left(\frac{8}{3} + 4 - 4\right) - \left(\frac{-1}{3} + 1 - \frac{1}{4}\right)$$

$$= \frac{8}{3} + \frac{1}{3} - \frac{3}{4}$$

$$= \frac{9}{4}.$$

23–32 Find the volume of the given solid.

24. Under the surface $z = 1 + x^2y^2$ and above the region enclosed by $x = y^2$ and $x = 4$



$x = y^2$ and $x = 4$ intersect when

$$y^2 = x = 4$$
$$\Rightarrow y = \pm 2$$

Using Type II region,

$$\text{Volume} = \iint_D (1 + x^2y^2)\, dA$$

$$= \int_{-2}^{2} \int_{y^2}^{4} (1 + x^2y^2)\, dx\, dy$$

$$= \int_{-2}^{2} \left[x + \tfrac{1}{3}x^3y^2\right]_{x=y^2}^{x=4} dy$$

$$= \int_{-2}^{2} \left[\left(4 + \tfrac{1}{3}(4)^3y^2\right) - \left(y^2 + \tfrac{1}{3}(y^2)^3y^2\right)\right] dy$$

$$= \int_{-2}^{2} \left(4 + \frac{64}{3}y^2 - y^2 - \frac{1}{3}y^8\right) dy$$

$$= \int_{-2}^{2} \left(4 + \frac{61}{3}y^2 - \frac{1}{3}y^8\right) dy$$

$$= \int_{-2}^{2} \left(4 + \frac{61}{3}y^2 - \frac{1}{3}y^8\right) dy$$

$$= \left[4y + \frac{61}{9}y^3 - \frac{1}{27}y^9\right]_{-2}^{2}$$

$$= \left(4(2) + \frac{61}{9}(2^3) - \frac{1}{27}(2^9)\right) - \left(4(-2) + \frac{61}{9}(-2)^3 - \frac{1}{27}(-2)^9\right)$$

$$= 8 + 8 + \frac{61(8)}{9} + \frac{61(8)}{9} - \frac{512}{27} - \frac{512}{27}$$

$$= \frac{432 + 2928 - 1024}{27}$$

$$= \frac{2336}{27}.$$

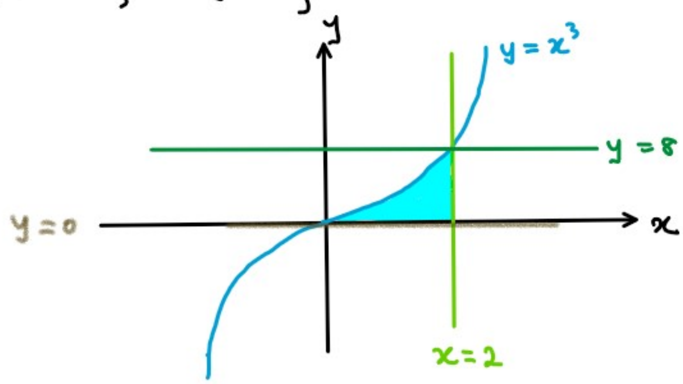
51–56 Evaluate the integral by reversing the order of integration.

56. $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy$ $\qquad D = \{(x,y) : \sqrt[3]{y} \le x \le 2,\ 0 \le y \le 8\}$

$y = x^3$ and $y = 8$ intersect when

$x^3 = 8 \Rightarrow x = 2$.

Thus, using a Type I region,



$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx\,dy = \int_0^2 \int_0^{x^3} e^{x^4} dy\,dx$$

$$= \int_0^2 y e^{x^4} \Big|_{y=0}^{y=x^3} dx$$

$$= \int_0^2 x^3 e^{x^4} dx$$

Let $u = x^4$. Then $du = 4x^3 dx$

$\Rightarrow dx = \frac{du}{4x^3}$

$x = 0 \Rightarrow u = 0$ and

$$= \int^{16} x^3 e^u \cdot \frac{du}{}$$

$4x^{-}$

$x = 0 \Rightarrow u = 0$ and

$x = 2 \Rightarrow u = 16$

$$= \int_0^{16} x^3 e^u \cdot \frac{du}{4x^3}$$

$$= \frac{1}{4} \int_0^{16} e^u \, du$$

$$= \frac{1}{4} e^u \Big|_{u=0}^{u=16}$$

$$= \frac{1}{4} \left(e^{16} - e^{0} \right)$$

$$= \frac{1}{4} \left(e^{16} - 1 \right).$$