Section 15.1: 15, 19, 27, 37, 47

15–26 Calculate the iterated integral.

15. $\int_1^4 \int_0^2 (6x^2y - 2x)\,dy\,dx$

$$\int_1^4 \underbrace{\int_0^2 (6x^2y - 2x)\,dy}\,dx = \int_1^4 \Big[3x^2y^2 - 2xy\Big]_{y=0}^{y=2} dx$$

$$= \int_1^4 \Big[\big(3x^2(2^2) - 2x(2)\big) - \big(3x^2(0^2) - 2x(0)\big)\Big] dx$$

$$= \int_1^4 (12x^2 - 4x)\,dx$$

$$= \Big[4x^3 - 2x^2\Big]_1^4$$

$$= \big(4(4^3) - 2(4^2)\big) - \big(4(1^3) - 2(1^2)\big)$$

$$= (256 - 32) - (4 - 2)$$

$$= 224 - 2$$

$$= 222.$$

19. $\int_{-3}^3 \int_0^{\pi/2} (y + y^2 \cos x)\,dx\,dy$

$$\int_{-3}^3 \underbrace{\int_0^{\pi/2} (y + y^2 \cos x)\,dx}\,dy = \int_{-3}^3 \Big[xy + y^2 \sin x\Big]_{x=0}^{\pi/2} dy$$

$$= \int_{-3}^3 \Big[\big(\tfrac{\pi}{2}(y) + y^2 \sin(\tfrac{\pi}{2})\big) - \big(0(y) + y^2 \sin 0\big)\Big] dy$$

$$= \int_{-3}^3 \left(\tfrac{\pi}{2}y + y^2\right) dy$$

$$\Big[\tfrac{\pi}{\ } y^2 + \tfrac{y^3}{\ }\Big]^3$$

$$= \left[\frac{\pi}{4} y^2 + \frac{y^3}{3}\right]_{-3}^{3}$$

$$= \left(\frac{\pi}{4}(3^2) + \frac{3^3}{3}\right) - \left(\frac{\pi}{4}(-3)^2 + \frac{(-3)^3}{3}\right)$$

$$= \left(\frac{9\pi}{4} + 9\right) - \left(\frac{9\pi}{4} - 9\right)$$

$$= \frac{9\pi}{4} - \frac{9\pi}{4} + 9 + 9$$

$$= 18.$$

27–34 Calculate the double integral.

27. $\iint\limits_R x \sec^2 y \, dA, \quad R = \{(x, y) \mid 0 \leqslant x \leqslant 2, 0 \leqslant y \leqslant \pi/4\}$

$$\iint\limits_R x\sec^2 y\,dA = \int_0^2 \underbrace{\int_0^{\pi/4} x\sec^2 y\,dy}\,dx$$

$$= \int_0^2 \left[x \tan y\right]_{y=0}^{y=\pi/4} dx$$

$$= \int_0^2 \left(x\tan\left(\frac{\pi}{4}\right) - x\tan 0\right) dx$$

$$= \int_0^2 x\,dx$$

$$= \left.\frac{x^2}{2}\right|_0^2$$

$$= \frac{2^2}{2} - \frac{0^2}{2}$$

$$= 2.$$

37. Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = \{(x, y) \mid -1 \leqslant x \leqslant 2, -1 \leqslant y \leqslant 1\}$.

$$4x + 6y - 2z + 15 = 0 \implies z = 2x + 3y + \frac{15}{2}$$

Since the region is a rectangle,

$$\text{Volume} = \int_{-1}^{2} \underbrace{\int_{-1}^{1} \left(2x + 3y + \frac{15}{2}\right) dy}\, dx$$

$$= \int_{-1}^{2} \left[2xy + \frac{3}{2}y^2 + \frac{15}{2}y\right]_{y=-1}^{y=1} dx$$

$$= \int_{-1}^{2} \left[\left(2x(1) + \frac{3}{2}(1^2) + \frac{15}{2}(1)\right) - \left(2x(-1) + \frac{3}{2}(-1)^2 + \frac{15}{2}(-1)\right)\right] dx$$

$$= \int_{-1}^{2} \left[\left(2x + \frac{3}{2} + \frac{15}{2}\right) - \left(-2x + \frac{3}{2} - \frac{15}{2}\right)\right] dx$$

$$= \int_{-1}^{2} (4x + 15)\, dx$$

$$= \left[2x^2 + 15x\right]_{-1}^{2}$$

$$= \left(2(2^2) + 15(2)\right) - \left(2(-1)^2 + 15(-1)\right)$$

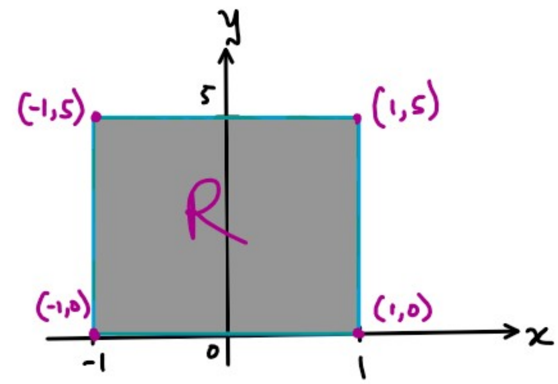
$$= (8 + 30) - (2 - 15)$$

$$= 38 + 13$$

$$= 51.$$

47–48 Find the average value of f over the given rectangle.

47. $f(x, y) = x^2 y$,
R has vertices $(-1, 0)$, $(-1, 5)$, $(1, 5)$, $(1, 0)$



Area of the rectangle

$$
\begin{aligned}
A(R) &= lb \\
&= (5-0)(1-(-1)) \\
&= 5(2) \\
&= 10
\end{aligned}
$$

Thus,

$$
\begin{aligned}
f_{ave} &= \frac{1}{A(R)} \iint_R f(x,y)\,dA \\
&= \frac{1}{10} \int_{-1}^{1} \underbrace{\int_0^5 x^2 y \, dy}\, dx \\
&= \frac{1}{10} \int_{-1}^{1} \left[\frac{x^2 y^2}{2} \right]_{y=0}^{5} dx \\
&= \frac{1}{10} \int_{-1}^{1} \left[\frac{x^2 (5^2)}{2} - \frac{x^2 (0^2)}{2} \right] dx \\
&= \frac{1}{10} \int_{-1}^{1} \frac{25x^2}{2} \, dx \\
&= \frac{25}{20} \left[\frac{1}{3} x^3 \right]_{-1}^{1} \\
&= \frac{25}{60} \left[1^3 - (-1)^3 \right] \\
&= \frac{25}{30} = \frac{5}{6}.
\end{aligned}
$$