Section 14.1: 1, 9, 22, 67

1. In Example 2 we considered the function $W = f(T, v)$, where W is the wind-chill index, T is the actual temperature, and v is the wind speed. A numerical representation is given in Table 1 on page 889.

(a) What is the value of $f(-15, 40)$? What is its meaning?

Table 1 Wind-chill index as a function of air temperature and wind speed

Wind speed (km/h)

Actual temperature (°C) T \ v	5	10	15	20	25	30	40	50	60	70	80
5	4	3	2	1	1	0	−1	−1	−2	−2	−3
0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

* From the table,

$f(-15, 40) = -27$

* It means the wind-chill index is $-27°C$ with no wind at $-15°C$ Temperature and 40 Km/h wind speed.

(b) Describe in words the meaning of the question "For what value of v is $f(-20, v) = -30$?" Then answer the question.

* The question is equivalent to:

At $-20°C$ temperature, what wind speed produces $-30°C$ wind-chill?

* From the table, we obtain

$v = 20$ Km/h.

(c) Describe in words the meaning of the question "For what value of T is $f(T, 20) = -49$?" Then answer the question.

* The question is equivalent to:

At what temperature is the wind-chill $-49°C$ if the wind speed is 20 Km/h?

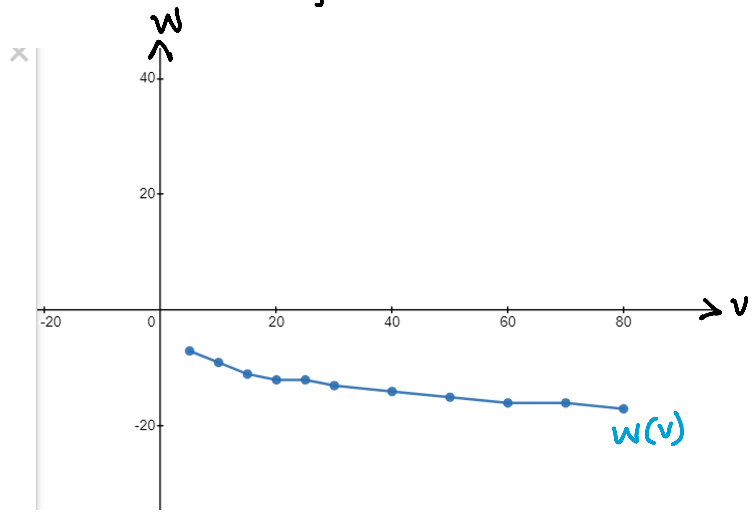
* From the table,

$T = -35°C$

(d) What is the meaning of the function $W = f(-5, v)$?
Describe the behavior of this function.

$W = f(-5, v)$ Describes the wind-chill W as a function of the wind speed v only at a fixed temperature of $-5°C$.

v	W
5	−7
10	−9
15	−11
20	−12
25	−12
30	−13
40	−14
50	−15
60	−16
70	−16
80	−17

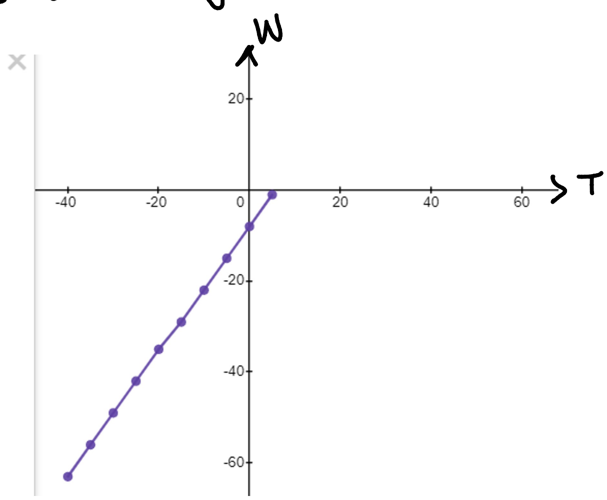


From the plot, we see that W decreases at an increasing rate as v increases.

(e) What is the meaning of the function $W = f(T, 50)$?
Describe the behavior of this function.

$W = f(T, 50)$ describes the wind-chill at a fixed wind speed of 50 Km/h as a function of temperature only.

T	W
5	−1
0	−8
−5	−15
−10	−22
−15	−29
−20	−35
−25	−42
−30	−49
−35	−56
−40	−63



From the plot, the wind-chill as a function increases linearly as the temperature T increases.

9. Let $g(x, y) = \cos(x + 2y)$.

(a) Evaluate $g(2, -1)$.

(b) Find the domain of g.

(c) Find the range of g.

(a) $g(2,-1) = \cos(2+2(-1)) = \cos(0) = 1$

(b) $x+2y$ is a real number for all x, y in $\mathbb{R}$ and the cosine function is defined for all real numbers. Hence, the domain of g is $\mathbb{R}^2$.

(c) the range of $x+2y$ is $\mathbb{R}$ and the range of the cosine function is $[-1,1]$. Hence, the range of g is $[-1,1]$.

13–22 Find and sketch the domain of the function.

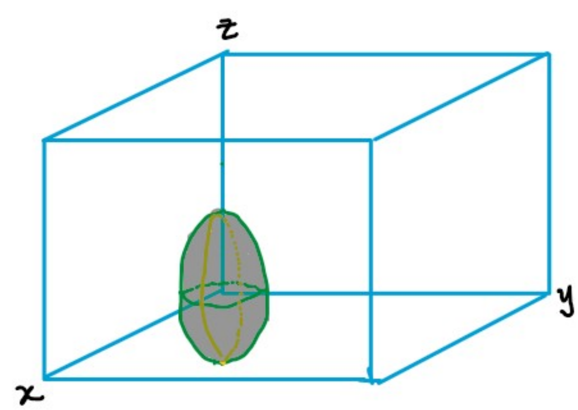
22. $f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2)$

Since ln only takes positive input, the domain of f is the set of x, y, z such that $16 - 4x^2 - 4y^2 - z^2 > 0$.

i.e.,

$$\text{Dom}(f) = \left\{ (x, y, z) : \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} < 1 \right\}$$

= points inside the ellipsoid $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} = 1$.



67–70 Describe the level surfaces of the function.

67. $f(x, y, z) = x + 3y + 5z$

For $f(x, y, z) = k$ fixed,

$x + 3y + 5z = k$ is a 3D plane with the normal vector $\langle 1, 3, 5 \rangle$.

Hence the level surfaces of f is a collection of parallel planes with the normal vector $\langle 1, 3, 5 \rangle$.