Section 14.4: 6, 25, 31

1–6 Find an equation of the tangent plane to the given surface at the specified point.

6. $z = \ln(x - 2y)$, $(3, 1, 0)$

The required equation is

$$z - 0 = f_x(3,1)(x-3) + f_y(3,1)(y-1)$$

But

$$f_x(x,y) = \frac{1}{x-2y} \Rightarrow f_x(3,1) = \frac{1}{3-2(1)} = 1$$

and

$$f_y(x,y) = \frac{-2}{x-2y} \Rightarrow f_y(3,1) = \frac{-2}{3-2(1)} = -2$$

Thus,

$$z = 1(x-3) - 2(y-1)$$

$$= x - 3 - 2y + 2$$

$$= x - 2y - 1$$

25–30 Find the differential of the function.

25. $z = e^{-2x} \cos 2\pi t$

The differential of z is given as

$$dz = \frac{\partial z}{\partial t} dt + \frac{\partial z}{\partial x} dx$$

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But

$$\frac{\partial z}{\partial t} = -2\pi e^{-2x} \sin(2\pi t) \quad \text{and} \quad \frac{\partial z}{\partial x} = -2e^{-2x} \cos(2\pi t)$$

Thus,

$$dz = \frac{\partial z}{\partial t} dt + \frac{\partial z}{\partial x} dx$$

$$= -2\pi e^{-2x} \sin(2\pi t)\, dt - 2e^{-2x} \cos(2\pi t)\, dx$$

31. If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, compare the values of Δz and dz.

$$z = 5x^2 + y^2 \Rightarrow z_x = 10x \quad \text{and} \quad z_y = 2y$$

Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (1.05, 2.1)$.

Then

$$\Delta x = x_2 - x_1 = 1.05 - 1 = 0.5 = dx$$

and

$$\Delta y = y_2 - y_1 = 2.1 - 2 = 0.1 = dy$$

Thus,

$$dz = z_x dx + z_y dy$$

$$= 10(1)(0.05) + 2(2)(0.1)$$

$$= 0.9$$

$$= 0.9$$

However, actual change in z

$$\Delta z = f(1.05, 2.1) - f(1, 2)$$

$$= \left[5(1.05)^2 + (2.1)^2\right] - \left[5(1)^2 + 2^2\right]$$

$$= 9.9225 - 9$$

$$= 0.9225$$