Section 14.5: 1, 5, 7, 15, 27, 31

1–6 Use the Chain Rule to find dz/dt or dw/dt.

1. $z = xy^3 - x^2y, \quad x = t^2 + 1, \quad y = t^2 - 1$

By the chain rule,

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= (y^3 - 2xy)(2t) + (3xy^2 - x^2)(2t)$$

$$= 2t(y^3 - 2xy + 3xy^2 - x^2)$$

Could expand in t only but no need

$x = t^2+1 \Rightarrow \frac{dx}{dt} = 2t$

$y = t^2-1 \Rightarrow \frac{dy}{dt} = 2t$

5. $w = xe^{y/z}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t$

By the chain rule,

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= e^{y/z}(2t) + \frac{x}{z}e^{y/z}(-1) - \frac{xy}{z^2}e^{y/z}(2)$$

$$= \left(2t - \frac{x}{z} - \frac{2xy}{z^2}\right)e^{y/z}$$

7–12 Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

7. $z = (x - y)^5, \quad x = s^2t, \quad y = st^2$

By definition,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= 5(x-y)^4(2st) - 5(x-y)^4(t^2)$$

$$= \underbrace{5(x-y)^4}(2st) - \underbrace{5(x-y)^4}(t^2)$$

$$= 5(x-y)^4(2st - t^2)$$

and

$$\frac{\partial z}{\partial t} = \underbrace{\frac{\partial z}{\partial x}}\frac{\partial x}{\partial t} + \underbrace{\frac{\partial z}{\partial y}}\frac{\partial y}{\partial t}$$

$$= 5(x-y)^4(s^2) - 5(x-y)^4(2st)$$

$$= 5(x-y)^4(s^2 - 2st)$$

15. Suppose f is a differentiable function of x and y, and $g(u, v) = f(e^u + \sin v, e^u + \cos v)$. Use the table of values to calculate $g_u(0, 0)$ and $g_v(0, 0)$.

	f	g	f_x	f_y
(0, 0)	3	6	4	8
(1, 2)	6	3	2	5

Explicitly,

$g(u,v) = f(x,y)$ where $x = e^u + \sin v$, $y = e^u + \cos v$.

So by the chain rule,

$$g_u(u,v) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$= f_x(e^u) + f_y(e^u)$$

$$= e^u f_x(x,y) + e^u f_y(x,y)$$

$x = e^u + \sin v$

$\Rightarrow x = e^0 + \sin 0$ at $(u,v) = (0,0)$

$$= e^0 f_x(1,2) + e^0 f_y(1,2)$$

$$= 1(2) + 1(5)$$

$$= 2 + 5$$

$$= 7$$

$\Rightarrow x = e^0 + \sin 0$ at $(u,v) = (0,0)$

$= 1$

$y = e^u + \cos v$

$\Rightarrow y = e^0 + \cos 0$ at $(u,v) = (0,0)$

$= 2$

So $(u,v) = (0,0) \Rightarrow (x,y) = (1,2)$

Similarly,

$$g_v(u,v) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

$$= f_x(x,y)(\cos v) + f_y(x,y)(-\sin v)$$

$$= f_x(1,2)(\cos 0) + f_y(1,2)(-\sin 0)$$

$$= 2(1) + 5(0)$$

$$= 2.$$

27–30 Use Equation 6 to find dy/dx.

27. $y \cos x = x^2 + y^2$ **28.** $\cos(xy) = 1 + \sin y$ ✓ Try this one!

Let $F(x,y) = y\cos x - x^2 - y^2 = 0$ Then $F_x(x,y) = -y\sin x - 2x$ and $F_y = \cos x - 2y$.

Thus, by equation 6,

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$= \frac{-(-y\sin x - 2x)}{\cos x - 2y}$$

$$= \frac{y\sin x + 2x}{\cos x - 2y}$$

31–34 Use Equations 7 to find $\partial z/\partial x$ and $\partial z/\partial y$.

31. $x^2 + 2y^2 + 3z^2 = 1$ **32.** $x^2 - y^2 + z^2 - 2z = 4$ ✓ Try this one!

Let $F(x,y,z) = x^2 + 2y^2 + 3z^2 - 1 = 0$. Then $F_x = 2x$, $F_y = 4y$ and $F_z = 6z$.

So by Equations 7,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$= \frac{-(2x)}{6z}$$

$$= -\frac{x}{3z}$$

and

$$\frac{\partial z}{\partial} = -\frac{F_y}{F_z}$$

$$= -\frac{4y}{6z}$$

$$= -\frac{2y}{3z}$$