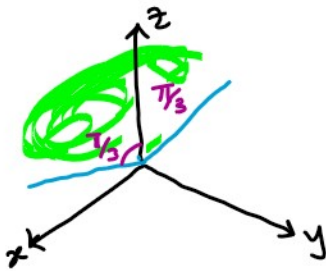
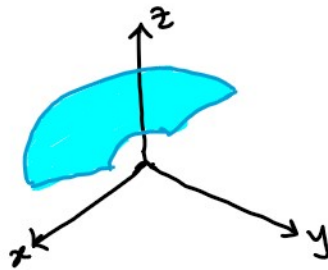


**11-14** Sketch the solid described by the given inequalities.

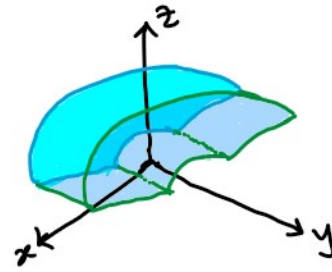
13.  $2 \leq \rho \leq 4, \quad 0 \leq \phi \leq \pi/3, \quad 0 \leq \theta \leq \pi$



$0 \leq \phi \leq \pi/3$



$0 \leq \phi \leq \pi/3$   
 $2 \leq \rho \leq 4$



$0 \leq \phi \leq \pi/3$   
 $2 \leq \rho \leq 4$   
 $0 \leq \theta \leq \pi$

16. (a) Find inequalities that describe a hollow ball with diameter 30 cm and thickness 0.5 cm. Explain how you have positioned the coordinate system that you have chosen.

The ball, as described is a spherical shell with outer radius 15cm and inner radius  $15 - 0.5 = 14.5$ cm.

If we center the ball at the origin, then in spherical coordinates, we describe the ball as

$14.5 \leq \rho \leq 15, \quad 0 \leq \phi \leq \pi, \quad 0 \leq \theta \leq 2\pi.$

(b) Suppose the ball is cut in half. Write inequalities that describe one of the halves.

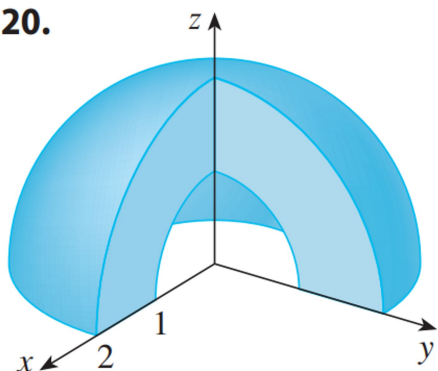
For the half of the ball above the  $x$ - $y$  plane,

$14.5 \leq \rho \leq 15, \quad 0 \leq \phi \leq \frac{\pi}{2}, \quad 0 \leq \theta \leq 2\pi$

is the spherical coordinate description.

**19–20** Set up the triple integral of an arbitrary continuous function  $f(x, y, z)$  in cylindrical or spherical coordinates over the solid shown.

20.



In spherical coordinates, the solid is given as  
 $E = \{(\rho, \theta, \phi) : 1 \leq \rho \leq 2, \frac{\pi}{2} \leq \theta \leq 2\pi, 0 \leq \phi \leq \frac{\pi}{2}\}$   
 Hence, the required integral is

$$\iiint_E f(x, y, z) dv = \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_1^2 f(x, y, z) \rho^2 \sin \phi d\rho d\theta d\phi$$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_1^2 f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi$$