Section 13.1: 4*, 9, 18, 31, 49

**3–6** Find the limit.

**3.** $\lim_{t\to 0}\left(e^{-3t}\,\mathbf{i} + \frac{t^2}{\sin^2 t}\,\mathbf{j} + \cos 2t\,\mathbf{k}\right)$

**4.** $\lim_{t\to 1}\left(\frac{t^2 - t}{t - 1}\,\mathbf{i} + \sqrt{t + 8}\,\mathbf{j} + \frac{\sin \pi t}{\ln t}\,\mathbf{k}\right)$

By definition,

$$\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\,i + \sqrt{t+8}\,j + \frac{\sin(\pi t)}{\ln t}\,k\right)$$

$$=\left(\lim_{t\to 1}\frac{t^2-t}{t-1}\right)i + \left(\lim_{t\to 1}\sqrt{t+8}\right)j + \left(\lim_{t\to 1}\frac{\sin(\pi t)}{\ln t}\right)k$$

But

$$\lim_{t\to 1}\frac{t^2-t}{t-1} = \lim_{t\to 1}\frac{t(t-1)}{t-1} = \lim_{t\to 1} t = 1,$$

$$\lim_{t\to 1}\sqrt{t+8} = \sqrt{\lim_{t\to 1} t+8} = \sqrt{1+8} = \sqrt{9} = 3,$$

and

$$\lim_{t\to 1}\frac{\sin(\pi t)}{\ln t} \overset{\text{L'H}}{=} \lim_{t\to 1}\frac{\pi\cos(\pi t)}{1/t} = \frac{\pi\cos\pi}{1/1} = -\pi$$

Hence

$$\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\,i + \sqrt{t+8}\,j + \frac{\sin(\pi t)}{\ln t}\,k\right)$$

$$=\left(\lim \frac{t^2-t}{\ldots}\right)i + \left(\lim \sqrt{t+8}\right)j + \left(\lim \frac{\sin(\pi t)}{\ln t}\right)k$$

$$= \underbrace{\left(\lim_{t\to 1} \frac{t^2-t}{t-1}\right)}\mathbf{i} + \underbrace{\left(\lim_{t\to 1} \sqrt{t+8}\right)}\mathbf{j} + \underbrace{\left(\lim_{t\to 1} \frac{\sin(\pi t)}{\ln t}\right)}\mathbf{k}$$

$$= \quad 1\mathbf{i} \quad + \quad 3\mathbf{j} \quad\quad -\pi\mathbf{k}$$

$$= 1\mathbf{i} + 3\mathbf{j} - \pi\mathbf{k}$$

*This problem is not contained in the list given by the professor.

**7–14** Sketch the curve with the given vector equation. Indicate with an arrow the direction in which $t$ increases.

**7.** $\mathbf{r}(t) = \langle \sin t, t \rangle$

**8.** $\mathbf{r}(t) = \langle t^2 - 1, t \rangle$

**9.** $\mathbf{r}(t) = \langle t, 2 - t, 2t \rangle$

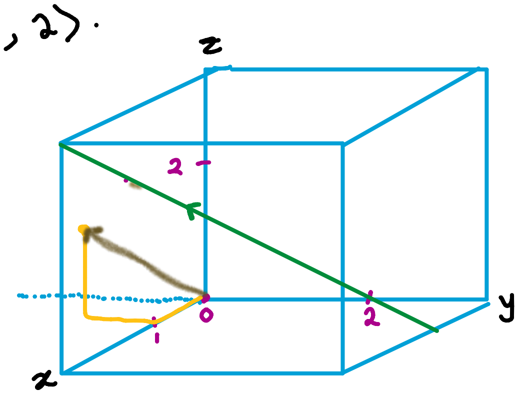
**10.** $\mathbf{r}(t) = \langle \sin \pi t, t, \cos \pi t \rangle$

9) $r(t) = \langle t, 2-t, 2t \rangle$.

The parametric equations are

$$x = t, \; y = 2-t, \; z = 2t,$$

which is the parametric equations of the line through the point $(0, 2, 0)$ with direction vector $\langle 1, -1, 2 \rangle$.




**17–20** Find a vector equation and parametric equations for the line segment that joins $P$ to $Q$.

**17.** $P(2, 0, 0)$, $Q(6, 2, -2)$

**18.** $P(-1, 2, -2)$, $Q(-3, 5, 1)$

(18) $P(-1, 2, -2)$, $Q(-3, 5, 1)$.

Let $r_0 = P(-1, 2, -2)$ be a fixed point on the line. Then a direction vector

$$v = \langle -3, 5, 1\rangle - \langle -1, 2, -2\rangle = \langle -2, 3, 3\rangle.$$

Thus, a vector equation of the line is

$$r = r_0 + t v, \quad 0 \le t \le 1 \text{ for a line segment}$$
$$= \langle -1, 2, -2\rangle + t\langle -2, 3, 3\rangle, \quad 0 \le t \le 1$$
$$= \langle -1-2t, 2+3t, -2+3t\rangle, \quad 0 \le t \le 1$$

So with $r = \langle x, y, z\rangle$

$$x = -1-2t, \quad y = 2+3t, \quad -2+3t$$

is the parametric equation of the line.

**31.** At what points does the curve $\mathbf{r}(t) = t\,\mathbf{i} + (2t - t^2)\,\mathbf{k}$ intersect the paraboloid $z = x^2 + y^2$?

Since $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, we have

$$x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = t\mathbf{i} + (2t - t^2)\mathbf{k}$$

$$\Rightarrow \quad x = t, \quad y = 0, \quad z = 2t - t^2.$$

Plugging into the paraboloid, we have

$$2t - t^2 = t^2 + 0^2$$
$$\Rightarrow \quad 2t(1-t) = 0$$
$$\Rightarrow \quad t = 0, 1.$$

Hence, the curve and the paraboloid intersect at $r(0)$ and $r(1)$.

i.e. at $(t, 0, 2t-t^2)\big| = (0, 0, 0)$ and $(t, 0, 2t-t^2)\big| = (1, 0, 1)$.

Hence, the curve ...

i.e., at $(t, 0, 2t-t^2)\Big|_{t=0} = (0,0,0)$ and $(t, 0, 2t-t^2)\Big|_{t=1} = (1,0,1)$.

**49.** If two objects travel through space along two different curves, it's often important to know whether they will collide. (Will a missile hit its moving target? Will two aircraft collide?) The curves might intersect, but we need to know whether the objects are in the same position *at the same time*. Suppose the trajectories of two particles are given by the vector functions

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \qquad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$$

for $t \geqslant 0$. Do the particles collide?

Assume the two particles collide at time $t_0$. Then $r_1(t_0) = r_2(t_0)$.

i.e.;

$$\langle t_0^2, 7t_0 - 12, t_0^2 \rangle = \langle 4t_0 - 3, t_0^2, 5t_0 - 6 \rangle.$$

$$\Rightarrow \begin{cases} t_0^2 = 4t_0 - 3 \\ 7t_0 - 12 = t_0^2 \\ t_0^2 = 5t_0 - 6 \end{cases} \Rightarrow \begin{cases} t_0^2 - 4t_0 + 3 = 0 \\ t_0^2 - 7t_0 + 12 = 0 \\ t_0^2 - 5t_0 + 6 = 0 \end{cases} \Rightarrow \begin{cases} (t_0 - 1)(t_0 - 3) = 0 \\ (t_0 - 3)(t_0 - 4) = 0 \\ (t_0 - 2)(t_0 - 3) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} t_0 = 1, 3 \\ t_0 = 3, 4 \\ t_0 = 2, 3 \end{cases} \Rightarrow t_0 = 3$$ since this is the only time the three coordinates hold true for $r_1(t_0) = r_2(t_0)$.

Hence, the particles intersect at time $t_0 = 3$ at the point

$$r_1(3) = r_2(3) = (t^2, 7t - 12, t^2)\Big|_{t=3}$$

$$= (9, 9, 9)$$