**Lecture Note 13, (Text book page 911)** Math 2630

## 14.3 Partial Derivatives

In general, if $f$ is a function of two variables $x$ and $y$, suppose we let only $x$ vary while keeping $y$ fixed, say $y = b$, where $b$ is a constant. Then we are really considering a function of a single variable $x$, namely, $g(x) = f(x, b)$. If $g$ has a derivative at $a$, then we call it the **partial derivative of $f$ with respect to $x$ at** $(a, b)$ and denote it by $f_x(a, b)$. Similarly, the **partial derivative of $f$ with respect to $y$ at** $(a, b)$, denoted by $f_y(a, b)$, is obtained by keeping $x$ fixed $(x = a)$ and finding the ordinary derivative at $b$ of the function $G(y) = f(a, y)$.

If $f$ is a function of two variables, its **first partial derivatives** are the functions $f_x$ and $f_y$ defined by

$$f_x(x, y) = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}$$

$$f_y(x, y) = \lim_{h \to 0} \frac{f(x, y + h) - f(x, y)}{h}$$

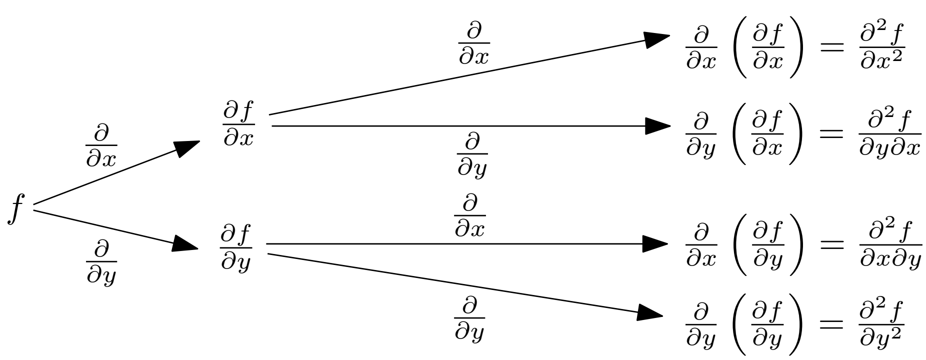
So while we find a partial derivative of a function $f$ with respect to a variable, we consider the other variables as constants and take the ordinary derivative of $f$.

**Example 1** Find $f_x(2, 1)$ and $f_y(2, 1)$ for function $f(x, y) = x^3 + x^2y^3 - 2y^2$.

**Example 2** Use implicit differentiation to calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for function $e^z = xyz$.

### Second-Order Partial Derivatives

The first partial derivatives $f_x(x, y)$ and $f_y(x, y)$ of a function $f(x, y)$ of the two variables $x$ and $y$ are also functions of $x$ and $y$. As such, we may differentiate each of the functions $f_x$ and $f_y$ to obtain the **second-order partial derivatives of $f$**.




**Example 3** Find all the second partial derivative of $v = \sin(s^2 - t^2)$.

**Example 4** For function $u = e^x \sin y$, show $u_{xx} + u_{yy} = 0$ which is called the Laplace's equation.

### Homework

15, 17, 19, 25, 33, 41, 44, 47, 53, 56, 64, 66,76 b, 76 c