TOPIC: Probability Density Function (pdf) Plots with Shading– In this tutorial, we show how to use MINITAB to shade in regions under the pdf which represent probabilities. Please review MINITAB TUTORIAL 1, before proceeding.

CDF: $F_X(a) = \int_{-\infty}^{a} F_X(x)dx = P(X \leq a)$

Reliability/survival: $R(b) = P(X > b) = \int_{b}^{\infty} f_X(x)dx = 1 - F_X(b)$

Between to points: $P(a \leq X \leq b) = \int_{a}^{b} f_X(x)dx = F(b) - F(a)$

If X is exponential (λ), then these functions are,

$$F_X(a) = 1 - e^{-\lambda a}, R(b) = e^{-\lambda x}, P(a \leq X \leq b) = e^{-\lambda a} - e^{-\lambda b}$$

Example (exponential): Suppose that the mean time between crash times on a computer system is 1 week. If the time between computer crashes is exponential distributed, then the distribution is exponential($\lambda = 1$). We wish to find the probability that the next crash will take over two weeks (assuming all crashes are independent).

Let X stand for the next computer crash (from a random crash environment), then $X \sim exp(\lambda = 1)$. From the properties of an exponential random variable, since $\lambda = 1$, $\mu_X = E(X) = 1, \sigma_X^2 = Var(X) = 1, \sigma_X = 1$, and $f_X(x) = e^{-x} I_{[0,\infty)}(x)$, $F_X(x) = 1 - e^{-x}$, and $R_X(x) = e^{-x}$, for $x > 0$. We would like to find $P(X > 2)$, which corresponds with the area beneath the pdf from 2 to ∞,

$$P(X > 2) = \int_{2}^{\infty} e^{-x} dx = R(2) = e^{-2}.$$

This pdf and this area/probability is shown Figure 1. The probability that the next crash is between 1 and 2 weeks is

$$P(1 < X < 2) = \int_{1}^{2} e^{-x} dx = F_X(2) - F_X(1) = e^{-1} - e^{-2} \approx 0.2325$$

The probability that the crash occurs within 0.5 weeks is,

$$P(X \leq 0.5) = \int_{0}^{0.5} e^{-x} dx = F_X(0.5) = 1 - e^{-0.5} \approx 0.3935$$

Go to "Graph" then select "Probability Distribution Plot". In the resulting dialogue box, choose "view probabilities", In the "Distribution" tab select the "Exponential" distribution and make sure the scale is 1 and the threshold is 0. Now, click on the "Shaded Area" tab, check the "X value" box. For the probability depicted in Figure 1, select "right tail" and enter the number 2 into the "X value" box.

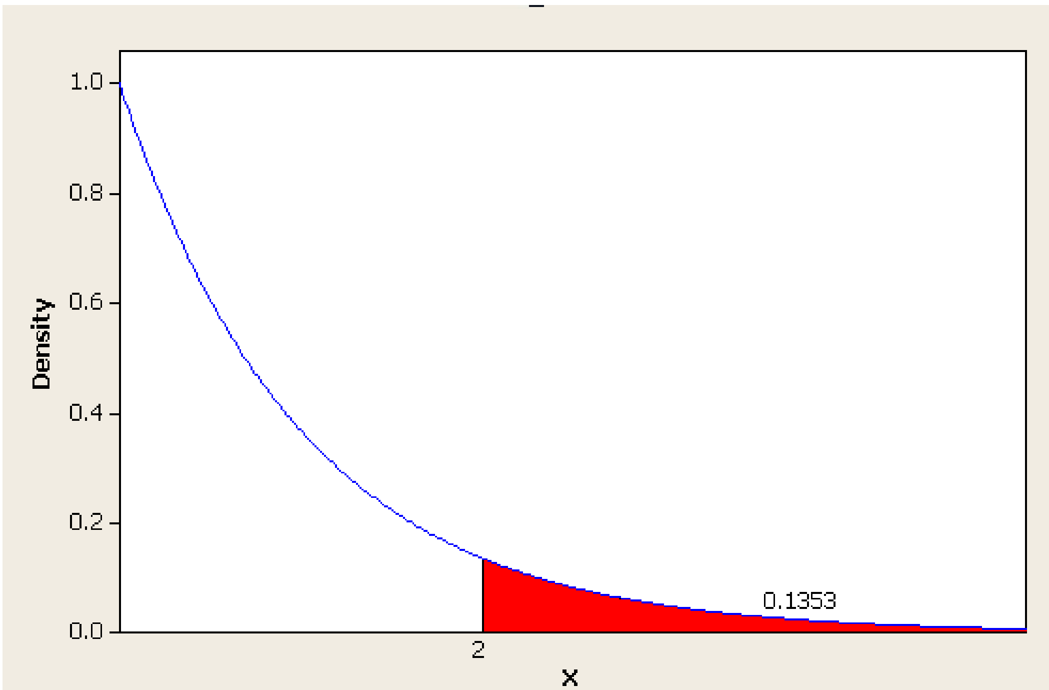

Figure 1: $P(X > 2) = \int_2^{\infty} e^{-x} dx = R(2) = e^{-2}$

Go to “Graph” then select “Probability Distribution Plot”. In the resulting dialogue box, choose “view probabilities”, In the “Distribution” tab select the “Exponential” distribution and make sure the scale is 1 and the threshold is 0. Now, click on the “Shaded Area” tab, check the “X value” box. For the probability depicted in Figure 2, select “Middle” and enter the number 1 for “X Value1”and 2 for “X value 2”

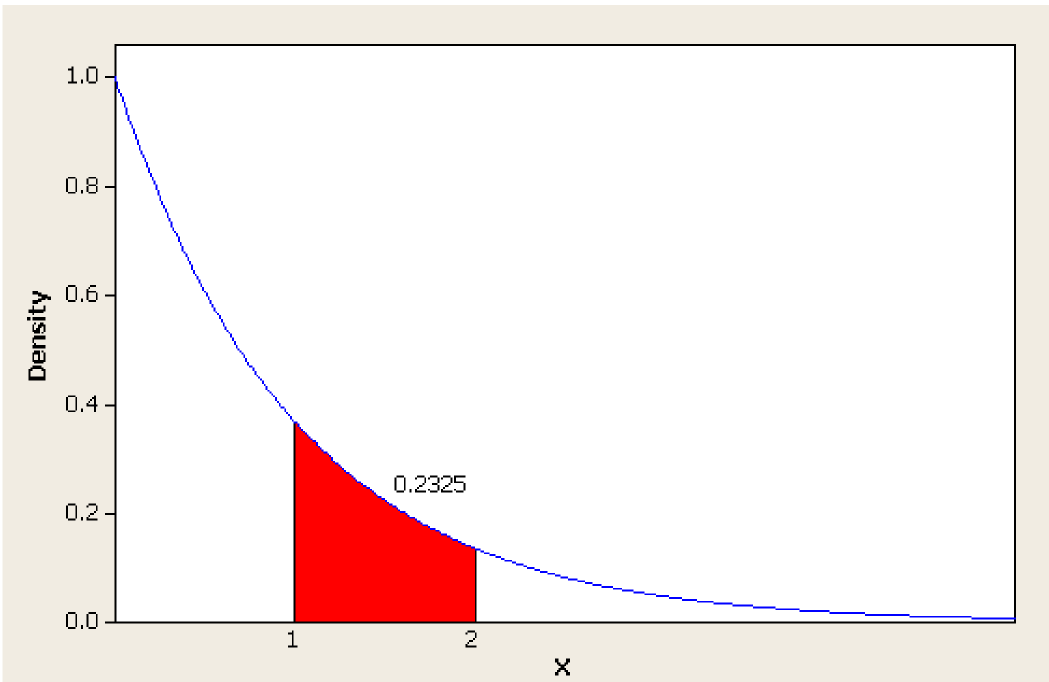

Figure 2: The probability that the next crash is between 1 and 2 weeks is $P(1 < X < 2) = \int_1^2 e^{-x}dx = F_X(2) - F_X(1) = e^{-1} - e^{-2} \approx 0.2325$

Go to "Graph" then select "Probability Distribution Plot". In the resulting dialogue box, choose "view probabilities", In the "Distribution" tab select the "Exponential" distribution and make sure the scale is 1 and the threshold is 0. Now, click on the "Shaded Area" tab, check the "X value" box. For the probability depicted in Figure 3, select "Left Tail" and enter the number 0.5 for "X Value' '

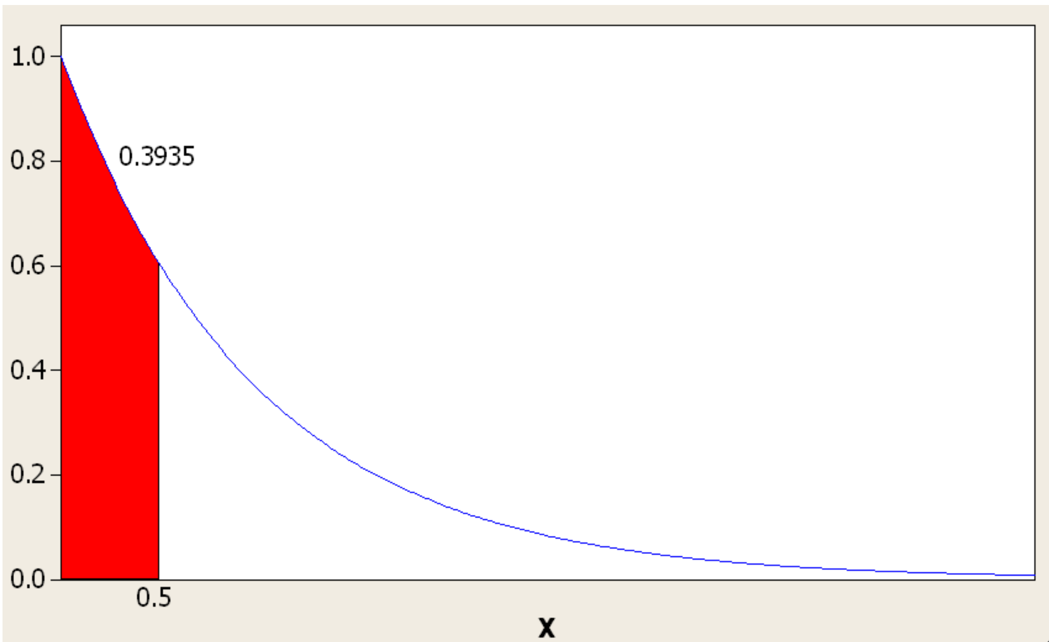

Figure 3: The probability that the next crash is less than 0.5 weeks is $P(X \leq 0.5) = \int_0^{0.5} e^{-x} dx = F_X(0.5) = 1 - e^{-0.5} \approx 0.3935$