STAT 7650 – Computational Statistics
Final Exam Practice Questions

1. **Root-Finding and Optimization**
   (a) Describe the key differences between root-finding and optimization problems in statistical computing.
   (b) Implement Newton's method for minimizing a smooth univariate function of your choice in `R`.
   (c) Plot the convergence of the function values and discuss the impact of initial values and stopping criteria.

2. **EM Algorithm**
   (a) Explain how the EM algorithm can be applied in problems involving latent variables.
   (b) Generate synthetic data for a two-component Gaussian mixture and implement one full EM iteration in `R`.
   (c) Analyze how the log-likelihood evolves across iterations and assess convergence numerically.

3. **Numerical Integration**
   (a) Discuss the tradeoffs among different numerical integration methods, especially in terms of error and computational complexity.
   (b) Write `R` functions to approximate a definite integral using both trapezoid and Gaussian quadrature.
   (c) Compare their performance for various integrands and integration intervals.

4. **Simulation of Random Variables**
   (a) Identify the main challenges in simulating random variables from non-standard distributions.
   (b) Implement at least two different methods to simulate from a skewed or heavy-tailed distribution (e.g., Cauchy, Beta).
   (c) Evaluate and compare the empirical distributions using plots and summary statistics.

5. **Basic Monte Carlo Methods**
   (a) Define the Monte Carlo method for approximating expectations and explain its statistical justification.
   (b) Use `R` to estimate an integral using Monte Carlo simulation and construct a confidence interval.

(c) Compare your result to that obtained using a built-in numerical integration function.

6. **Importance Sampling**

   (a) Explain the motivation and formulation of importance sampling in computational statistics.

   (b) Use `R` to implement importance sampling for estimating a probability involving a tail event.

   (c) Compare the variance of this estimator to that of a basic Monte Carlo estimator.

7. **Markov Chain Monte Carlo (MCMC)**

   (a) Describe the idea of a Markov chain and its role in MCMC.

   (b) Implement a simple random walk Metropolis sampler in `R` for a posterior density you define.

   (c) Examine convergence diagnostics such as trace plots, autocorrelations, and running averages.

8. **Gibbs Sampling**

   (a) Summarize the key steps of the Gibbs sampler and contrast it with Metropolis-Hastings.

   (b) Construct a bivariate distribution with known conditionals and implement the Gibbs sampler in `R`.

   (c) Visualize the joint samples and marginal histograms, and comment on mixing behavior.

9. **Stochastic Approximation**

   (a) Discuss the rationale and application of stochastic approximation in computational optimization.

   (b) Implement a basic Robbins-Monro type algorithm in `R` for estimating a quantile.

   (c) Evaluate the stability and convergence rate through repeated trials and trajectory plots.

10. **Simulated Annealing**

   (a) Identify use cases for simulated annealing and describe its conceptual foundation.

   (b) Implement simulated annealing in `R` for a univariate function with multiple local maxima.

   (c) Compare the optimization result to that of deterministic methods like `optim` or grid search.

Practice Questions for Final Exam
Based on Givens & Hoeting (Chapters 2–8)

1. Chapter 2: Problem 2.1
2. Chapter 2: Problem 2.5
3. Chapter 3: Problem 3.2 (only do the below parts instead of the parts in the book)
   (a) Implement the Simulated Annealing algorithm for a given combinatorial problem.
   (b) Vary the cooling schedule and discuss solution quality.
   (c) Visualize solution path and energy levels over iterations.
4. Chapter 4: Problem 4.3
5. Chapter 5: Problem 5.3
6. Chapter 6: Problem 6.2
7. Chapter 7: Problem 7.1