STAT 7650: Homework 3
(Due: Tuesday, 02/25/2025)

Note: Show all your work for the necessary steps to receive full credit.

Please turn in the HW on paper, hand-written and/or typed. For computational problems, return only the relevant parts of the output with comments/annotations. From the code you are using to answer the problems, turn in the relevant output, and the figures (if requested), preferably printed from the output. No need to turn in your code or long lists of generated samples.

Please disclose any us of AI in your solutions. Regardless of you use it or not, make sure you submit your own work, not copy from other source(s). Any suspicion of AI use will result in automatic 0 or substantial point loss in any question.

Q1. Exercise 2.2 in Givens & Hoeting. Implement Newton's method using the scripts available on Canvas. Also, in part (d) tabulate only the local mode and the interval of attraction, e.g., if x^* is a local mode (i.e., local max), find the longest interval $[x^*-\Delta_1, x^*+\Delta_2]$ with $\Delta_1, \Delta_2 > 0$ for which for all initial points x_0 in this interval, the algorithm converges to x^*.

Q2. (a) Do **Exercise 2.4** in Givens & Hoeting. (The Gamma(α, β) distribution in this question uses the shape-scale version, i.e., α is the shape and β is the scale parameter. Use the `R` function `pgamma` to evaluate the gamma cumulative distribution function. You are allowed to use the `uniroot` function in `R` in solving this problem.)
(b) Repeat the problem for Gamma$(0.5, 1)$ distribution.

Q3. Do only parts (a) and (b) of **Exercise 2.6** in Givens & Hoeting.

Q4. Example (from class slides): Gamma Distribution MLE:
Given $X_1, \ldots, X_n \stackrel{iid}{\sim}$ Gamma(α, β), where both α and β are unknown parameters to be estimated. Density function is

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x \geq 0.$$

And the log-likelihood function is (effectively)

$$\ell(\alpha, \beta) = n\alpha \log \beta - n \log \Gamma(\alpha) + \alpha \sum_{i=1}^{n} \log X_i - \beta \sum_{i=1}^{n} X_i.$$

(a) Find first and second (partial) derivatives of $\ell(\alpha, \beta)$.

(b) Choose an optimization method to estimate the MLE and write the updating equation for it.

(c) Estimate the MLE using the method you chose in part(b) (for this generate 25 points from Gamma$(\alpha = 2, \beta = 3)$ distribution and treat this as your data set. Use `set.seed(123)` if you are using `R`).

(d) Answer parts (b) and (c) using an AI tool, compare your results to your manual answers above.