SECTIONS 12.5–12.6

WARM UP

- FIND ANGLE BETWEEN $\overset{\vec{v}}{\langle 1,0,1,2\rangle}$, $\overset{\vec{w}}{\langle 5,7,1,0\rangle}$

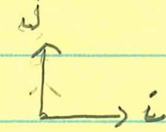
SOLN: $\vec{v}\cdot\vec{w}/|v||w| = \cos\theta = 6/\sqrt{6}\cdot 5\sqrt{3} \Rightarrow \theta = \cos^{-1}\left(2/5\sqrt{2}\right)$.

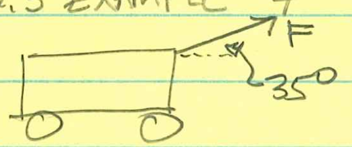
- COMPUTE $\langle 1,0,1\rangle \times \langle 5,7,1\rangle$ AND $\langle 5,7,1\rangle \times \langle 1,0,1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 5 & 7 & 1 \end{vmatrix} = i(-7) - j(-4) + k(7) = \langle -7,4,7\rangle. \qquad = -\langle -7,4,7\rangle.$$

TWO APPLICATIONS FROM LAST TIME

12.3 EXAMPLE 7 WORK $=\overrightarrow{\text{FORCE}}\cdot\overrightarrow{\text{DISTANCE}}$





wagon pulled 100 m. using 70 N FORCE

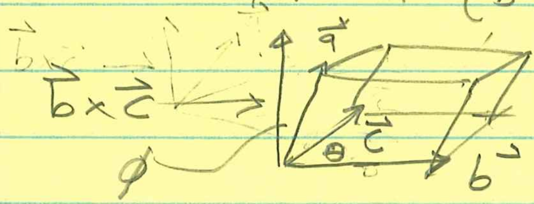
$\vec{D} = 100i$, $\vec{F} = 70\cos 35\, i + 70 \sin 35\, j$

$\vec{F}\cdot\vec{D} = 7000 \cos 35$ N/meters ? Joules

12.4 TRIPLE SCALAR PRODUCT: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

SINCE $|\vec{b}\times\vec{c}| = |b||c|\sin\theta$

WE GET $\vec{a}\cdot(\vec{b}\times\vec{c}) = |a||b||c|\sin\theta\ \cos\phi$

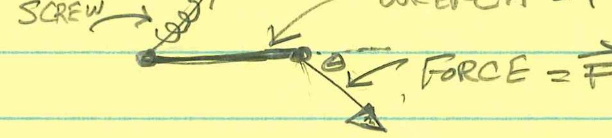


$|\vec{a}|\cos\phi$ = height of // piped

$|b||c|\sin\theta$ = area of base.

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c})$ = volume of // piped.

12.4 EXAMPLE 6 <u>TORQUE</u>



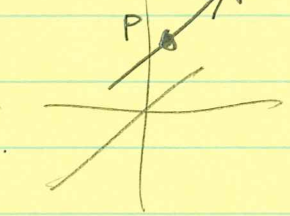
TORQUE $\vec{\tau} = \vec{r}\times\vec{F}$. SCREW GETS DRIVEN $\perp$ TO PLANE OF WRENCH + FORCE. "$\vec{j}$"

$$\left.\begin{array}{l} \theta = 75^\circ \\ \vec{r} = .25\text{ m} \\ \vec{F} = 40\text{ N} \end{array}\right\} \Rightarrow \tau = \begin{vmatrix} i & j & k \\ 0 & .25 & 0 \\ 0 & \alpha & \beta \end{vmatrix} = \tfrac{1}{4}\beta\, i$$

$F = \underbrace{40\cos 75}_{\alpha}\, j - \underbrace{40 \sin 75}_{\beta}\, K$

§ EQNS OF LINES + PLANES

L = Line is given by any point $\vec{P}$, plus a direction $\vec{V}$



e.g. line thru origin, // to $\langle 1,2,3\rangle$
is $(0,0,0) + t(1,2,3)$
line thru $(1,1,1)$ // to $\langle 1,2,3\rangle$
is $(1,1,1)+t(1,2,3) = \underbrace{(1+t, 1+2t, 1+3t)}_{\text{parameteric}}$

Genrally, $L = p + tv$
t: think of as time, a parameteric

Symmetric eqn for L: solve for t

e.g. $x = 1+t \Rightarrow t = x-1$
$y = 1+2t \Rightarrow t = \frac{y-1}{2}$
$z = 1+3t \Rightarrow t = \frac{z-1}{3}$

$\underbrace{x-1 = \frac{y-1}{2} = \frac{z-1}{3}}_{\text{"symmetric"}}$

$\boxed{\text{Class}}$ Find parametric, symmetric eqns for line L thru points $\underset{p_1}{(2,4,-3)}$ and $\underset{p_2}{(3,-1,1)}$. When does L hit x-y plane $(z=0)$.

SOLN: $\vec{V_2}$ $\vec{V} = p_2 - p_1 = \langle 1,-5,4\rangle$ so $L = p_1 + t\vec{V}$
$= (2+t, 4-5t, -3+4t)$

x-y plane $\Rightarrow z=0$
$\Rightarrow -3+4t=0$
$\Rightarrow t = 3/4$

NOW PLUG IN TO GET x,y coords.

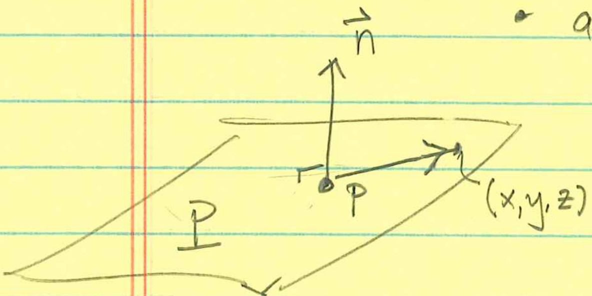
$x = 2+t \Rightarrow t = x-2$
$y = 4-5t \Rightarrow t = y-4/-5$
$z = -3+4t \Rightarrow t = z+3/4$

symmetric: $x-2 = \frac{y-4}{-5} = \frac{z+3}{4}.$

PLANES

TO GET A PLANE, WE NEED

- a point p on $\mathbb{P}$, $p = (p_1, p_2, p_3)$
- a normal vector to $\mathbb{P}$, $\vec{n} = \langle n_1, n_2, n_3 \rangle$



$\boxed{\text{Key}}$ the vector from p to (x,y,z) must be $\perp$ to $\vec{n}$ $\boxed{\text{iff}}$ $(x,y,z) \in \mathbb{P}$

$$\iff \langle x-p_1, y-p_2, z-p_3 \rangle \perp \vec{n}$$

$$\iff \underbrace{\qquad\qquad\qquad} \cdot \vec{n} = 0 \qquad \square$$

$\boxed{\text{Class}}$ Find eqn of plane thru $(2,4,-1)$ and $\perp$ to $\langle 2,3,4 \rangle$

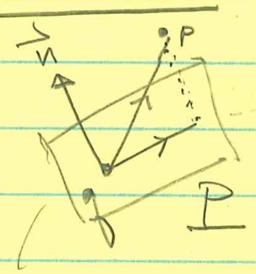
$$\Rightarrow (x-2, y-4, z+1) \cdot (2,3,4) = 0 \iff \underbrace{2x+3y+4z}_{\text{notice coeffs } = (n_1, n_2, n_3)} = 12$$

EXAMPLE FIND LINE OF INTERSECTION AND ANGLE BETWEEN $x+y+z=1$, $x-2y+3z=1$

SOLN: $n' = \langle 1,1,1 \rangle$, $n'' = \langle 1,-2,3 \rangle$

- angle between normals $= \cos^{-1} \dfrac{(1,1,1)\cdot(1,-2,3)}{\sqrt{3}\,\sqrt{14}} = \cos^{-1}\left(\dfrac{2}{\sqrt{42}}\right)$
- line of intersection $\Rightarrow z = 1-x-y$, sub in
  $x - 2y + 3(1-x-y) = 1 \Rightarrow x = 2 + \frac{5}{2}y$, backsub

Key **EXAMPLE** Distance from point p to a plane $\mathbb{P}$



Idea:
- find $q \in \mathbb{P}$, let $\vec{v} = \vec{qp}$
- find projection of $\vec{v}$ onto unit normal
- take length

Class: Find dist from $(2,6,1)$ to $x+2y+3z=3$

<u>Soln</u> $\vec{n} = \langle 1, 2, 3 \rangle$ (read coords off x, y, z coefts of normal to P)

unit $\vec{n} = \langle 1, 2, 3 \rangle / \sqrt{14}$

How get a $q \in \underline{P}$? set $x = 0 = y$, solve for $z \Rightarrow (0, 0, 1)$

$\Rightarrow \vec{qp} = (2\,6\,1) - (0\,0\,1) = \langle 2\,6\,0 \rangle$. $\boxed{\text{proj}_{\langle n \rangle} \langle 2, 6, 0 \rangle}$

$\parallel$

$$\boxed{1} = \frac{\langle 1, 2, 3 \rangle / \sqrt{14} \cdot \langle 2, 6, 0 \rangle}{\sqrt{14}} \cdot \frac{\vec{n}}{|\vec{n}|}$$

§12.6. QUADRIC SURFACES — GRAPH OF A DEG 2 POLY

$0 = Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J$

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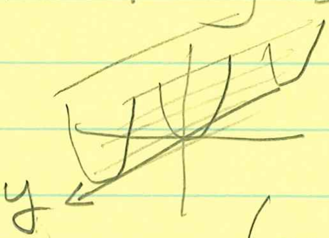
KEY

<u>Example</u> $y = z^2$



cylinder (all lines L are $\parallel$ to fixed L)

(parabolic cylinder)

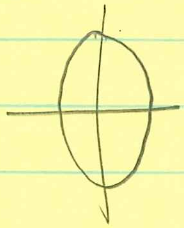
<u>Standard Form</u> $Ax^2 + By^2 + Cz^2 + J = 0$; $Ax^2 + By^2 + Iz = 0$ } translate, rotate

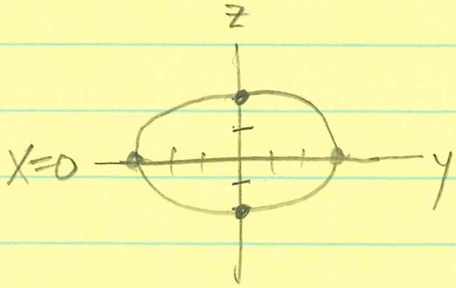
<u>Key</u>: <u>Traces</u> = set a variable to 0

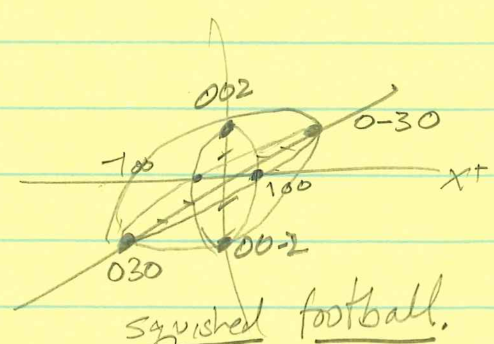
<u>EXAMPLE 3</u> $x^2 + y^2/9 + z^2/4 = 1$

All cross sections ellipses

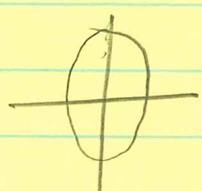
$z = 0$







$y = 0$

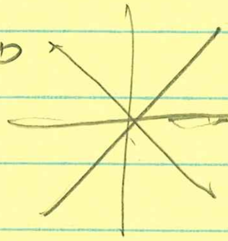


$y^2/9 + z^2/4 = 1$

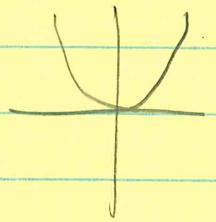
squished football.

EXAMPLE 5 $z = y^2 - x^2$

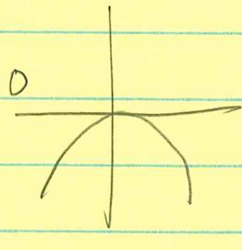
$z=0$



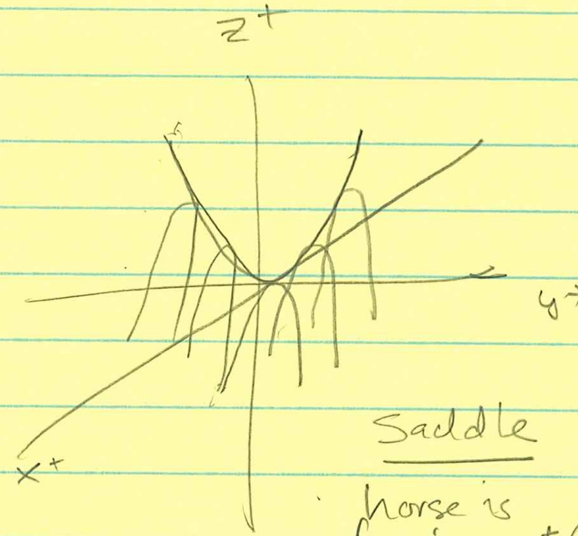
$x=0$



$y=0$



$\Rightarrow$



Saddle

horse is facing $y^+ (y^-)$

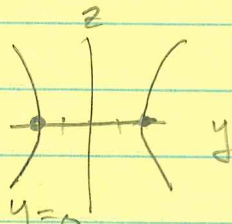
EXAMPLE 7 $4x^2 - y^2 + 2z^2 + 4 = 0$

Class (1) divide by -4 $\Rightarrow \boxed{-x^2 + \frac{y^2}{4} - \frac{z^2}{2} = 1}$

$\boxed{x=0}$

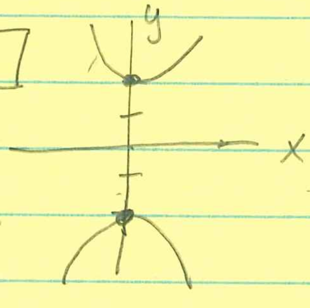
hyperbola

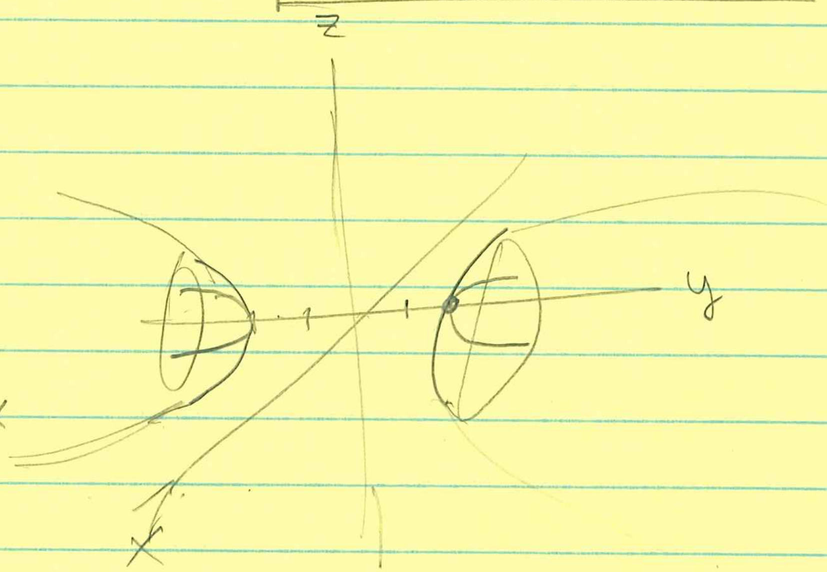
no solns $y=0$



$\boxed{z=0}$

no solns $y=0$





$\boxed{y=0}$ no solns!

Key:
- IDENTIFY GENERAL SHAPE
- TAKE TRACES
- PLOT POINTS
- DO REPS!