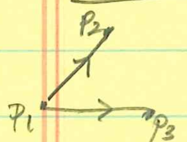
(0)

SECTION 13.1-13.2

WARM UP: FIND EQN OF PLANE THRU $\overset{p_1}{(1,1,1)}, \overset{p_2}{(1,2,3)}, \overset{p_3}{(2,2,2)}$
AND DISTANCE TO POINT $(3,5,7) = p$.

SOLN normal $= (p_3 - p_1) \times (p_2 - p_1)$ $\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (i - 2j + k) \cdot (x-3, y-5, z-7) = \boxed{x - 2y + z = 0}$

unit normal is $\langle 1,-2,1\rangle / \sqrt{6} = \vec{n}$



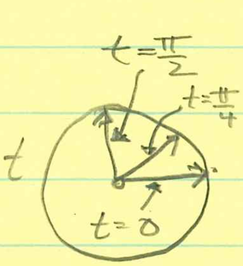
distance: $\left| \text{proj}_n \overrightarrow{p_1 p} \right| = \dfrac{\frac{\langle 1,-2,1\rangle}{\sqrt{6}} \cdot \langle 2,4,6\rangle}{} = 0$

CHECK $(3,5,7)$ on plane? $3 - 2(5) + 7 = 0$ ✓

§ 13.1 Vector function: $\mathbb{R} \to \{\text{Vectors}\}$

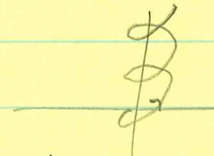
"point vs. vector"

e.g.: $t \to (\cos t, \sin t) = i \cos t + j \sin t$



Space curve = set of values of points in space

EX: $\cos t\, \vec{i} + \sin t\, \vec{j} + t \vec{k}$:



helix

$\boxed{\text{Class}}$ 17. Find vector + para eqns $\langle\ \rangle$ $(\ \ )$
for segment joining $(1,-1,2), (4,1,7)$

SOLN: $\vec{v} = p_2 - p_1 = \langle 3,2,5\rangle$; $p_1 + t\vec{v} = (1+3t, -1+2t, 2+5t)$

Notice: $t=0$ at p_1, $t=1$ at p_2.

$\boxed{\text{Limits ☹}}$ $\lim_{t\to a} f_1(t)\vec{i} + f_2(t)\vec{j} + f_3(t)\vec{k} = \left(\lim_{t\to a} f_1(t)\right) i + \dots$

$\boxed{\text{Class}}$ find $\lim_{t \to 0} (1+t^3) i + t e^{-t} j + \dfrac{\sin t}{t} k$

$\downarrow$ 0 $\quad \downarrow$ 0 $\quad \downarrow$ L'Hop $\rightsquigarrow \dfrac{\cos t}{1} = 1$

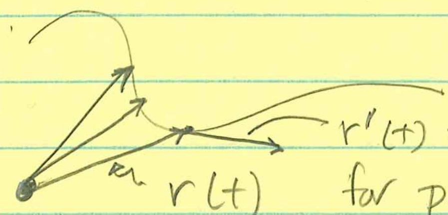
$\Rightarrow$ limit is $0\vec{i} + 0\vec{j} + 1\vec{k} = \vec{k}$.

ϕ 13.2

13.1–2

- We'll write $\vec{r}(t) = f_1(t)\vec{i} + f_2(t)\vec{j} + f_3(t)\vec{k}$ for a vector valued function, think of tip of vector as at position $(f_1(t), f_2(t), f_3(t))$ at time t.

- <u>DEF</u>: $\frac{d\vec{r}}{dt}$ $(\vec{r}\,'(t)) = f_1'(t)\,i + f_2'(t)\,j + \dots$ (differentiate each fn)



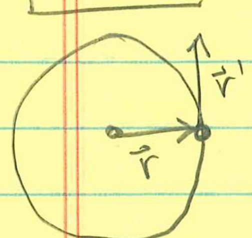
$r'(t)$ = velocity

$r(t)$ for <u>point</u> at position $r(t)$

$\boxed{\text{Class}}$ $\vec{r}(t) = \cos t\; i + \sin t\; j$, find + draw $\vec{r}\,'(t)$

$\vec{r}\,'(t) = -\sin t\; i + \cos t\; j$

☆



$t=0$ $\vec{r}(0) = i = \langle 1, 0\rangle$, $\vec{r}\,'(0) = j = (0,1)$ ✓

$$\boxed{\text{DEF: Unit tangent vector} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}}$$

$\boxed{\text{Class}}$ For ☆, show $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$ for <u>all</u> times t. Can you guess why? <u>Hint</u> $|r(t)| = 1$ for all t

<u>Soln</u>: $\vec{r}(t)\cdot\vec{r}\,'(t) = -\cos t\sin t + \sin t\cos t = 0$.

From hint: $|\vec{r}(t)| = $ const c. But $|r(t)| = r(t)\cdot r(t)$, and so

$r(t) = (f_1(t), \dots, f_n(t)) \Rightarrow$

$\vec{r}\cdot\vec{r} = (f_1^2, \dots f_n^2)$

$0 = d(c) = d(r\cdot r) = (2f_1f_1', \dots 2f_nf_n') = 2\vec{r}\cdot\vec{r}\,' \Rightarrow \vec{r}(t)\cdot r'(t) = 0$

13.1–2

(ME)

<u>Ex 3</u> Find eqns (parametric) for tan line to helix
$r(t) = (2\cos t, \sin t, t)$ @ $(0, 1, \frac{\pi}{2})$. ✓ <u>velocity</u>

<u>Soln:</u> $r' = (-2\sin t, \cos t, 1)$ @ $\frac{\pi}{2} = (-2, 0, 1)$

<u>tan line</u> $(0, 1, \pi/2) + t(-2, 0, 1)$ ∎

<u>Rules for diff</u> (p 858) $d(\vec{u}(t) + \vec{v}(t)) = \vec{u}'(t) + \vec{v}'(t)$ etc.

"Works the way you'd like". $d(\vec{u} \cdot \vec{v}) = \vec{u} \cdot \vec{v}' + \vec{v} \cdot \vec{u}'$

<u>Same with $\int$.</u> $d(\bar{u} \times \bar{v}) = \bar{u} \times \bar{v}' + \bar{u}' \times \bar{v}$

<u>Class</u> 23. Find para. eqns for tan line to
$r(t) = (t^2+1, 4t^{1/2}, e^{t^2-t})$ @ $(2, 4, 1) = p$ (so $t = 1$)

<u>Soln:</u> $r'(t) = (2t, 2t^{-1/2}, (2t-1)e^{t^2-t})$ @ $p = (2, 2, 1)$

tan line is $(2, 4, 1) + t(2, 2, 1)$ ∎

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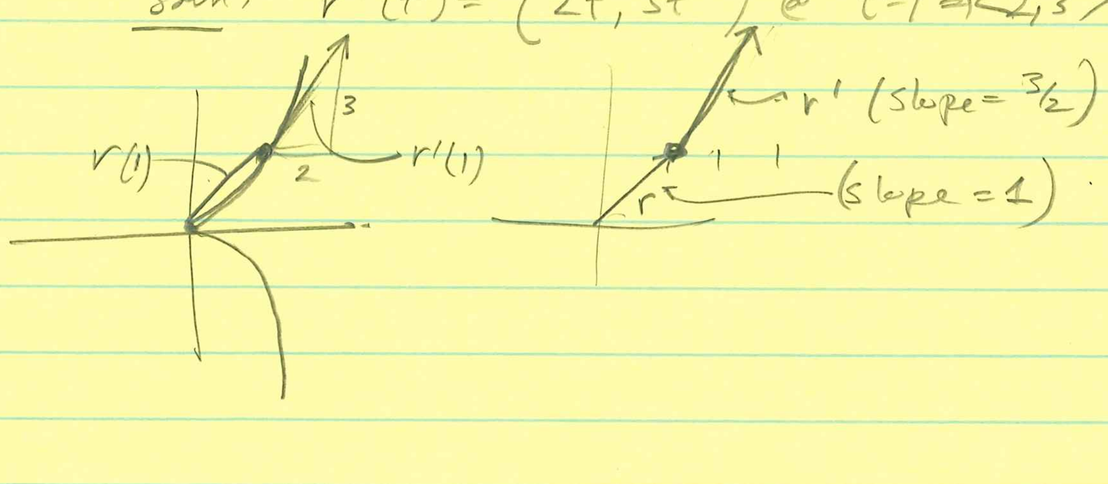
4. Sketch position $\bar{r}(t)$ and tangent vector $\bar{r}'(t)$ for (t^2, t^3) @ $t = 1$
$\vec{r}(1) = \langle 1, 1 \rangle$

<u>Soln:</u> $r'(t) = (2t, 3t^2)$ @ $t = 1 \Rightarrow \langle 2, 3 \rangle$

$y^2 = x^3$
$y = x^{3/2}$



~~SKIP~~ ~~VB~~ ~~WARM UP~~

13.1–2

18. Find unit tangent vector $\boxed{\text{at } t=0}$

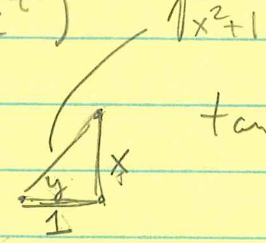
$r(t) = \left(\tan^{-1}(t), 2e^{2t}, 8te^{t}\right)$

recall $d(\tan^{-1}x)$

WARM UP NEXT TIME

PAUSE DO THIS.

$y = \tan^{-1}(x) = \tan y = x \Rightarrow$



$\tan = \frac{o}{a}$

$\sec^2 y \, dy = dx \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2} = \cos^2(y)$

$= \cos^2\left(\tan^{-1}(x)\right)$

$\Rightarrow \cos y = \frac{1}{\sqrt{x^2+1}} \Rightarrow \cos^2 = \frac{1}{x^2+1}$

$r'(t) = \left(\frac{1}{1+t^2}, 4e^{2t}, 8te^{t} + 8e^{t}\right)$ @ $t=0 \Rightarrow \boxed{\frac{\langle 1, 4, 8\rangle}{\sqrt{101}}}$

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21. $\vec{r}(t) = (t, t^2, t^3)$. Find $\vec{r}'(t)$, $\vec{T}(t)$, $\vec{r}''$, $\vec{r}' \times \vec{r}''$

$r'(t) = (1, 2t, 3t^2)$

$|r'(t)| = \sqrt{1+4t^2+9t^4}$ $\Rightarrow \vec{T}(1) = \frac{\langle 1,2,3\rangle}{\sqrt{14}}$ ← Didn't need to do!

$r''(t) = (0, 2, 6t)$

$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = i(6t^2) - j(6t) + k(2)$

velocity   accel.

$= \langle 6t^2, -6t, 2\rangle$

Check: $\cdot r'$ get $6t^2 - 12t^2 + 6t^2 = 0$ ✓

$\cdot r''$ get $-12t + 12t = 0$ ✓

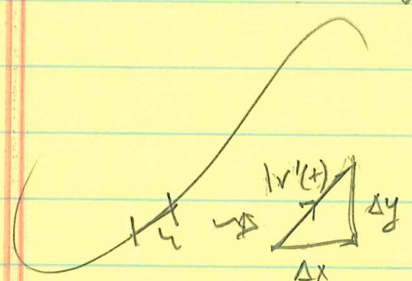
13.1–2

Look Ahead:

Curvature measures how fast $\vec{T}(t)$ is changing w.r.t arclength

$$\kappa = \frac{|T'(t)|}{|ds/dt|}$$

$$s = \int_a^t |r'(t)|\,dt$$



Fund. Thm Calc: $\frac{ds}{dt} = |r'(t)|$

So $\kappa = \dfrac{|T'(t)|}{|r'(t)|}$

As you can guess, this is messy

BUT it is easy to prove (p 865)

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$\boxed{\text{Class}}$ Find Curvature of (t, t^2, t^3)

Soln: $\bar{r}'(t) \times \bar{r}''(t) = \langle 6t^2, -6t, 2 \rangle$

so $|\quad " \quad| = \sqrt{36t^4 + 36t^2 + 4}$ (A)

$r'(t) = \langle 1, 2t, 3t^2 \rangle$ so $|r'(t)| = \sqrt{1 + 4t^2 + 9t^4}$ (B)

$\Rightarrow \kappa = \dfrac{(A)}{(B)^3}$ ▮