§ 14.3 Partials (Good news — rest of semester, one section/lecture!)

① warm up — a_N, a_T (13.4)

② warm up — lims (14.2)

13.4 (19.) $r(t) = (t^2, 5t, t^2 - 16t)$. when is speed a minimum?

Soln: $r'(t) = (2t, 5, 2t-16)$

$$|r'(t)| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$$
$$= \sqrt{8t^2 - 64t + 281}$$

minimize $\Rightarrow \frac{1}{2}(8t^2-64t+281)^{-1/2} \cdot (16t-64) = 0$

$\dfrac{8t-32}{\sqrt{8t^2-64t+281}} = 0 \Rightarrow t = 4$. min or max?

A: Second deriv test!

13.4, (41). Find a_N, a_T for

$r = ((t^2+1), t^3)$ $\qquad \vec{a} = a_T T + a_N N$

Cross	N
dot	T

Soln: $a_N = |r' \times r''| / |r'|$

$a_T = r' \cdot r'' / |r'|$.

$r' = (2t, 3t^2)$, $r'' = (2, 6t)$

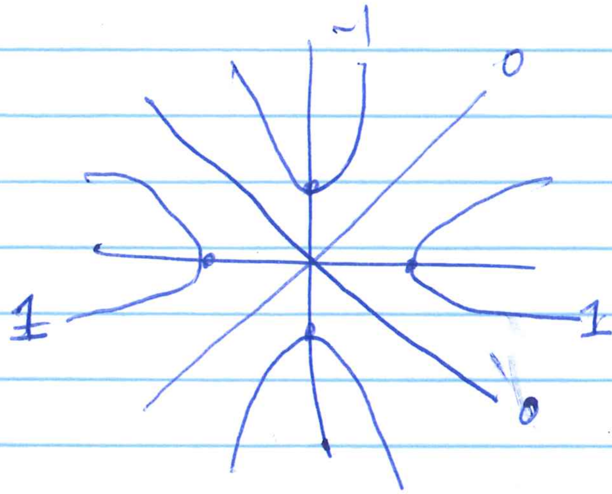
$r' \cdot r'' = 4t + 18t^3$, $r' \times r'' = \begin{vmatrix} i & j & k \\ 2t & 3t^2 & 0 \\ 2 & 6t & 0 \end{vmatrix} = 12t^2 - 6t^2 k = 6t^2$

$\Rightarrow |r' \times r''| = 6t^2$, $|r'| = \sqrt{4t^2 + 9t^4}$

$\Rightarrow a_N = 6t^2 / \sqrt{4t^2+9t^4}$, $a_T = 4t + 18t^3 / \sqrt{4t^2+9t^4}$.

14.2.95 Draw level curves for $x^2 - y^2 = k$

$= (x-y)(x+y)$



$k = 0$

$\Rightarrow x - y = 0$ or $x + y = 0$

$k = 1 \Rightarrow y = 0 \quad x = \pm 1$

$x = 0$, No soln

$k = -1 \quad x = 0, y = \pm 1$

$y = 0$ No soln

<u>Know</u> they are hyperbolas

§ 14.3 Partial Derivs, $z = f(x,y)$

Def: $\frac{\partial f}{\partial x}$ (OR f_x) (a,b) point $\triangleq \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$.

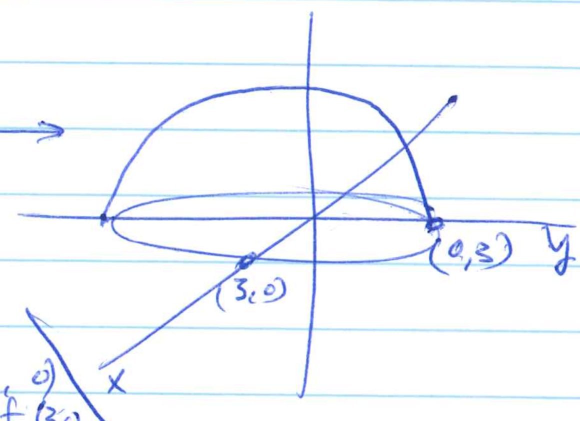
idea: hold y value fixed @ b, approach a along x-values.

Notice this is calc 1, because y is fixed!

Same def for f_y (but x held fixed).

Picture $z = 3 - x^2 - y^2$

{ top ½ of sphere, radius $\sqrt{3}$ center at origin



$\frac{\partial f}{\partial x}(3,0) = \lim_{h\to 0} \frac{f(3+h,0) - f(3,0)}{h}$

$= \lim_{h\to 0} \frac{3-(3+h)^2 - (3-9)}{h} = \frac{3-(9+6h+h^2)+6}{h} = \frac{-6h-h^2}{h}$

$= -6-h$

$\boxed{= -6}$

Notice: diff $3-x^2-y^2$ treating y as const

$= \boxed{-2x}$ at $(3,0)$ $= -6$!

This is the general rule!

Class Example 1 (Book)

find f_x, f_y if $f(x,y) = x^3 + x^2y^3 - 2y^2$ and $f_x(2,1), f_y(2,1)$.

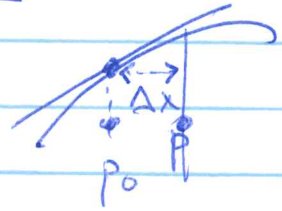
Soln

$f_x = 3x^2 + 2xy^3$ @ $(2,1) = 12+4 = 16$

$f_y = 3x^2y^2 - 4y$ @ $(2,1) = 12-4 = 8$ //

Can (should) do w/ lims + check ///

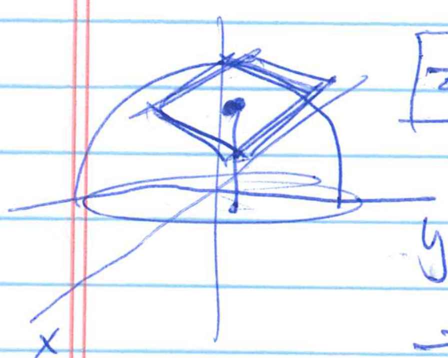
Intuition Calc 1 we had Newton's Method



$$f(p) \sim f(p_0) + \underbrace{\frac{dy}{dx}(p)}_{\text{slope}} \cdot \underbrace{\Delta x}_{\text{change in } x}$$

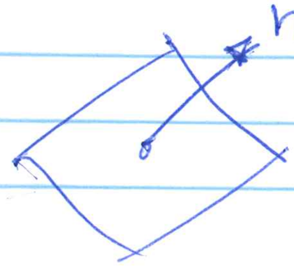
Approximate function ($y=f(x)$) w/ tangent line.

Calc III : Approx funct $z=f(x,y)$ with tan plane



$\boxed{z = 3 - x^2 - y^2}$ what does this look like at $x=1=y$ = p

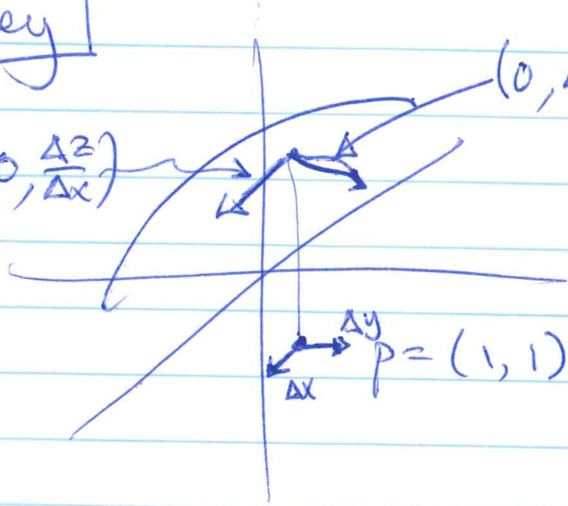
To get tan plane at p, we need a point (easy; $(1,1,f(p)=1)$) and the normal $\vec{n}$



Key

$(0, \Delta y, \frac{\Delta z}{\Delta y}) = \vec{V}_2$

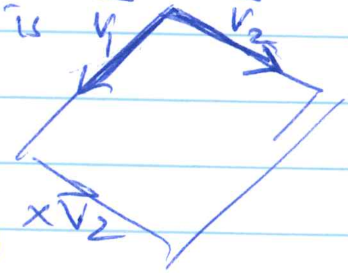
$\vec{V}_1 = (\Delta x, 0, \frac{\Delta z}{\Delta x})$



$f_x(p)$ = change in z; for small Δx

$\Rightarrow$ same for $f_y(p)$

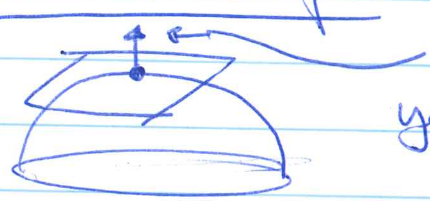
$\Rightarrow$ Tangent plane is $\vec{V}_1$ $\vec{V}_2$ so



normal is $\vec{V}_1 \times \vec{V}_2$

$$= \begin{vmatrix} i & j & k \\ 1 & 0 & f_x(p) \\ 0 & 1 & f_y(p) \end{vmatrix} = \langle -f_x(p), -f_y(p), 1 \rangle$$

Check on example! $f_x(0,0) = 0 = f_y(0,0)$



$\Rightarrow n = \langle 0, 0, 1 \rangle$

yee!

tangent plane to $z = 3 - x^2 - y^2$ at $(0,0,3)$

is $(x-0, y-0, z-3) \cdot (0,0,1) = 0 \Rightarrow \underline{\underline{z - 3 = 0}}$

Last bit Higher partials.

We can keep going: $f(x,y) = x^3y^2 + 2\cos x$

f_{yxx} — first, second, third

$f_y = 2x^3y$

$f_x = 3x^2y^2 - 2\sin x$ $\qquad$ $f_{yx} = 6x^2y$

$f_{xy} = 6x^2y$ $\qquad$ $f_{yxx} = 12xy.$

$f_{xyy} = 6x^2$ etc.

$\boxed{\text{Theorem}}$ If f_{xy}, f_{yx} exist and are continuous, then they are equal.

$\boxed{\text{Who cares?}}$ Solving PDE'S.

Heat equation: $U_{xx} + U_{yy} = 0$ (Laplace)

Class: Show $e^x \sin y$ solves $\boxed{\text{Heat eqn.}}$

$U_x = e^x \sin y = U_{xx}$. $U_y = e^x \cos y$. $U_{yy} = -e^x \sin y$

So $U_{xx} + U_{yy} = 0$.

$\boxed{\text{Wave Eqn}}$ $U_{tt} = a^2 U_{xx}$.

e.g.: ~~wave~~ particle on vibrating string. a ← string density + tension

$\boxed{\text{Class}}$ Find all second partials

53: $x^4y - 2x^3y^2$ $\quad$ $f_y = x^4 - 4x^2y$

$f_x = 4x^3y - 6x^2y^2$, $f_{xx} = 12x^2y - 12xy^2$

$f_{xy} = 4x^3 - 12x^2y$, $f_{yy} = -4x^2$,

54 $\quad$ Similarly to above.