Announcements ; • 14 practice problems are posted
• Test covers up to 14.4
• Bring ID

{Today: 45 min 14.5 / 30 min review.}

§ Last time The differential
of $w = f(x,y,z) = dw = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

Think: for a little change (dx, dy, dz)) we get a change dw.
or $\Delta w = (f_x, f_y, f_z) \cdot (\Delta x, \Delta y, \Delta z)$.

warm up } #35 / estimate amount of tin in can, $d=8$, $h=12$, .04 cm thick.

Soln: $SA = 2\pi r^2$ (bottom+top) $+ 2\pi r h$ (sides)
$dSA = 2\pi\,(2r\,dr + r\,dh + h\,dr)$
$r=4, h=12, dr=dh=.04$
$\Rightarrow 2\pi(2\cdot 4\cdot .04 + 4\cdot .04 + 12\cdot .04) \cong 1.92\pi$ //

§ $\boxed{\text{14.5 Chain Rule}}$

Calc 1 $\frac{d}{dx}(f(g(x)) = f'(g(x))\cdot g'(x)\,dx$

Suppose $z = f(x,y)$ with $x = x(t)$, $y = y(t)$.
we could plug in, get f as a fn of t, diff.

BUT better! $dz = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

Example (me) $f(x,y) = xy$

$x(t) = t^7 + 1$

$y(t) = t^6 + t.$

find $\frac{\partial f}{\partial t}$. **Soln:** $f_x = y,\ f_y = x.$ $\frac{dx}{dt} = 7t^6,\ \frac{dy}{dt} = 6t^5 + 1$

$$\Rightarrow \frac{\partial f}{\partial t} = y \cdot 7t^6 + x \cdot (6t^5 + 1)$$

$$= (t^6+t)(7t^6) + (t^7+1)(6t^5+1)$$

Check on own: get same answer if you plug in $f(x(t),y(t)) = (t^7+1)(t^6+t)$ and diff.

Class $w = x e^{y/z}$, $x = t^2$, $y = 1-t$, $z = 1+2t$

Find $\frac{dw}{dt}$.

Soln: $e^{y/z} \cdot 2t + x \frac{1}{z} e^{y/z}(-1) + x\left(\frac{-y}{z^2}\right) e^{y/z} \cdot 2$

+ plug in for x, y, z

//

More vars $u = u(x_1, \ldots, x_n)$

$x_1 = f_1(t_1, \ldots t_m)$

$\vdots$

$x_n = f_n(t_1, \ldots t_m)$

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1} \cdot \frac{\partial x_1}{\partial t_i} + \cdots \frac{\partial u}{\partial x_n} \cdot \frac{\partial x_n}{\partial t_i}$$

Example 6 (back)

$$g(s,t) = f(\overset{x}{\overset{\shortparallel}{s^2-t^2}},\ \overset{y}{\overset{\shortparallel}{t^2-s^2}})$$

and f is differentiable.

Show $t g_s + s g_t = 0$

$$g_s = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} = f_x \cdot 2s + f_y(-2s)$$

$$g_t = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} = f_x(-2t) + f_y(2t)$$

So $\left.\begin{aligned} t g_s &= f_x(2st) + f_y(-2st) \\ s g_t &= f_x(-2st) + f_y(2st) \end{aligned}\right\}$ sum, get zero.

Class

$$w(s,t) = F(u(s,t),\ v(s,t))$$

Given

$u(1,0) = 2$	$v(1,0) = 3$
u_s " $= -2$	v_s " $= 5$
u_t " $= 6$	v_t " $= 4$

$F_u(2,3) = -1$, $F_v(2,3) = 16$

Find w_s, w_t @ $(s,t) = (1,0)$ $\Rightarrow \underbrace{(\overset{u(1,0)}{\overset{\shortparallel}{2}}, \overset{v(1,0)}{\overset{\shortparallel}{3}})}_{\text{Need vals at (2,3)}}$

Soln

$$w_s = F_u \frac{du}{ds} + F_v \frac{dv}{ds}$$

$$= \overset{\shortparallel}{-1}(-2) + \overset{\shortparallel}{16}(5) = 52 \quad \boxed{\text{etc}}$$

14.5

Implicit Fns + Diff

$F(x,y)=0$ implicity defines y as a fn. of x.

e.g. $x^2+y^2=6 \Rightarrow y=\sqrt{6-x^2}$.
so can think of as

$$F(x, f(x)) = 0 \qquad \text{Remember!}$$

$$\Rightarrow \frac{\partial F}{\partial x}\cdot\frac{dx}{dx}+\frac{\partial F}{\partial y}\frac{dy}{dx}=0 \Rightarrow \boxed{\frac{dy}{dx}=\frac{-F_x}{F_y}}$$

Do same for $F(x,y,z)$, (treat z as a fn of x,y)

get $\frac{dz}{dx}=\frac{-F_x}{F_z}$ $(F_z\neq 0)$.

$\boxed{\text{ME}}$ $x^2+y^2+z^2-3xyz=0$ Find $\frac{\partial z}{\partial x}$.

soln: ~~F_x~~ $\underbrace{2x-3yz}_{F_x}\cdot\underset{\underset{1}{\shortparallel}}{\frac{dx}{dx}}+(2z-3xy)\frac{dz}{dx}=0$

$$\Rightarrow \frac{dz}{dx}=\frac{-2x+3yz}{2z-3xy}.$$

$\boxed{\text{CLASS}}$ $yz-\ln(x+y)=0$ find $\frac{dz}{dy}$.

$$\underbrace{z-\frac{1}{x+y}}_{F_y}+\underbrace{y}_{F_z}\frac{dz}{dy}=0 \Rightarrow \frac{dz}{dy}=\frac{-z+\frac{1}{x+y}}{y}$$

$\boxed{\text{BEGIN T1 REVIEW}}$