14.1–2

Set up, don't evaluate

WARM UP (LONG!)

Find velocity $=\vec{v}$, accelleration $=\vec{a}$, curvature, TNB and a_N, a_T for (13.4.7) $\bar{r}(t) = \langle t, t^2, 2\rangle$

$\boxed{\text{Soln}}$ $\vec{v}(t) = \bar{r}\,'(t) = \langle 1, 2t, 0\rangle$, $\vec{r}\,''(t) = \langle 0, 2, 0\rangle = \vec{a}(t)$

$|\bar{r}\,'(t)| = \sqrt{1+4t^2}$

(PROBABLY 15 MINUTES)

$\bar{r}\,' \times \bar{r}\,'' = \begin{vmatrix} i & j & k \\ 1 & 2t & 0 \\ 0 & 2 & 0 \end{vmatrix} = 2\vec{k}$, $|r' \times r''| = \sqrt{0^2+0^2+2^2} = 2$

$\vec{T} = \dfrac{\langle 1, 2t, 0\rangle}{\sqrt{1+4t^2}}$, $\vec{N} = \dfrac{\left\langle -\frac{1}{2}(1+4t^2)^{-\frac{1}{2}}\cdot 8t,\ (1+4t^2)^{\frac{1}{2}}\cdot 2 - 2t\cdot\frac{1}{2}(1+4t^2)^{-\frac{1}{2}}\cdot 8t,\ 0\right\rangle}{1+4t^2}$

$\vec{B} = \vec{T} \times \vec{N}$ (UGH)

$a_T = \dfrac{\bar{r}\,'\cdot r''}{|r'|} = \dfrac{4t}{\sqrt{1+4t^2}}$, $a_N = \dfrac{|r' \times r''|}{|r'|} = \dfrac{2}{\sqrt{1+4t^2}}$

§ 14.1 Functions of several vars

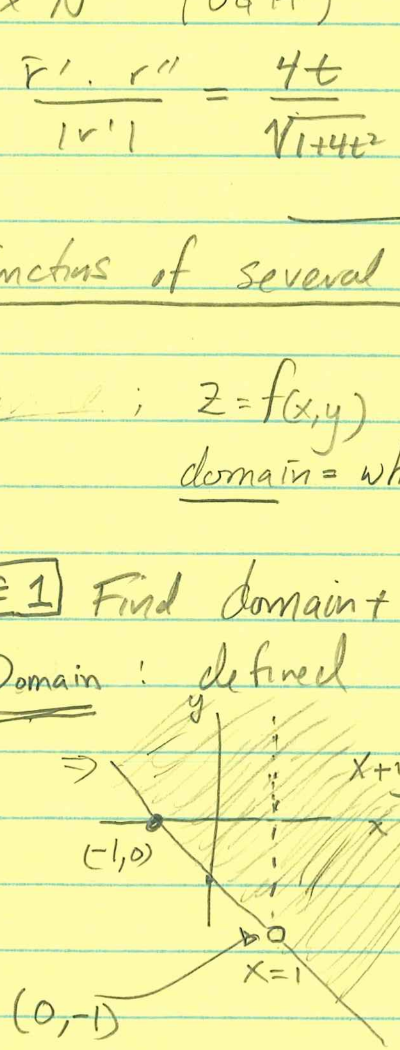
; $z = f(x,y)$ could be temp. at point (x,y)

domain = where defined, range = values hit.

$\boxed{\text{EXAMPLE 1}}$ Find domain + range of $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Domain: defined when $x \neq 1$, $x+y+1 \geq 0$

$x+y \geq -1$

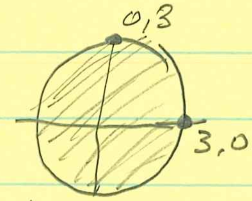


Range: $\sqrt{x+y+1}$ can take all positive values. But $x-1$ can be $+1$ or -1. So range = all real #s.

14.1–2

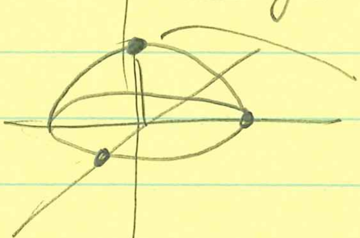
$\boxed{\text{EXAMPLE 4}}$ Find domain, range, and sketch $z = \sqrt{9-x^2-y^2}$

Soln: domain $= (x^2+y^2) \le 9 \Rightarrow$



points on or inside circle radius 3 @ (0,0)

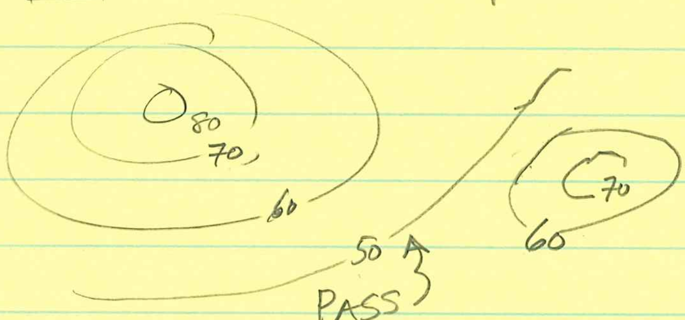
range: any z value between 0 and 3.

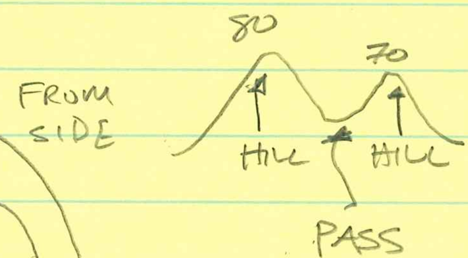


$x=0, y=0 \Rightarrow z=3$ top half of sphere w/ $r=3$ //

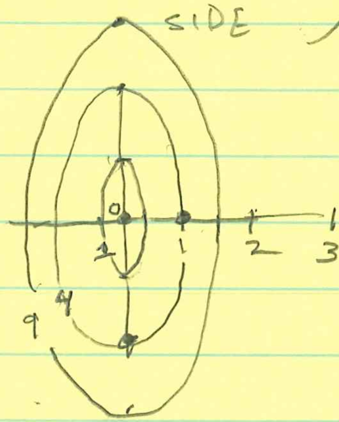
Key idea: Level curves = (topo. map).

Def: curves in plane with $f(x,y) = \text{const.} = k$



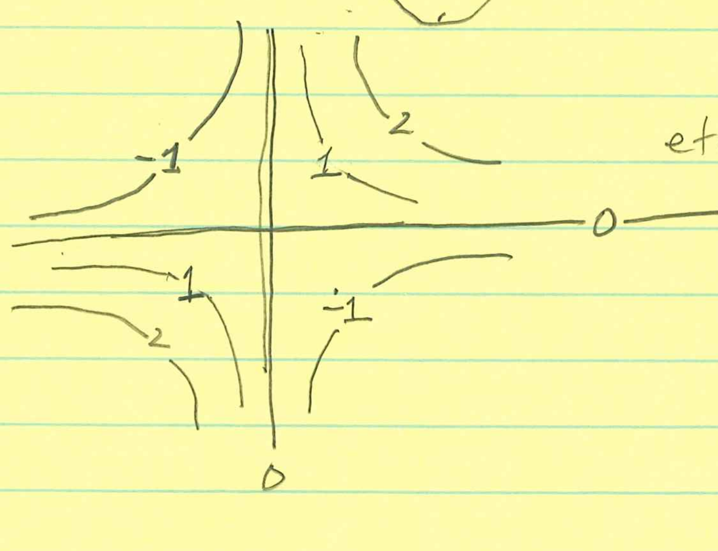


$\boxed{\text{ME}}$ $\left.\begin{array}{l} z = 4x^2+y^2, \\ K = 0, 1, 4, 9 \end{array}\right\} \Rightarrow$



$\boxed{\text{Class}}$ $z = xy$

$-1, 0, 1, 2, 3$



etc.

14.1 – 2

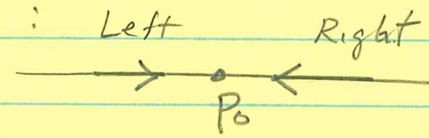
§ 14.2 Limits

$\boxed{\text{DEF}}$: $\lim\limits_{\substack{(x,y)\to(a,b) \\ \overset{\shortparallel}{p} \quad \overset{\shortparallel}{p_0}}} f(x,y) = L$ if $\overset{\text{for all}}{\forall}\ \varepsilon > 0\ \overset{\text{exists}}{\exists}\ \delta > 0$

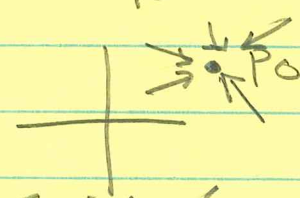
so that $|p - p_0| < \delta \Rightarrow |f(p) - L| < \varepsilon$

"When you get really close to p_0, $f(p)$ gets close to L"

$\boxed{\text{KEY KEY KEY}}$: In calc 1, you could only approach from left or right:



But in 2D, we can approach from an ∞ # of directions



Strategy to show lim Does Not Exist (DNE)

come in from different directions

$$\lim_{p\to(0,0)} \frac{x^2-y^2}{x^2+y^2} \begin{cases} \text{along line } (t,0) \Rightarrow \frac{t^2}{t^2} = 1 \\ \qquad\qquad\qquad\qquad\qquad\quad \neq \\ \text{along line } (0,t) \Rightarrow \frac{-t^2}{t^2} = -1 \end{cases}$$

$\boxed{\text{DEF}}$ $f(x,y)$ is <u>CONTINUOUS</u> at p_0 if

$\underline{\lim\limits_{p\to p_0} f(x,y) = L}$ <u>and</u> $L = f(p_0)$.

<u>This is just like calc 1</u>

- Polynomials cont everywhere
- Rational fns $\left(\frac{\text{poly}}{\text{poly}}\right)$ cont. when defined
- prev. example — not defined at $(0,0)$

14.1–2

ME: 13. $\lim\limits_{p \to (0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$

idea: polar $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$

as $p \to (0,0)$, $r \to 0$ and θ can be anything.

So, this is

$$\lim_{r \to 0} \frac{r\cos\theta \cdot r\sin\theta}{r} = \lim_{r \to 0} r\cos\theta\sin\theta = 0$$

//

CLASS Use polar coords to find

$$\lim_{p \to (0,0)} \frac{x^3+y^3}{x^2+y^2} = \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r}$$

$$= \lim_{r \to 0} \frac{r^3(\cos^3+\sin^3)}{r}$$

$$= \lim_{r \to 0} r^2(\cos^3+\sin^3) = 0$$

CLASS 6. $\lim\limits_{p \to (2,-1)} \dfrac{x^2y+xy^2}{x^2-y^2}$. Soln: defined at $(2,-1)$, evaluate $= \dfrac{-2}{3}$

HARD 10. $\lim\limits_{p \to (0,0)} \dfrac{5y^4\cos^2 x}{x^4+y^4}$

along $(0,t) \to \dfrac{5t^4}{t^4} = 5$

along $(t,0) \to \dfrac{0}{t^4} = 0$

DNE