WARM UP

(7.) Use LaGrange to find max / min of

$f(x,y,z) = 2x + 2y + z$ subject to $g(x,y,z) = x^2 + y^2 + z^2 = 9$.

Soln $\nabla f = \langle 2, 2, 1 \rangle = \lambda \nabla g = \lambda \langle 2x, 2y, 2z \rangle$

$\Rightarrow \left.\begin{array}{l} 2 = 2x\lambda \\ 2 = 2y\lambda \\ 1 = 2z\lambda \end{array}\right\}$ $x = 1/\lambda$, $y = 1/\lambda$, $z = 1/2\lambda$

$\Rightarrow$ PLUG IN TO g: $\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 9$

$\Rightarrow \frac{4}{4\lambda^2} + \frac{4}{4\lambda^2} + \frac{1}{4\lambda^2} = 9$

PLUG BACK IN $\Rightarrow \frac{9}{4\lambda^2} = 9 \Rightarrow 4\lambda^2 = 1$, $\lambda^2 = \frac{1}{4}$, $\lambda = \pm\frac{1}{2}$

So, with $\lambda = 1/2$, get $(x,y,z) = (2,2,1)$

$\lambda = -1/2$ get $(x,y,z) = (-2,-2,-1)$

Check value of $f(2,2,1) = 2 \cdot 2 + 2 \cdot 2 + 1 \cdot 1 = 9$

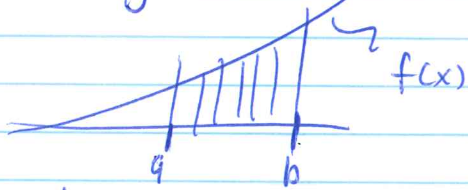
$f(-2,-2,-1) = 2(-2) + 2(-2) + 1(-1) = -9$

CONCLUDE $(2,2,1)$ maxes f

$(-2,-2,-1)$ minimizes f. //

§ 15.1 Double integrals over rectangles

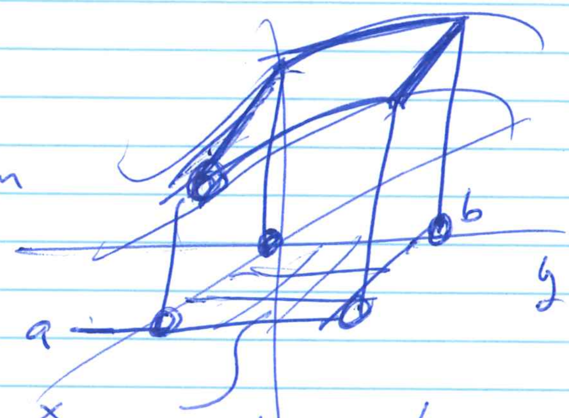
Calc 2



point in i^{th} interval.

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum f(x_i^*)\Delta x$$

(Almost) Same thing for integrating over a rectangular region



partition the rectangle R

Def: $$\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

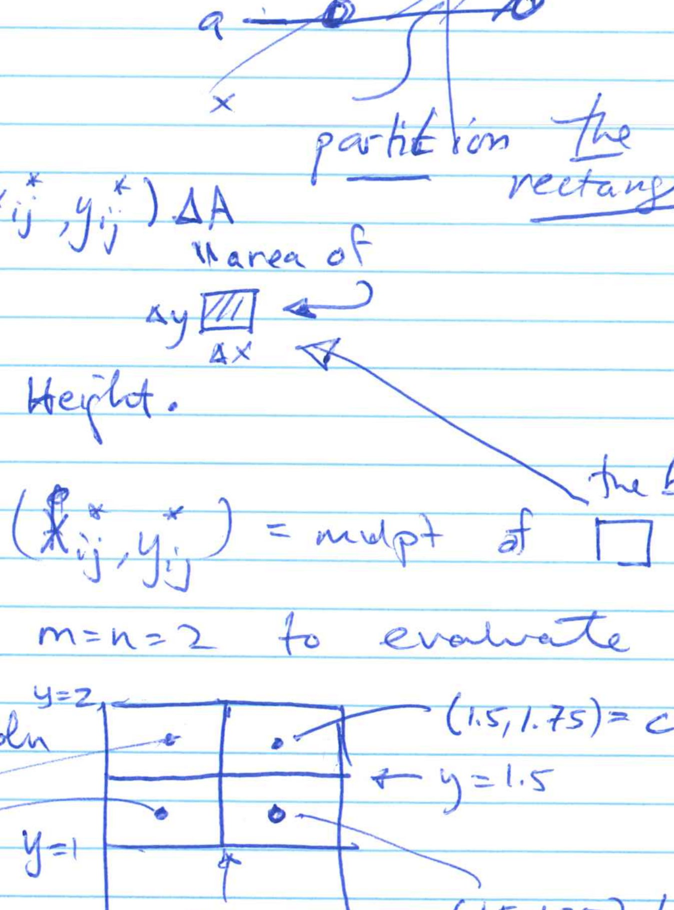
area of Δy Δx the box.

Intuition: Volume = Base x Height.

Midpoint Rule: Use (x_{ij}^*, y_{ij}^*) = midpt of □

Use mid pt rule with $m=n=2$ to evaluate

$$☆ = \int_{x=0}^{2}\int_{y=1}^{2} (x-3y^2)\,dA.$$ Soln

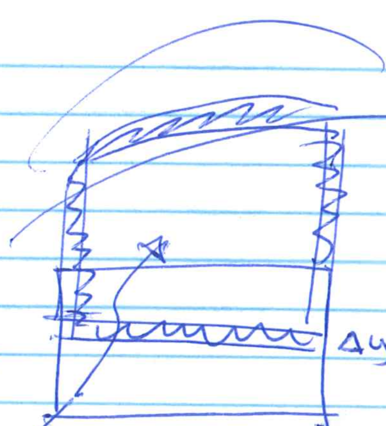


$$☆ \approx .5f(a) + .5f(b) + .5f(c) + .5f(d)$$ $(= -11.875)$

15.1, 2

The way we actually do it:

$$\int_{x=0}^{2} \underbrace{\int_{y=1}^{2} (x-3y^2)\,dy}_{\text{integrate wrt } dy,\ x \text{ const}}\,dx.$$

Slice region  Δy

"bread slice", thickness = Δy.

$$\int_{x=0}^{2} \left(\Big|_{y=1}^{2} xy - y^3 \right) dx$$

$$= (2x-8) - (x-1)$$

$$\int_{x=0}^{2} (x-7)\,dx = \Big|_0^2 \frac{x^2}{2} - 7x = 2 - 14 = \boxed{-12}$$

ACTUAL VALUE

"iterated integrals"

Fubini's Thm: Over a rectangle, can switch order of integration (f continuous)

$\boxed{\text{RMK}}$: Average Value of fn $= \dfrac{1}{\text{Area of } R} \iint_R f(x,y)\,dA$

$\boxed{\text{Class}}$

$$\int_{x=0}^{3}\int_{y=1}^{2} x^2 y\,dy\,dx$$

$$= \int_{x=0}^{3} \left(\Big|_{y=1}^{2} x^2 \frac{y^2}{2} \right) dx$$

$$= \int_{x=0}^{3} \tfrac{3}{2} x^2\,dx = \Big|_0^3 \frac{x^3}{2} = \frac{27}{2}$$

$$\int_{y=1}^{2}\int_{x=0}^{3} x^2 y\,dx\,dy$$

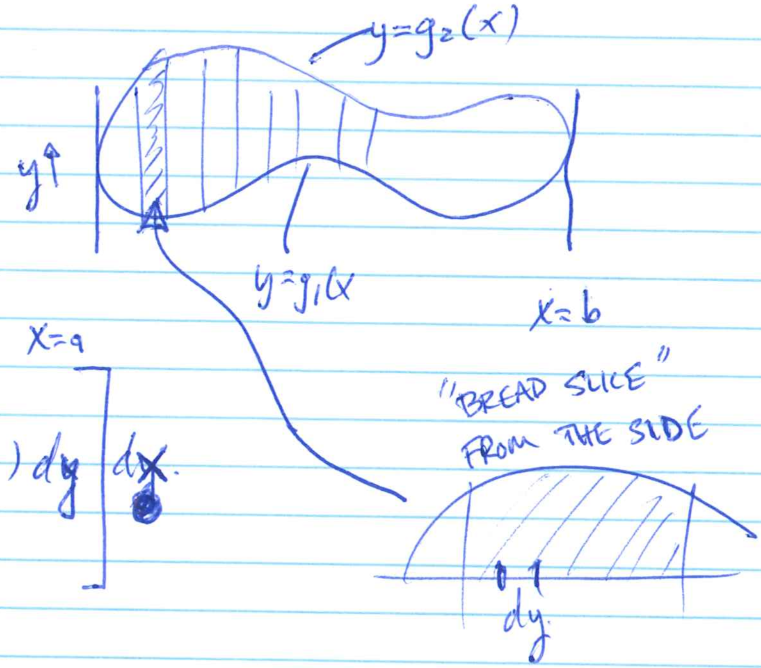
$$= \int_1^2 \Big|_0^3 \frac{x^3}{3} y\,dy = \int_1^2 9y\,dy$$

$$= \Big|_1^2 \tfrac{9}{2} y^2$$

$$= 18 - \tfrac{9}{2} = \tfrac{27}{2}$$

15.1, 2

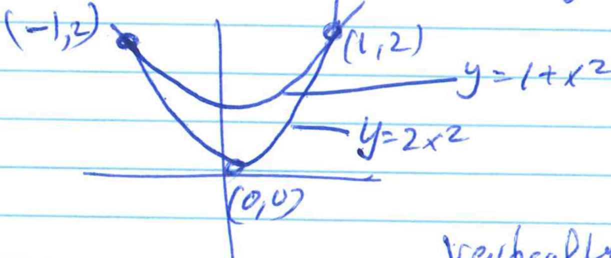
§ 15.2 $\left(\iint_{\text{general region}}\right)$ Tricky, but useful



$$\int_{x=a}^{b}\left[\int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy\right]dx.$$

Example: $\iint_R (x+2y)\,dA$, R bounded by $y=2x^2$, $y=1+x^2$.

SOLN FIRST, FIND R:

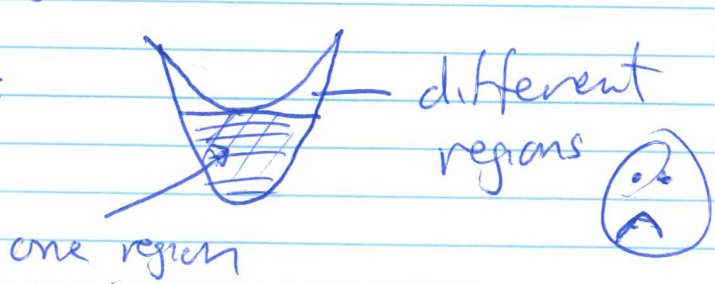


PAY ATTENTION: if we slice ||| vertically, only need 1 integral

MOST IMPORTANT PART OF TODAY!

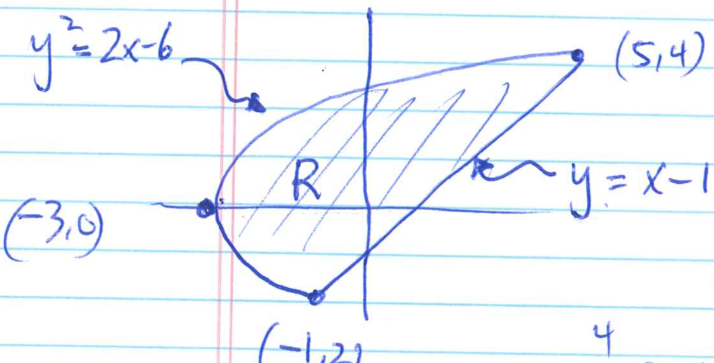
$$\int_{x=-1}^{1}\int_{y=2x^2}^{1+x^2}(x+2y)\,dy\,dx. \quad (\text{easy})$$

but if slice horizontally ≡



15.1, 2 ④

$\boxed{\text{CLASS}}$ (Ex 3 in book)



Find $\iint_R xy \, dA$

Soln: if slice |||, need 2 integrals so slice ≡

$$\int_{y=2}^{4} \int_{x=\frac{y^2+6}{2}}^{x=y+1} xy \, dx \, dy \quad \text{(easy to do)}$$

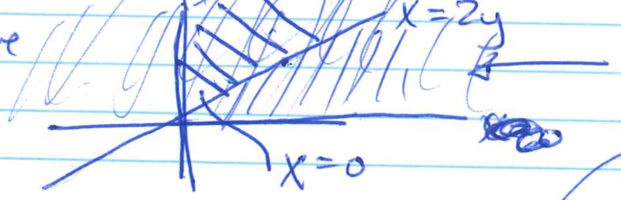
↑ LH limit

//

Notice if interior integrand f (ie. $\iint f \, dA$) is height, then we are computing volume.

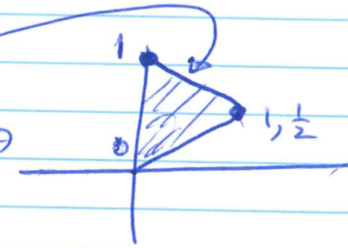
Example find volume of tetrahedron bounded by $x + 2y + z = 2$, $x = 2y$, $x = 0$, $z = 0$.

Soln: $z = 0$ is our slab = base. In x-y plane, we have



looks infinite

but when $z = 0$, $x + 2y = 2$ ⇒



with "roof" $z = 2-(2x+y)$

$$= \iint_{\triangleright} 2-(2x+y) \, dA = \int_{x=0}^{1} \int_{y=\frac{x}{2}}^{\frac{2-x}{2}} 2-(2x+y) \, dy \, dx$$

↑ easier