Lecture 2

MATH OF PAGE RANK

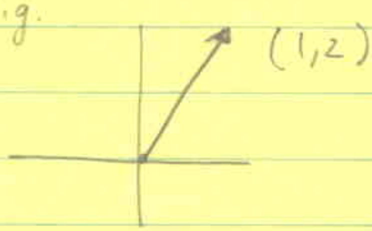
1. vectors + basis.
2. Δ basis
3. Linear Transform.

$\boxed{\text{This is a dry day}}$

<u>Def:</u> a vector space is a collection of objects (vectors), with a pair of operations: vector addition and scalar mult.

We'll work with <u>real</u> vector spaces, so for us a vector will be an n-tuple of $\mathbb{R}$

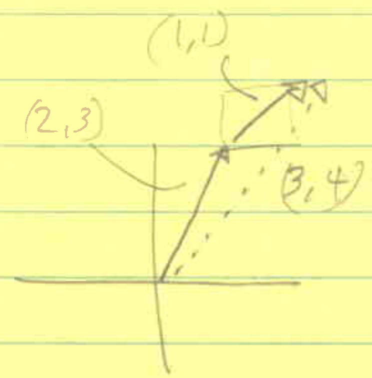
e.g.



arrow, tail @ (0,0)
head @ (1,2)

add usual way

$(1,2) + (1,1) = (2,3)$



<u>Def:</u> a set of vectors $\{v_1 \ldots v_k\}$ is Linearly dep. if $\exists_{0 \neq} c_i \in \mathbb{R}$ s.t. $\sum c_i v_i = 0$

<u>ex:</u> $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}$ is LD, b/c

$$\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

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$\{u_1 \ldots u_k\}$

Def a set of vectors spans a vector space V if any $v \in V$ can be written $\sum c_i u_i = v$

V is like a dictionary, Basis like a set of letters
spans = enough letters to write every word
LID = don't have redundant letters.

—//—

in $\mathbb{R}^2$ $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ is a basis

$\to (a,b) = a(1\ 0) + b(0\ 1)$
span, and if
$a(1\ 0) + b(0\ 1) = (0,0)$ then $a = b = 0$

—//—

skip! do next part first.

If V is a vector space, a linear transform L is a rule $V \to V$

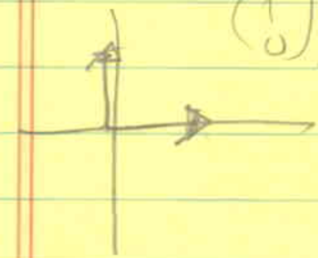
so that $L(av_1 + v_2) = aL(v_1) + L(v_2)$

"Sums split up
Scalars pull out"

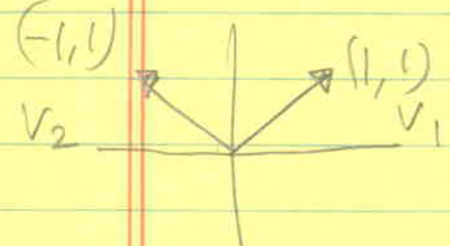
Lecture 2 "Basis is a frame of reference"

<u>Hardest Concept of the class</u>

$\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ basis for $\mathbb{R}^2$



B_1



is also a basis

B_2

When we write a vector, from now on it will be with respect to a basis.

How do we write $\begin{pmatrix}1\\0\end{pmatrix}$ as a combo of

$\underset{v_1}{\begin{pmatrix}1\\1\end{pmatrix}}$ and $\underset{v_2}{\begin{pmatrix}-1\\1\end{pmatrix}}$?

Solve $a\begin{pmatrix}1\\1\end{pmatrix} + b\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$\Rightarrow \left.\begin{matrix} a-b=1 \\ a+b=0 \end{matrix}\right\}$ Gaussian elim!!!

$2a = 1$

$a = 1/2$

$b = -1/2$

$\left(\frac{1}{2}, -\frac{1}{2}\right)_{B_2} := \frac{1}{2}v_1 - \frac{1}{2}v_2$

$= (1,0)_{B_1}$

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WANTED: Translator matrix

$$(v)_{B_2} = \left(\Delta_{B_1 \to B_2}\right)(v)_{B_1}$$

idea: v_1, v_2 basis B_1

w_1, w_2 basis B_2

say $v_1 = \alpha w_1 + \beta w_2$

$v_2 = \gamma w_1 + \Delta w_2$

then if $(v)_{B_1} = \begin{pmatrix} a \\ b \end{pmatrix}_{B_1}$

$$\begin{aligned}
&= a v_1 + b v_2 \\
&= a(\alpha w_1 + \beta w_2) \\
&\quad + b(\gamma w_1 + \Delta w_2) \\
&= (a\alpha + b\gamma) w_1 + (a\beta + b\Delta w_2) \\
&= \begin{bmatrix} \alpha & \gamma \\ \beta & \Delta \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix}
\end{aligned}$$

Mission Solve $\left[\begin{array}{c|c|c} w_1 & w_2 & v_1 \end{array}\right]$

$\left[\begin{array}{c|c|c} w_1 & w_2 & v_2 \end{array}\right]$

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Solve for old in terms of new

Example: say old is $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

new is $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}$

Solve $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$x = \frac{1}{2}$

$y = \frac{1}{2}$

Solve $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$

$y = -1/2$

$x = 1/2$

$= \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & -\frac{1}{2}\end{bmatrix}$

$\Delta_{B_1 \to B_2}$

check:

$\Delta \begin{pmatrix}1\\0\end{pmatrix}_{B_1} = \begin{pmatrix}1/2\\1/2\end{pmatrix}_{B_2}$ ✓

$= \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

= Finally, give def of Linear transform.

Thm: Any linear transform can be represented, wrt a basis, as matrix multiplication.