1. Evaluate $\int_{-1}^{0}\int_{0}^{1} \cos(y^3+y)\,dy\,dx + \int_{0}^{3}\int_{\sqrt{x/3}}^{1} \cos(y^3+y)\,dy\,dx.$

Taking the current integrals and sketching we get what is shown on the right where $0 \leqslant y \leqslant 1$ and where $-1 \leqslant x \leqslant 3y^2$. Changing the order of integration we get (with appropriate u-substitution)

$$\int_0^1\int_{-1}^{3y^2} \cos(y^3+y)\,dx\,dy = \int_0^1 x\cos(y^3+y)\Big|_{x=-1}^{x=3y^2} dy$$

$$= \int_0^1 (3y^2+1)\cos(y^3+y)\,dy = \int_0^2 \cos u\,du = \sin u\Big|_0^2 = \sin 2.$$

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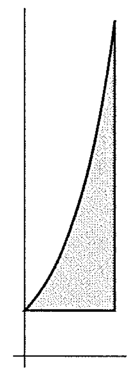
1. Evaluate $\int_{1}^{e^2}\int_{\ln y}^{2} e^{(e^x-x)}\,dx\,dy.$

Taking the current integral and sketching we get what is shown on the right where $0 \leqslant x \leqslant 2$ and where $1 \leqslant y \leqslant e^x$. Changing the order of integration we get (with appropriate u-substitution)



$$\int_0^2\int_1^{e^x} e^{(e^x-x)}\,dy\,dx = \int_0^2 ye^{(e^x-x)}\Big|_{y=1}^{y=e^x} dx$$

$$= \int_0^2 (e^x-1)e^{(e^x-x)}\,dx = \int_1^{e^2-2} e^u\,du = e^u\Big|_1^{e^2-2} = e^{(e^2-2)} - e.$$

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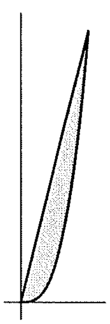
1. Evaluate $\int_{0}^{4}\int_{y/4}^{\sqrt[3]{y/4}} \sin(2x^2-x^4)\,dx\,dy.$

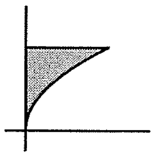
Taking the current integral and sketching we get what is shown on the right where $0 \leqslant x \leqslant 1$ and where $4x \leqslant y \leqslant 4x^3$. Changing the order of integration we get (with appropriate u-substitution)



$$\int_0^1\int_{4x^3}^{4x} \sin(2x^2-x^4)\,dy\,dx = \int_0^1 y\sin(2x^2-x^4)\Big|_{y=4x^3}^{y=4x} dx$$

$$= \int_0^1 (4x-4x^3)\sin(2x^2-x^4)\,dx = \int_0^1 \sin(u)\,du = -\cos(u)\Big|_0^1 = 1-\cos 1.$$

1. Evaluate $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 10xe^{y^5}\, dy\, dx.$



If we change the order of integration to $dx\, dy$ then we can see that $0 \le y \le 1$, and on the left we have $x = 0$. On the right hand side we have $y = \sqrt{x}$ or $x = y^2$. Combining we get
$\displaystyle\int_0^1 \int_0^{y^2} 10xe^{y^5}\, dx\, dy = \int_0^1 5x^2 e^{y^5}\Big|_{x=0}^{x=y^2} dy = \int_0^1 5y^4 e^{y^5}\, dy$ and let $u = y^5$ with $du = 5y^4\, dy$ to become $\displaystyle\int_0^1 e^u\, du = e^u\Big|_0^1 = \boxed{e-1}$

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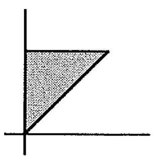
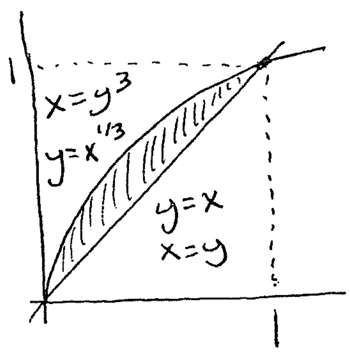
1. Evaluate $\displaystyle\int_0^1 \int_x^1 (2+6x)\cos(y^2+y^3)\, dy\, dx.$



If we change the order of integration to $dx\, dy$ then we can see that $0 \le y \le 1$, and on the left we have $x = 0$. On the right hand side we have $y = x$ or $x = y$. Combining we get
$\displaystyle\int_0^1 \int_0^y (2+6x)\cos(y^2+y^3)\, dx\, dy =$
$\displaystyle\int_0^1 (2x+3x^2)\cos(y^2+y^3)\Big|_{x=0}^{x=y} dy = \int_0^1 (2y+3y^2)\cos(y^2+y^3)\, dy$ and let $u = y^2 + y^3$ with $du = (2y+3y^2)\, dy$ to become
$\displaystyle\int_0^2 \cos u\, du = \sin u\Big|_0^2 = \boxed{\sin 2}$

swap order of integration

4. Find $\int_0^1 \int_x^{\sqrt[3]{x}} 24\sqrt{1-y^4}\, dy\, dx$. (Note $\sqrt[3]{x} = x^{1/3}$.)



$$\int_0^1 \int_x^{x^{1/3}} 24\sqrt{1-y^4}\, dy\, dx = \int_0^1 \int_{y^3}^{y} 24\sqrt{1-y^4}\, dx\, dy$$

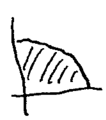
$$= \int_0^1 24\sqrt{1-y^4}\, x \Big|_{x=y^3}^{x=y}\, dy$$

$$= \int_0^1 24\sqrt{1-y^4}\,(y-y^3)\, dy$$

$$= \int_0^1 24\sqrt{1-y^4}\, y\, dy - \int_0^1 24\sqrt{1-y^4}\, y^3\, dy$$

$u = y^2$, $du = 2y\,dy$ $\qquad$ $u = y^4$, $du = 4y^3\,dy$

$$= 12\int_0^1 \sqrt{1-u^2}\, du - 6\int_0^1 u^{1/2}\, du$$



1/4 of circle of radius 1 so gives $\pi/4$

$$= 12\left(\frac{\pi}{4}\right) - 6\cdot\frac{2}{3}u^{3/2}\Big|_0^1$$

$$= \boxed{3\pi - 4}$$

score