Week 2 problem bank

1. Given the spheres $(x-7)^2+(y+3)^2+z^2=49$ and $(x+3)^2+(y-7)^2+(z+5)^2=100$, find the equation of the largest sphere that lies inside *both* spheres.
2. Find all values a so that $\langle 2,-1,1\rangle + a\langle -4,1,-2\rangle$ is a unit vector.
3. Find the projection of $\vec{u} = -\mathbf{i}+5\mathbf{j}+3\mathbf{k}$ onto the vector $\vec{v} = -\mathbf{i}+\mathbf{j}-3\mathbf{k}$.
4. Determine all values a so that the vectors $\langle a^2-a-1, a^2, 2-a\rangle$ and $\langle 2a, 3-2a, -1\rangle$ are orthogonal.
5. Find $\cos\theta$ where θ is the angle between the vectors $\langle 1,2,-2\rangle$ and $\langle 4,0,3\rangle$.
6. Find all values a so that $\cos\theta = \frac{1}{6}$, where θ is the angle between the vectors $\langle 1,2,1\rangle$ and $\langle a, 1-a, a-1\rangle$.
7. Find a vector which is perpendicular to both $\langle 3,1,4\rangle$ and $\langle 1,5,9\rangle$.
8. Find $\left(t\mathbf{i}+2\mathbf{j}+t^2\mathbf{k}\right)\times\left(\mathbf{i}+2t\mathbf{k}\right)$.
9. [F10-F] Find the area of the triangle with $(1,-1,-2)$, $(-2,0,-1)$ and $(0,-2,1)$ as vertices.
10. The volume of a tetrahedron is $\frac{1}{6}$ of the volume of a parallelepiped whose sides are formed using the vectors coming out of one corner of the tetrahedron. Find the volume of the tetrahedron with corners at $(1,1,1)$, $(1,5,5)$, $(2,1,3)$, and $(2,2,1)$.

Week 2 additional practice

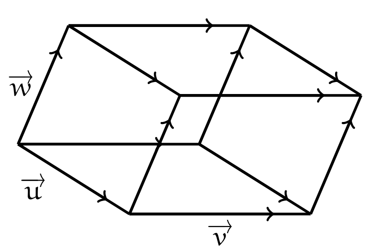
The following problems are drawn from a variety of sources, including previous exams and reviews. These are good practice to prepare for the upcoming exam. The more problems that one is *able* to answer, the better.

1. [S16-F] $ABCD$ is a parallelogram where the vertices are labeled counterclockwise. The coordinates of the points A, B and D are given by $A=(1,0,0)$, $B=(2,1,-2)$ and $D=(3,3,4)$.

   (a) Compute the diagonal vector $\overrightarrow{AC}$ and the co-ordinates of the point C.

   (b) Find the area of the parallelogram $ABCD$.

2. [S19-E1] Find the total *surface area* (**not** volume) from all *six sides* of the parallelepiped with the vectors corresponding to the edges being: $\vec{u}=\langle 3,1,0\rangle$, $\vec{v}=\langle 3,1,\sqrt{10}\rangle$, and $\vec{w}=\langle -1,0,0\rangle$.



3. Given that $\vec{u}$ and $\vec{v}$ are nonzero vectors, show that $0 \leqslant \dfrac{\left|\text{proj}_{\vec{u}}\left(\text{proj}_{\vec{v}}(\vec{u})\right)\right|}{|\vec{u}|} \leqslant 1$. Determine when you get 0 and when you get 1.

4. Let A, B, C, D be the four corners of a parallelogram with point C "opposite" from A. Given that $A=(3,1,-2)$, $B=(4,1,5)$, and $D=(1,4,3)$, determine the location of C. Also find the area of the parallelogram $ABCD$.

5. Let $\vec{u}=\langle 1,-2,2\rangle$ and let $\mathbf{r}(t)=\langle e^{2t}-2\sin^2(t)-4, e^t+\cos^2(t)+t, t-1\rangle$. Find a formula for $\text{proj}_{\vec{u}}\left(\mathbf{r}(t)\right)$ *and* determine all times when the projection is the zero vector $\vec{0}$.

6. [F19-E1] Find the area of a triangle formed by the three points $(1,2,0)$, $(1,2,1)$, $(5,-1,1)$.