Week 4 problem bank

1. Sketch the domain of the function $f(x,y) = \dfrac{\ln(x-y^2)}{\sqrt{4-x^2-y^2}}$.
2. Sketch and label the level curves for the function $f(x,y) = y/(x^2+1)$ for the values $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$.
3. Determine $\lim\limits_{(x,y)\to(0,0)} \dfrac{x\sin x}{x^2+y^4}$.
4. Determine $\lim\limits_{(x,y)\to(1,1)} \dfrac{x\sqrt{|y|}\sin x}{x^2+y^4}$.
5. Determine $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2}$.
6. Find $h_t(s,t)$ for $h(s,t) = \ln(1+st^2) + 2\sqrt{s}\arctan(\sqrt{s}\,t)$.
7. Find $f_x(0,1)$ and $f_y(0,1)$ for the function $f(x,y) = \sin x + y^2\cos x + y^4\arctan(x(y^2-1)) + \ln(2e^{\sin x}-1)\sec(xy)\tan(y-1)$.
8. Find all the second partial derivatives for the function $g(x,y) = xe^{xy}$.
9. Given $h(x,y) = x^3y - xy^3$ determine $h_{xx}(x,y) + h_{yy}(x,y)$, simplify as much as possible.
10. Determine all values b so that $f(x,y) = e^{2x-by}$ satisfies $2f_{xx}(x,y) + f_{xy}(x,y) = f_{yy}(x,y)$.

Week 4 additional practice

The following problems are drawn from a variety of sources, including previous exams and reviews. These are good practice to prepare for the upcoming exam. The more problems that one is *able* to answer, the better.

1. [F18-E2] Find $\lim\limits_{(x,y)\to(0,0)} x^2\sin^2\big(\ln(|x|+|y|)\big)$.
2. [S18-E2] Sketch the domain of $f(x,y) = \dfrac{\ln(x-1)+\sqrt{y-x}+\ln(4-y)}{x^2+2x+1}$.
3. Given that

$$z = y\sin(2x) + (1+x^2)^{\sin(x)}e^{x^2} + \cos(x)\arctan(y^4+1)\ln(y^6+137)$$

find $\dfrac{\partial^2 z}{\partial x\partial y}(0,1)$ (i.e., find the second order mixed partial derivative evaluated at the point $(0,1)$).

Hint: recall that derivatives are linear and so we can work the problem through for each part and then combine the answers together, depending on which part we are working on we can use a different approach.

4. Find the following limits, or show they do not exist:
   (a) As $(x,y)\to(0,0)$ of $\dfrac{\sec(x+y)\ln(2-x)}{\sqrt{9-\pi e^{x^2+y^2}}}$.
   (b) As $(x,y)\to(0,0)$ of $x^2\sin^2(\ln(|x|+|y|))$.
5. [F18-E2] Sketch the domain for the function below. Shade the interior of the region and make sure to indicate whether a boundary is included (solid line) or not included (dashed line). You only need to provide the sketch; no other work will be graded.

$$f(x,y) = \sqrt{xy-1}\ln(y-x^2)$$

6. [S19-E2] Consider the following function:

$$f(x,y) = \begin{cases} \dfrac{x^2+y^2}{2x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   (a) Sketch and label the *level curves* for $f(x,y)$ for the values (or "heights") of -1 and 2.
   (b) Determine whether the $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists and if so give its value, if not explain (briefly!) why the limit does not exist.
7. Sketch the domain of the function $h(x,y) = \dfrac{\ln\left(y-(x-3)^2\right)}{\sqrt{9-y}-\sqrt{3-x}}$
8. Sketch and label the level curves for the function $z = \dfrac{4y}{x^2+y^2+1}$ for the values (or heights) of $2, 1, 0, -1, -2$.
9. Evaluate the following limits.
   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(xy)}{x^2+y^2}$
   (b) $\lim\limits_{(x,y)\to(0,0)} xy\sin\left(\frac{1}{x^2+y^2}\right)$
   (c) $\lim\limits_{(x,y)\to(1,1)} \dfrac{\sin(xy)}{x^2+y^2}$