Week 8 problem bank

1. Let $\iint_{[a,b]\times[c,d]} f(x,y)\,dA$ denote the integral of $f(x,y)$ over the region with $a \leqslant x \leqslant b$ and $c \leqslant y \leqslant d$. Find $\iint_{[2,3]\times[0,1]} f(x,y)\,dA$ given the following:
$\iint_{[0,1]\times[0,2]} f(x,y)\,dA = 4$,
$\iint_{[0,2]\times[0,1]} f(x,y)\,dA = -2$,
$\iint_{[0,3]\times[1,2]} f(x,y)\,dA = 3$, $\iint_{[1,3]\times[0,2]} f(x,y)\,dA = 7$.
2. Find $\displaystyle\int_0^1\int_0^2 \frac{y}{1+x^2}\,dy\,dx$.
3. Let R be the bounded region between $y = x$ and $y = x^2$. Write $\iint_R f(x,y)\,dA$ as an iterated integral in both ways, i.e., $dx\,dy$ and $dy\,dx$.
4. By changing the order of integration write the following as a *single* integral
$$\int_{-1}^{0}\int_0^{3+3x} f(x,y)\,dy\,dx + \int_0^1\int_0^3 f(x,y)\,dy\,dx + \int_1^2\int_{x^2-1}^{3} f(x,y)\,dy\,dx.$$
5. Change the order of integration for $\displaystyle\int_0^4\int_{x^2-8}^{4\sqrt{x}} f(x,y)\,dy\,dx$.
6. Find the following $\displaystyle\int_0^1\int_0^1 \cos(e^y)\,dy\,dx + \int_1^e\int_{\ln x}^1 \cos(e^y)\,dy\,dx$.
7. Find $\displaystyle\iint_R \frac{2}{1+x^2}\,dA$ where R is the triangular region with vertices at $(0,0)$, $(2,0)$ and $(2,2)$.
8. Find $\displaystyle\int_1^2\int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx$.
9. Find the volume of the solid region consisting of points $x^2 \leqslant z \leqslant 4-y^2$.
10. Set up, but do not evaluate, an integral in polar coordinates which finds the volume for the set of points satisfying $x^2+y^2 \leqslant z \leqslant 2y$.

The quiz will consist of two randomly chosen problems (up to small variations; so make sure to learn processes).

Week additional practice

The following problems are drawn from a variety of sources, including previous exams and reviews. These are good practice to prepare for the upcoming exam. The more problems that one is *able* to answer, the better.

1. [F16-F] (a) Reverse the order of integration of $\int_0^4\int_{\sqrt{y}}^2 x^3e^{xy}\,dx\,dy$.
(b) Evaluate the integral in (a).
2. [F15-F] Change the order of integration and then find the exact value of
$$\int_0^1\int_y^{y^{1/3}} 4\sin(2x^2-x^4)\,dx\,dy.$$
3. [F15-F] Change the order of integration to rewrite the following sum as one integral, and then find the exact value of
$$\int_0^4\int_0^{y/4} e^{(x-2)^3}\,dx\,dy + \int_4^8\int_{\sqrt{y-4}}^{y/4} e^{(x-2)^3}\,dx\,dy.$$
4. [S15-F] (a) Sketch the region of integration of $\displaystyle\int_0^2\int_{x^2}^4 xe^{y^2}\,dy\,dx$, (b) reverse the order of integration and (c) evaluate the integral.
5. [F14-F] Change the order of integration of the following double integral
$$\int_0^2\int_{x^2}^{6-x} f(x,y)\,dy\,dx.$$
6. [F13-F] Let R be the region between the origin and the curve $r = \theta$ for $0 \leqslant \theta \leqslant \pi$. A lamina based on R has density $\delta(x,y) = 1$. Find
$$\iint_R (x^2+y^2)\delta(x,y)\,dA,$$
i.e., the moment of inertia of R with respect to the z-axis.
7. [F12-F] Consider the double integral.
$$\int_0^1\int_{y^2}^{2-y} f(x,y)\,dx\,dy$$
Draw the region of integration and change the order of integration.
8. [S12-F] Evaluate
$$\iint_S xy\,dA,$$
where S is the region in the first quadrant inside $x^2+y^2 = 9$ and outside $x^2+y^2 = 4$.
9. Change $\displaystyle\int_0^1\int_{-\sqrt[3]{x}}^{x^2} f(x,y)\,dy\,dx + \int_1^2\int_{5x-6}^{x^2} f(x,y)\,dy\,dx$ to a sum of two integrals where each is integrated $dx\,dy$.
10. Rewrite the following as a *single* integral by changing the order of integration.
$$\int_0^1\int_{-\frac{1}{2}\sqrt{4-y}}^{-\sqrt{1-y}} f(x,y)\,dx\,dy + \int_1^4\int_{-\frac{1}{2}\sqrt{4-y}}^{\frac{1}{2}\sqrt{4-y}} f(x,y)\,dx\,dy + \int_0^1\int_{\sqrt{1-y}}^{\frac{1}{2}\sqrt{4-y}} f(x,y)\,dx\,dy$$
11. Find $\displaystyle\int_0^1\int_0^1 e^{2x}\cos(e^xy)\,dx\,dy$.
12. Find the following $\displaystyle\int_0^1\int_0^1 \cos(e^y)\,dy\,dx + \int_1^e\int_{\ln x}^1 \cos(e^y)\,dy\,dx$.
13. Find $\displaystyle\int_0^1\int_x^{x^{1/3}} 2e^{y^2}\,dy\,dx$.
(Hint: $\int ue^u\,du = (u-1)e^u + C$)

14. Rewrite the following as a single iterated integral by changing the order of integration.

$$\int_{-2}^{0}\int_{-\sqrt{\frac{1}{2}y+1}}^{\sqrt{\frac{1}{2}y+1}} f(x,y)\,dx\,dy + \int_{0}^{6}\int_{\frac{1}{2}y-1}^{\sqrt{\frac{1}{2}y+1}} f(x,y)\,dx\,dy$$

15. Find the following:

$$\int_{-1}^{1}\int_{0}^{2}\Big(e^{y^{16}+y^2}\sin(x^{61}) + (y-1)^9\arctan(x^4+16\cos(x))+2\Big)\,dy\,dx.$$

16. Given the following:

$$\int_{1}^{4}\int_{\frac{4-y}{3}}^{\sqrt{y}} f(x,y)\,dx\,dy = 2$$

$$\int_{0}^{2}\int_{x^2}^{\sqrt{8x}} f(x,y)\,dx\,dy = 9$$

$$\int_{0}^{4}\int_{0}^{y^2/8} f(x,y)\,dx\,dy = 4$$

there are exactly two regions R_1 and R_2 besides the three given above for which $\iint_{R_1} f(x,y)\,dA$ and $\iint_{R_2} f(x,y)\,dA$ can be determined. For the *smaller* of these two regions set up an iterated integral and determine the value.

17. Find $\int_0^1\int_{x^{2/3}}^1 8x\cos(y^4)\,dy\,dx$.

18. Before setting out on their quest to reclaim the gold, Thorin Oakenshield needed to start making arrangements to transport all of the gold back to his home in the Blue Mountains (dwarves are known for their optimism in accomplishing impossible tasks). To get an estimate for the number of pack animals needed to transport the treasure he needs to do some quick computations on the amount of treasure for which the volume gives an accurate measure (treasure has a constant density). Given that Smaug's lair where the treasure is kept can be described as a semicircular room with a radius of 10 meters, the highest point on the treasure is at the midpoint of the straight wall forming one side of the semicircle and the height at distance r from this point is $\dfrac{10}{1+r^2}$ meters, then find the total volume.

19. Change the order of integration for

$$\int_{0}^{4}\int_{x^2-8}^{4\sqrt{x}} f(x,y)\,dy\,dx$$

20. Set up (but do not integrate) an integral in polar coordinates to which finds the volume of the set of points satisfying $y \geqslant 0$, $z \geqslant x^2+y^2$ and $z \leqslant 2y$.

21. Find $\displaystyle\int_{0}^{1}\int_{\sqrt{y}}^{1} 3e^{x^5}\,dx\,dy$.

22. [S19-E3] Find $\displaystyle\int_{0}^{1}\int_{x}^{\sqrt[3]{x}} 24\sqrt{1-y^4}\,dy\,dx$. (Note $\sqrt[3]{x}=x^{1/3}$.)

23. Find $\displaystyle\int_{0}^{1}\int_{\arctan(y)}^{\frac{1}{4}\pi} \cos\big(\ln(\sec(x))\big)\,dx\,dy$.

24. [S18-E3] Evaluate $\displaystyle\int_{0}^{1}\int_{\sqrt{x}}^{1} 10xe^{y^5}\,dy\,dx$.

25. [S18-E3] Evaluate $\displaystyle\int_{0}^{1}\int_{x}^{1}(2+6x)\cos(y^2+y^3)\,dy\,dx$.

26. [F17-E3] Evaluate $\displaystyle\int_{-1}^{0}\int_{0}^{1}\cos(y^3+y)\,dy\,dx + \int_{0}^{3}\int_{\sqrt{x/3}}^{1}\cos(y^3+y)\,dy\,dx$.

27. [F17-E3] Evaluate $\displaystyle\int_{1}^{e^2}\int_{\ln y}^{2} e^{(e^x-x)}\,dx\,dy$.

28. [F17-E3] Evaluate $\displaystyle\int_{0}^{4}\int_{y/4}^{\sqrt[3]{y/4}} \sin(2x^2-x^4)\,dx\,dy$.

29. [S17-F] Find the inteegral of the function $f(x,y)=\cos(y^3)$ over the region bounded by $y=2\sqrt{x}$, $y=8$ and $x=0$.

30. Evaluate $\displaystyle\int_{1}^{e}\int_{\ln y}^{1}\cos(e^x-x)\,dx\,dy$.

31. Let R be the region $\sqrt{y^2}\leqslant x\leqslant\sqrt{1-y^2}$. Find $\displaystyle\iint_R \frac{x}{x^2+y^2+1}\,dA$.

32. Find $\displaystyle\int_{0}^{1}\int_{x}^{\sqrt{x}} 30x\cos(5y^3-3y^5)\,dy\,dx$.