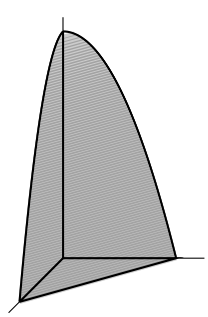
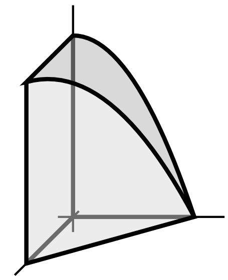
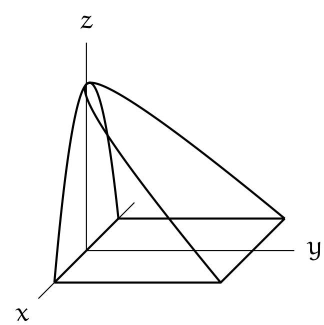
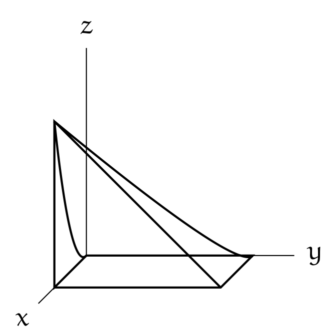
Week 9 surface area and triple integrals

1. Rewrite the iterated integral $\int_0^2 \int_0^{4-x^2} \int_0^{4-y} f(x,y,z)\,dz\,dy\,dx$ as an iterated integral with order of integration $dx\,dz\,dy$.
2. For $\int_0^6 \int_0^{4-(2/3)x} \int_0^{2-(1/3)x-(1/2)y} f(x,y,z)\,dz\,dy\,dx$ change order of integration to $dx\,dy\,dz$.
3. Find $\int_0^1 \int_0^1 \int_0^1 y^2 x e^{xyz}\,dx\,dy\,dz$.
4. Find the surface area of the cone $z = \sqrt{x^2+y^2}$ above the region R given by $4 \leqslant x^2+y^2 \leqslant 25$.

Week 9 additional practice

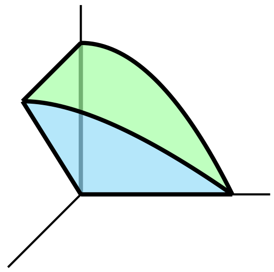
The following problems are drawn from a variety of sources, including previous exams and reviews. These are good practice to prepare for the upcoming exam. The more problems that one is *able* to answer, the better.

1. [F13-F] Find $\int_0^1 \int_0^1 \int_0^1 y^2 z e^{xyz}\,dy\,dz\,dx$. (Hint: consider changing the order of integration.)
2. [S12-F] Rewrite the iterated integral
$$\int_0^2 \int_0^{4-x^2} \int_0^{4-y} z\sqrt{4-y}\,dz\,dy\,dx$$
as an iterated integral with order of integration $dx\,dz\,dy$ and evaluate the integral.
3. Let S be the solid in the positive orthant ($x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$) bounded by the surface $z = 4-(x+y)^2$ (see picture below). Set up the bounds for the following two integrals over S:
$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dx\,dy\,dz \quad \text{and}$$
$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dz\,dy\,dx$$

4. Change the order of integration of $\int_0^1 \int_0^1 \int_0^{1-x^2} f(x,y,z)\,dz\,dy\,dx$ to the order $dy\,dx\,dz$.
5. Find the surface area for the solid bounded above by the surface $z = 4$, outside the cylinder $x^2+y^2 = 1$, and bounded below by $z = x^2+y^2$ which has density $\delta(x,y,z) = 2$.
6. [S19-E3] Write an iterated *triple* integral (**but do not evaluate**) to find the volume of the solid with $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$ and bounded by the surfaces $x+y = 2$ and $z = 3 - \frac{3}{4}y^2$.
   (a) Where $dV = dz\,dy\,dx$.
   (b) Where $dV = dx\,dy\,dz$.

7. [F17-E3] Let S be the solid $-2 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 4$, $0 \leqslant z \leqslant 4-x^2$ and $y+z \leqslant 4$.
   Set the bounds for the integral of the function $f(x,y,z)$ over S.
   (a) $dy\,dz\,dx$
   (b) $dx\,dy\,dz$


8. [F17-E3] Let S be the solid $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 4$, $0 \leqslant z \leqslant x^2$ and $y+z \leqslant 4$.
   Set the bounds for the integral of the function $f(x,y,z)$ over S.
   (a) $dy\,dz\,dx$
   (b) $dx\,dy\,dz$


9. Set up, but do NOT evaluate, an integral that finds the surface area of the surface $z = x^2y + xe^y$ over the region R where $y^2 \leqslant x \leqslant 4$.
10. Let S be the solid with $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$ and bounded by the surfaces $z = x$ and $z = 1-y^2$ (see below). Set up integrals $\iiint_S f(x,y,z)\,dV$ with
    (a) $dV = dy\,dz\,dx$, and

(b) $dV = dz\,dx\,dy$.



11. Rewrite the following as a *single* integral.

$$\int_0^1 \int_0^1 \int_0^{1-y} f(x, y, z)\,dx\,dz\,dy + \int_0^1 \int_{1-x}^{2-x} \int_0^{2-x-y} f(x, y, z)\,dz\,dy\,dx$$

12. You are creating a special plate to be installed in some machinery at your company. The plate is modeled by a rectangle with $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant b$ (where b can be set to any desired value); moreover the density of the plate satisfies $\delta(x, y) = x^a y$ where a can also be set to any specified value. Given you want to construct a plate which has a center of mass located at $(\frac{1}{3}, 2)$, determine the correct choices for a and b.

13. [F19-E3] Rewrite the integral $\displaystyle\int_0^1 \int_0^{2x} \int_0^{4-y^2} f(x, y, z)\;dz\,dy\,dx$ as an iterated integral with order of integration $dx\,dy\,dz$.