Week 11 problem bank

1. Given a wire which is bent in the shape of a helix $(\sin(3t), \cos(3t), 4t)$ for $0 \leqslant t \leqslant 2\pi$ and where the density of the wire is $\delta(x, y, z) = z$, find the mass of the wire.
2. Evaluate $\int_C (x^3 + y)\, ds$ where C is the curve given by $x = 3t$ and $y = t^3$ for $0 \leqslant t \leqslant 1$.
3. Find the work done by moving a particle along the curve C given that $\mathbf{F} = \langle x^3 - y^3, xy^2 \rangle$ and the curve is $x = t^2$ and $y = t^3$ for $-1 \leqslant t \leqslant 0$.
4. Find an f so that $\mathbf{F} = \nabla f$ where $\mathbf{F} = \left\langle 2xyz + 3x^2z + e^x, x^2z + y, x^2y + x^3 + \dfrac{1}{1+z^2} \right\rangle$.
5. Evaluate the following line integral for $C = (\cos t, \sin t, t)$ and $0 \leqslant t \leqslant 2\pi$: $\displaystyle\int_C yz\, dx + xz\, dy + (xy + 2z)\, dz.$
6. Let C be the curve that travels on the unit circle counterclockwise from $(1, 0)$ to $(0, 1)$. Calculate $\displaystyle\int_C \left(2xe^y \cos(x^2) - y\right) dx + \left(e^y \sin(x^2) + x\right) dy.$
7. Let C be the curve $\left(\cos(\pi t^2) + t^{17}, e^{t(1-t)} - \sin(\pi t/2)\right)$ for $0 \leqslant t \leqslant 1$ and let $\mathbf{F} = \left(3y^2 + \cos(x+y)\right)\mathbf{i} - \cos(x+y)\mathbf{j}$. Find $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\, ds$. (Hint: recall that $\mathbf{n} = \langle \frac{dy}{ds}, -\frac{dx}{ds} \rangle$.)
8. Let S be the region $x^2 + y^2 \leqslant 1$ and $(x+y)^2 \leqslant 1$. Let $C = \partial S$ be the boundary of the region oriented counter-clockwise. Find $\displaystyle\oint_C \left(y\cos x - e^{-x} - y\right) dx + \left(\sin x + x + e^y\right) dy.$
9. Let C be the closed curve $(x-2)^2 + (y-2)^2 = 1$ oriented *clockwise* and let D be the closed curve consisting of three line segments joining $(0, 0)$, $(6, 0)$, and $(0, 6)$ (in that order). Note that C lies inside the region bounded by D. Find $\displaystyle\oint_{C \cup D} \left(e^y - \frac{y(x-1)^2}{1+x^2}\right) dx + \left(xe^y + \ln(1+x^2)\right) dy.$
10. Let G be the part of the surface $z = y^2 - x^3$ lying above R which consists of $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 1$. Find the flux for $\mathbf{F} = 2y\arctan(z^2)\mathbf{i} + 3x^2\arctan(z^2)\mathbf{j} + z\mathbf{k}$ through G where $\mathbf{n}$ is the upward pointing normal.

Week 11 additional practice

The following problems are drawn from a variety of sources, including previous exams and reviews. These are good practice to prepare for the upcoming exam. The more problems that one is *able* to answer, the better.

1. [F10-F] Evaluate the line integral $\displaystyle\oint_C 2xy\, dx + x^2\, dy$ where C is the curve on the cardiod $r = 2 + \cos\theta$ traveled counterclockwise.
2. While waiting for a movie you pass the time by using a line integral to determine how many people are in line. The wall against which people are lined up is given by the curve C with $\left(x(t), y(t)\right) = (6t^2, 4\sqrt{3}t^3)$ for $0 \leqslant t \leqslant 1$ with distances measured in meters. Also the number of people in the line at a given (x, y) is $\frac{5}{2}x + \frac{13}{2}$ people per meter. Determine the number of people in line.
3. Determine if $\mathbf{F} = \left\langle 3x^2y^2 + e^x, 2x^3y + \sin y \right\rangle$ is conservative. If so find an f so that $\mathbf{F} = \nabla f$.
4. Evaluate the following line integral where C is the curve $y = 5x^7 + 4x\cos(3\pi x)$ with $0 \leqslant x \leqslant 1$: $\displaystyle\int_C (y + 2x)\, dx + (x - 2y)\, dy.$
5. [S18-F] Evaluate $\displaystyle\int_C (1 - 2xy)\, dx + (y - x^2)\, dy$, where C consists of the line segment from $(0, 0)$ to $(1, 0)$ followed by the line segment from $(1, 0)$ to $(2, -1)$.
6. [F17-F] (a) Is the vector field $\langle 2xe^{x^2} + z, 1 + e^y, x + 2z \rangle$ conservative? Justify your answer.

   (b) Let $C = \left((t-1)e^t, t + \cos(\frac{\pi}{2}t)\ln(1+t^2), (t+1)^{(t+1)} - 1\right)$ be a parametric curve with $0 \leqslant t \leqslant 1$. Find the following
   $$\int_C \left\langle 2xe^{x^2} + z, 1 + e^y, x + 2z \right\rangle \cdot \mathbf{T}\, ds.$$
7. Find a potential function for the vector field
   $$\mathbf{F}(x, y, z) = \langle 2xy + z^2, x^2, 2xz + \pi\cos(\pi z) \rangle.$$
   Further, compute the flow of $\mathbf{F}$ along the curve parameterized by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ for $0 \leqslant t \leqslant \pi$.
8. Find the work done by the vector field
   $$\mathbf{F}(x, y) = \langle 2e^{\sin x} + y^2, x^2 + 2\tan^{-1}(y^2 + 1) \rangle$$
   along the closed curve formed by $y = 0$, $x = 2$ and $y = x^3/4$, oriented counter-clockwise.
9. Calculate the surface integral of $g(x, y, z) = \sqrt{x^2 + y^2 + 1/4}$ over the part of the paraboloid $z = 9 - x^2 - y^2$ lying above the xy-plane.
10. [F16-F] Let $\mathbf{F} = (x^3 - 2x^2 + y^2)\mathbf{i} + (y^3 + 4xy)\mathbf{j}$ and C the path along the x-axis from $(-1, 0)$ to $(1, 0)$ and then along the semicircle $y = \sqrt{1 - x^2}$ back to $(-1, 0)$. Use Green's Theorem to evaluate

    (a) The counterclockwise circulation $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$.

    (b) The outward flux $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$.
11. [S16-F] Calculate
    $$\int_C 3x^2y\, dx + (x^3 + 1)\, dy + 9z^2\, dz$$
    where C is the curve parameterized by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leqslant t \leqslant 1$.

12. [F15-F] After a heavy snowstorm you decide to build a small "fort" in preparation for a snowball fight with some friends. The fort consists of a curved wall built out of snow to offer something to hide behind. The wall you build will lie on top of the curve C with $\big(x(t), y(t)\big) = (3t^2, 2\sqrt{3}t^3)$ for $0 \leqslant t \leqslant 1$ with distances measured in feet. The height of the wall at a given (x, y) is $\frac{1}{2}x + \frac{5}{2}$ feet. You will need one cubic foot of snow for each square foot of surface area of the wall. Use a line integral to determine how many cubic feet of snow you will need to build this wall, i.e., find $\int_C (\frac{1}{2}x + \frac{5}{2})\, ds$.

13. [S15-F] Let $\mathbf{F} = (x^2 - 2y^2)\mathbf{i} + (2y - 2xy)\mathbf{j}$ and C the path along the semicircle $y = \sqrt{4 - x^2}$ from $(2, 0)$ to $(-2, 0)$ and then along the x-axis from $(-2, 0)$ back to $(2, 0)$. Use Green's Theorem to find

    (a) the counterclockwise circulation $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$.

    (b) the outward flux $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$.

14. [F14-F] Evaluate $\int_C M\, dx + N\, dy + P\, dz$ where C is given by

$$\mathbf{r}(t) = (2\cos(t))\mathbf{i} + (2\sin(t))\mathbf{j} + (4t)\mathbf{k}, \quad 0 \leqslant t \leqslant 2\pi$$

    and $\mathbf{F} = -yz\mathbf{i} + xz\mathbf{j} + (x + y)\mathbf{k}$.

15. [F13-F] For $0 \leqslant t \leqslant 1$ let C be the parameteric curve $\big(t\cos(2\pi t), \sin(\frac{1}{2}\pi t)\big)$. Find

$$\int_C (2xy + 2x)\, dx + (x^2 + e^y)\, dy.$$

16. [F13-F] Let R be the region in the plane with $-1 \leqslant x \leqslant 1$ and $-1 - \sqrt{1 - x^2} \leqslant y \leqslant 1 + \sqrt{1 + x^2}$. (This corresponds to a 2×2 rectangle centered at the origin with half circles added to the top and bottom.) Let $C = \partial R$, i.e., the boundary of R, oriented counterclockwise and find

$$\oint_C (y^4 + y)\, dx + (y^4 + 2x)\, dy.$$

    (Hint: look for symmetry.)

17. [F12-F] Consider the vector field $\mathbf{F}(x, y) = (y - \frac{1}{x^2})\mathbf{i} + (x + \frac{1}{y^2})\mathbf{j}$.

    (a) Is the vector field $\mathbf{F}(x, y)$ conservative? Justify your answer.

    (b) Let C be the path from the point $(1, 1)$ to the point $(4, 1)$ which consists of straight line segments from $(1, 1)$ to $(2, 3)$ and from $(2, 3)$ to $(4, 1)$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

18. [S12-F] Let C be the closed curve which consists of straight line segments between the points $(-3, 0)$, $(3, 0)$ and $(0, 3)$ where C is oriented counterclockwise. Find

$$\oint_C \left(y^2 + \cos(x^3) - x^2 y\right) dx + \left(2xy + \frac{1}{1 + e^{2y}}\right) dy.$$

19. [S12-F] Given the vector field $\mathbf{F}(x, t) = (\cos(x) + y)\mathbf{i} + (e^y + x)\mathbf{j}$ and the curve C parameterized by $x(t) = \frac{\pi}{2}\cos(t)$ and $y(t) = \sin(t)$ with $0 \leqslant t \leqslant \pi$, compute the following line integral.

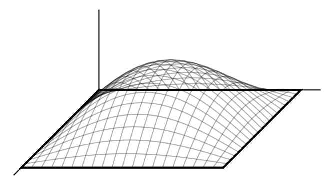
$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

20. Let C be the closed oriented curve that starts at $(0, 0)$, goes to $(2, 0)$, then goes to $(2, 1)$, and finally returns to $(0, 0)$. Determine the value of

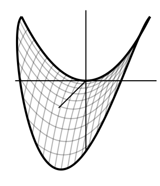
$$\oint_C \arctan(x^7)\, dx + x\cos(y(2 - y))\, dy.$$

21. Let G be the surface $z = \sin(x)\sin(y)$ with upward pointing normals and where $0 \leqslant x \leqslant \pi$ and $0 \leqslant y \leqslant \pi$ (see picture below). Find

$$\iint_G \big(\nabla \times ((5z - y^2)\mathbf{i} + x\cos(z)\mathbf{j} + xy\mathbf{k})\big) \cdot \mathbf{n}\, d(SA).$$



22. Compute the flux through the surface $z = y^2 - x^2$ where $x \geqslant 0$ and $x^2 + y^2 \leqslant 1$ (see picture below) given it has upward pointing normals and that $F = 2x\mathbf{i} - y\mathbf{j} + 2y^2\mathbf{z}$.



23. Let C be the parameterized curve $\big(t^7, \frac{1}{2}(1 - \cos(7\pi t)), \frac{4}{\pi}\arctan(t)\big)$ with $0 \leqslant t \leqslant 1$. Find

$$\int_C (1 + \sin z)\, dx + (2 + z^3 e^y)\, dy + (2z + x\cos z + 3z^2 e^y)\, dz.$$

24. Having successfully commandeered the MAV and stripped it of all of its useless equipment (who needs navigational computers or heat shields on rockets anyways), you blast off into orbit as the rescue ship draws near. You are now within sight of the ship, but unfortunately not quite close enough for them to retrieve you. The idea of waving as they pass after working so hard to stay alive is not inviting, so you decide to put a small puncture into your glove to have some of your remaining air escape that will then propel you forward "iron man"-style to bridge the remaining short distance.

    Your trajectory can be modeled by the curve in three space $(t\cos(2t), t\sin(2t), \frac{4}{3}t^{3/2})$ for time $0 \leqslant t \leqslant 1$. Given that the amount of oxygen you use at any given point along the curve is $1 - \frac{9}{16}z^2$ units per distance along the trajectory (i.e., as you start you will use many units but as you get close you will need less), use a line integral to find the number of units of oxygen this short flight uses.

25. Let $C(t) = \left(3\ln(1+(e-1)t^3), 2\cos(\frac{7}{2}\pi t)\right)$ for $0 \leqslant t \leqslant 1$. Find

$$\int_C (2x + ye^{xy})\,dx + (4y + xe^{xy})\,dy.$$

26. Aldrich Killian, the man behind the Mandarin, has successfully infiltrated the highest level of government and has sent one of his minions aboard Air Force One to abduct the President. To keep Iron Man from interfering with his plans he sends a large number of the crew and passengers into a free fall from the plane. With only moments to act, Iron Man quickly soars into action and works to save as many people as he can, to do so he enters into a spiral, well more of a helix, that can be described by the curve C given parametrically as

$$\left(\cos(t^2), \sin(t^2), 4 - t^2\right) \quad \text{for } 0 \leqslant t \leqslant 2.$$

As he moves along this curve he is able to grab more people. The number of people that he is able to pick up can be found by $\int_C \sqrt{2}\,z\,ds$, i.e., the closer to the ground he gets the more difficult it is to add more people to his group. Determine the number of people that Iron Man will be able to save.

27. Let C be the curve in three dimensions with the following parameterization for $0 \leqslant t \leqslant 2$:

$$\left(\sin(\pi t^{71}), t^2 - 3t + 2, t^3 - 3t\right).$$

Find the value of

$$\int_C \left(\cos(x+y) + ze^{xz}\right)dx + \cos(x+y)\,dy + (xe^{xz} + 2)\,dz.$$

28. While separated from your vertically challenged travelling companions you find yourself lost in an underground network of caves. While fumbling around in the dark you come across a ring. While you cannot see it clearly you can tell that it has an unusual property. Namely the density is not perfectly symmetric around the center. Upon further reflection and weighing it in your hand you realize that if you model the ring by a unit circle centered at the origin (so one possible parameterization would be $\mathbf{r}(t) = \langle \cos t, \sin t\rangle$) then the density of a point (x, y) on the ring is $\delta(x, y) = x + y + 2$ this throws the center of mass slightly off from the center and thus is a most unusual ring (subsequent experimentation will reveal that it has even more interesting properties than its unusual density). Use the information to determine the center of mass of the ring.

Hint: There is a line of symmetry. Where? $\int_0^{2\pi} \sin^2 t\,dt = \int_0^{2\pi} \cos^2 t = \pi$, $\int_0^{2\pi} \sin t \cos t\,dt = 0$.

29. Let C be the curve in three dimensions with the following parameterization for $0 \leqslant t \leqslant 3$.

$$x(t) = \cos(\pi t) \qquad y(t) = e^{41\sin(\pi t)}$$
$$z(t) = \begin{cases} t^3 + t^2 - 2t & \text{if } 0 \leqslant t \leqslant 1 \\ t^2 - 1 & \text{if } 1 \leqslant t \leqslant 3 \end{cases}$$

Find the value of

$$\int_C \frac{2xz}{1+z^2}\,dx + 3y^2 \sin z\,dy\left(\ln(1+x^2) + y^3\cos z\right)dz.$$

30. Let C be the curve consisting of the straight line segments

$$(-2,-2) \to (3,-2) \to (3,3) \to (-2,3) \to (-2,-2)$$

and let D be the curve consisting of the straight line segments

$$(0,0) \to (0,1) \to (1,1) \to (1,0) \to (0,0).$$

Find

$$\int_{C\cup D} \left(xy^2 - \frac{\sin x}{1+x^2} + 52\right)dx + \left(2x + e^{y+\sin y} + x^2 y\right)dy.$$

31. Let C_1 be the curve which connects the following points (in the given order with straight line segments)

$$(1,1) \to (1,-1) \to (-1,-1) \to (-1,1) \to (1,1),$$

similarly let C_2 be the curve which connects the following points (in the given order with straight line segments)

$$(3,0) \to (0,3) \to (-3,0) \to (0,-3) \to (3,0).$$

Let C be the union of the curves C_1 and C_2. Find

$$\oint_C (xy^2 - y + \cos(x^2))\,dx + (y^7 + x^2 y + x^2)\,dy$$

32. As reigning champion of *The Ultimate Extreme Roller Coaster Happy Fun Hour* you have recently been hired to help design rides in an amusement park. One of the rides that you are working on involves a large bowl-like surface on which people are strewn into. In particular, the surface is $z = \frac{1}{2}(x^2 + y^2)$ for $0 \leqslant z \leqslant 2$. You are working to ensure the supports in place will be sufficient to support the weight. In particular, the design calls for the density to increase with the height, i.e., $\delta = z$. Help estimate the total mass of the surface by finding $\iint_S \delta\,d(SA)$.

33. Let C be the closed curve consisting of the straight line segments that visit the following points in the listed order:

$$(0,0) \to (2,1) \to (4,0) \to (5,2) \to (4,4) \to (2,3) \to (0,4) \to (-1,2) \to (0,0)$$

Find $\oint_C (y^2 + \cos(x^3) - xy)\,dx + (2xy + \dfrac{1}{1+e^{2y}})\,dy$.

(Hint: draw the region; apply Green's theorem; take a moment to think about how to interpret integral.)

34. Let $C = \left((t^{14}-1)e^t + \cos(\pi t), 3t^2 - \frac{8}{\pi}\arctan t\right)$, for $0 \leqslant t \leqslant 1$. Find

$$\int_C \left(ye^{xy} + \cos x\right) dx + \left(xe^{xy} + \frac{2y}{y^2+1}\right) dy.$$

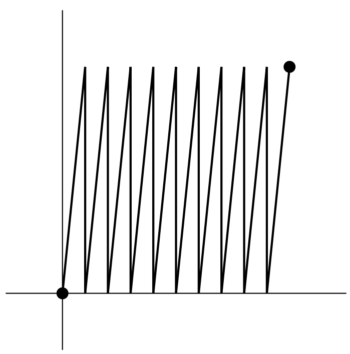
35. [F18-E3] Evaluate

$$\oint_C \left(\sin\left(x^2+3x\right) + 5y - e^{\cos x}\right) dx + \left(2x + \ln\left(y^2+1\right)\right) dy$$

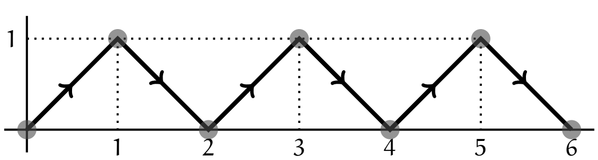
where C is the path composed of line segments in the xy-plane connecting $(3,0)$ to $(0,5)$ to $(-1,0)$ to $(2,-1)$ back to $(3,0)$.

36. [F18-E3] A sawtooth curve C (see below) consists of 19 line segments *starting* at $(0,0)$ and *ending* at $(1,1)$. Find the value b, given that

$$\int_C \left(-\pi\sin(\pi x) + 4e^{1-x}\arctan(y)\right) dx + \left(by - \frac{4e^{1-x}}{1+y^2}\right) dy = 5 - \pi.$$

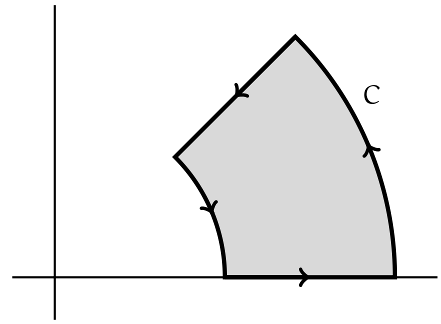


37. [S19-E3] Find $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F} = \langle 2x - ye^{xy}, -xe^{xy}\rangle$, and C is the parameterized curve $\left(t, 3 - |t-1| + |t-2| - |t-3| + |t-4| - |t-5|\right)$ where $0 \leqslant t \leqslant 6$ (see picture below).



38. [S19-E3] Let C be the boundary of the region defined by polar inequalities $1 \leqslant r \leqslant 2$ and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$ having counterclockwise orientation.

Determine the value of
$\oint_C \arctan\left(\frac{y}{x}\right) dx + 3\ln(x^2+y^2)\,dy.$

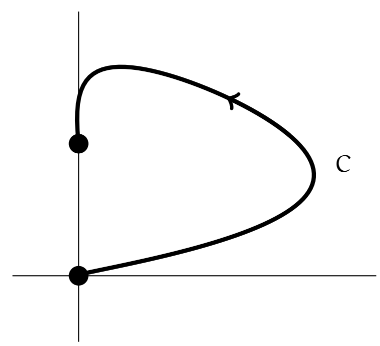


39. [S19-E3] Find the flux through the surface using upward normals defined by $f(x,y) = \sqrt{3x^2+3y^2}$ over the region $1 \leqslant x^2+y^2 \leqslant 4$ given by the vector field $\langle x, y, xy^2\rangle$.

40. [F18-F] Evaluate the following line integral for the curve $C = \left(e^{t^2-t} - \cos(2\pi t), 2\sin\left(\frac{\pi}{2}t^2\right) - t^9\right)$ where $0 \leqslant t \leqslant 1$ (shown below):

$$\int_C \left(2xy + e^{-x^2}\right) dx + \left(x^2 + y\cos\left(\frac{\pi}{2}y^2\right)\right) dy$$

(Hint: Are we allowed to use a different C?)



41. [S19-F] Find $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$ where $\mathbf{F} = \langle\sqrt{z}, -2x, \sqrt{y}\rangle$ and C is the parameterized curve from $(0,0,0)$ to $(2,4,16)$ given by (t, t^2, t^4) where $0 \leqslant t \leqslant 2$.

42. Let C be the closed curve in the plane that starts at $(6,0)$, then goes to $(6,6)$, then goes to $(-2,-2)$ then goes to $(-2,0)$ and finally returns to $(6,0)$. Find $\oint_C (e^y - y)dx + (xe^y + \arctan(y+\pi))dy.$

43. Let $C = \left(\sin(t), t\cos(t), \frac{1}{2}t^2\right)$ for $0 \leqslant t \leqslant 2\pi$. Find $\int_C e^x\,dx + dy + (x+2z)\,dz.$

44. Find $\int_C (2 + yz^{-1/2})\,ds$ where $C = \left(\frac{1}{3}t^3, 2(t-1)e^t, e^{2t}\right)$ for $0 \leqslant t \leqslant 1$.

45. [S18-E3] Consider the vector field $\mathbf{F}(x, y, z) = (4z^2)\mathbf{i} + (\cos z)\mathbf{j} + (8xz - y\sin z + 1)\mathbf{k}$.

a) Show that $\mathbf{F}$ is conservative by finding a potential function f such that $\mathbf{F} = \nabla f$.

b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ for $0 \leqslant t \leqslant \frac{1}{2}\pi$.

46. [S18-E3] Let C be the curve that goes (in order) from $(0, 0)$ to $(2, 0)$ to $(2, 2)$ and then returns to $(0, 0)$. Find

$$\oint_C \tan^{-1}(e^x)\, dx + xy\cos(y^2 - \tfrac{1}{3}y^3)\, dy.$$

47. [S18-E3] Find the flux of the vector field $\mathbf{F}(x, y, z) = \langle \frac{1}{2}y, \frac{1}{2}x, z \rangle$ in the negative z direction (downward $\mathbf{n}$) through the part of the surface $z = 4 - x^2 - y^2$ that lies above the xy-plane.

48. [S18-E3] Find the flux of the vector field $\mathbf{F}(x, y, z) = \langle x\sqrt{9 - x^2 - y^2}, y\sqrt{9 - x^2 - y^2}, z^2 \rangle$ in the negative z direction (downward $\mathbf{n}$) through the hemisphere $z = \sqrt{9 - x^2 - y^2}$.

49. [S17-F] Find a potential function for the vector field

$$\mathbf{F}(x, y, z) = \langle 2xy + z^2, x^2, 2xz + \pi\cos(\pi z) \rangle,$$

Further, compute the flow of $\mathbf{F}$ along the curve parameterized by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ for $0 \leqslant t \leqslant \pi$.

50. [S17-F] Find the work done by the vector field

$$\mathbf{F}(x, y) = \langle 2e^{\sin x} + y^2, x^2 + 2\tan^{-1}(y^2 + 1) \rangle$$

along the closed curve formed by $y = 0$, $x = 2$ and $y = x^3/4$, oriented counter-clockwise.

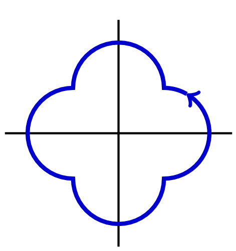
51. Let C be the curve (t, t^2) with $0 \leqslant t \leqslant 1$. Find $\int_C (y + x^2)\, dx + e^{x^2}\, dy$.

52. For $0 \leqslant t \leqslant \frac{1}{2}\pi$, let C be the curve $(2\cos(t) + \frac{1}{10}\sin(20t), \sin(t) + \frac{1}{10}\sin(20t))$. Find

$$\int_C \left(ye^x - 2x\right) dx + \left(e^x + \frac{2y}{1 + y^2}\right) dy$$

53. Let R be the region in the plane consisting of a 2×2 square centered at the origin with semicircles of radius 1 glued on each side; let C be the boundary of the curve of R traversed *counterclockwise* (see below). Find

$$\oint_C \big(\sin(2x) + \arctan(y)\big)\, dy - 4y\sin^2(x)\, dx$$



54. You have recently been hired to help design rides in an amusement park. One of the rides that you are working on involves a large bowl-like surface. In particular, the surface is $z = \frac{1}{2}(x^2 + y^2)$ for $0 \leqslant z \leqslant 2$ with distances measured in meters. You are working to ensure the supports in place will be sufficient to support the weight. In particular, the design calls for the density to increase with the height, i.e., $\delta(x, y, z) = z$ kg/meter^2. Find the total mass of the surface.

55. Let G be the part of the surface $z = x + y^3$ lying above R which is the triangle in the plane with corners at $(0, 0)$, $(1, 1)$ and $(0, 1)$. Find the flux for $\mathbf{F} = \big\langle \sin(2x - x^2), e^{xy}, 8xy \big\rangle$ through G where $\mathbf{n}$ is the upward pointing normal.

56. (a) Show that the following vector field is conservative:

$$\mathbf{F} = \left\langle \frac{1}{x^2 + 1} + 2xye^{x^4y^2}, x^2e^{x^4y^2} + \frac{2y}{y^2 + 1} \right\rangle.$$

(b) Let C be $(\sin(7\pi t), t^8 - t^5 + t^3 + \cos(8\pi t) - 1)$ for $0 \leqslant t \leqslant 1$. For $\mathbf{F}$ from (a), find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

57. Let G be the surface $f(x, y) = \frac{1}{3}x^3 + \frac{1}{2}y^2$ over the region $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 2$. Find $\iint_G (24zy - 12y^3)\, d(SA)$.

58. Let C be the curve which starts at $(0, 0, 0)$ and ends at $(1, 1, 1)$ parameterized by (t^b, t^{2b}, t^{3b}) where $b > 0$ is some parameter. Determine the *minimum* value of $\int_C (2xyz + 3x^2)\, dx + (x^2z + ze^y)\, dy + (x^2y + e^y)\, dz$.

59. [F19-E3] Evaluate $\oint_C (3y - \arctan(x^6))dx + (7x - \cos(\sqrt{y - 1}))dy$ where C is the circle $x^2 + y^2 = 1$ traversed counterclockwise.

60. [F19-E3] Find the work done by the force field $\mathbf{F} = (x + y)\,\mathbf{i} + xy\,\mathbf{j}$ in moving a particle along the curve C given by $x = 2t$, $y = t^2 - 1$, $0 \leqslant t \leqslant 2$.

61. [F19-E3] Let G be the part of the surface $z = e^{3x^2 - 2y^3}$ lying above the region R, where R is described by $y \geqslant 0$ and $1 \leqslant x^2 + y^2 \leqslant 4$. Find the flux for $\mathbf{F} = y^2\mathbf{i} + x\mathbf{j} + \dfrac{x + y}{x^2 + y^2}\mathbf{k}$ through G where $\mathbf{n}$ is the upward pointing normal.

62. [F19-F] Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ when $\mathbf{F} = \left\langle \frac{2x}{y}, \frac{1 - x^2}{y^2} \right\rangle$ and C is the path $(1 + \cos(\frac{\pi t}{2}), 2 - \cos(\frac{\pi t}{2}))$ for $0 \leqslant t \leqslant 1$.

63. [F19-F] A thin sheet of metal has the shape of the surface $z = 2x + y^2$, restricted to the portion that lies over the triangle with corners $(0, 0)$, $(2, 0)$ and $(2, 1)$ in the xy-plane. Assuming that the density is $\delta(x, y, z) = 1 + xy + z^2$, set up, **but do not evaluate**, the integral for calculating the total mass of the sheet. The answer needs to be an integral, with bounds, only in terms of x and y.