

Unit Root Test against the Exponential STAR Model

Hyeonwoo Kim

Let's consider a univariate exponential smooth transition autoregressive of order 1 (ESTAR(1))model.

$$y_t = y_{t-1} + \gamma y_{t-1} [1 - \exp(-\theta y_{t-1}^2)] + \varepsilon_t, \quad t = 1, \dots, T \quad (1)$$

, where $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. Equivalently, (1) can be rewritten as follows.

$$y_t = \{1 + \gamma [1 - \exp(-\theta y_{t-1}^2)]\} y_{t-1} + \varepsilon_t \quad (2)$$

Under the null of a unit root, we would have restrictions $\theta = 0$, and therefore,

$$H_0 : \Delta y_t = \varepsilon_t \quad (3)$$

Under the alternative of stationarity, θ should be strictly positive, and $0 < \exp(-\theta y_{t-d}^2) < 1$. One interesting fact is that y_t would follow a unit root process locally around $y_{t-1} = 0$. If y_{t-1} is very big, and thus $\exp(-\theta y_{t-1}^2) \approx 0$, y_t would become linear AR(1) process approximately with the stable coefficient $1 + \gamma$ as long as $-2 < \gamma < 0$. This test is based on the latter case.

The unit root test with the null of $\theta = 0$ is not feasible, since γ is not identified under the null hypothesis. Kapetanios *et al.* (2004), therefore, transformed (2) as follows.

$$\Delta y_t = \gamma y_{t-1} [1 - \exp(-\theta y_{t-1}^2)] + \varepsilon_t \quad (4)$$

, where $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. As a unit root test against stationary ESTAR(1), they proposed following t -statistic.

$$t_{NLADF} = \frac{\delta_{LS}}{s_{LS}} \quad (5)$$

, where δ_{LS} and s_{LS} is the OLS estimate and its standard error obtained from following regression.

$$\Delta y_t = \delta y_{t-1}^3 + error \quad (6)$$

, which is an Taylor series approximation of (4). It can be proven that t_{NLADF} (nonlinear ADF) has following asymptotic distribution.

$$\frac{\left[\frac{1}{4} W(1)^2 - \frac{3}{2} \int_0^1 W(r)^2 dr \right]}{\sqrt{\int_0^1 W(r)^6 dr}} \quad (7)$$

, where $W(r)$ is the standard Brownian motion defined on $r \in [0, 1]$. In order to deal with nonzero mean or trend data, we can use OLS residuals on a constant or constant and linear time trend. That is, we can use demeaned or demeaned and detrended data for the process in (5) and (6).

With serially correlated errors, we can use same t -statistics in (5) using OLS estimate and standard error from following regression.

$$\Delta y_t = \sum_{j=1}^p \phi_j \Delta y_{t-j} + \delta y_{t-1}^3 + error \quad (8)$$

Finally, the authors tabulated the critical values by stochastic simulations for the unit root test in both serially uncorrelated and correlated cases as follows.

Asymptotic Critical Values			
fractile	NLADF	NLADF _m	NLADF _{mτ}
1%	-2.82	-3.48	-3.93
5%	-2.22	-2.93	-3.40
10%	-1.92	-2.66	-3.13

The authors show that the NLADF test has superior power under the alternative of nonlinear stationarity via Monte Carlo simulation. And they also showed that using NLADF test we could reject the unit root in real interest rate more often, while we are unable to do so with linear ADF test.

Reference

Kapetanios, George, Youngcheol Shin, and Andy Snell (2003), "Testing for a Unit Root against Nonlinear STAR Models," *Journal of Econometrics*.