# How to Mistake a Trivial Fact About Probability For a Substantive Fact About Justified Belief

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It is sometimes thought that the lottery paradox and the paradox of the preface demand a uniform solution (Hunter (1996) and Foley (1992), for example). Let us see why that appears to be the case.

I am justified in believing that my lottery ticket—call it $t_1$—will not win, on statistical grounds. Those grounds apply equally to any other ticket, so I am justified in believing of any other ticket $t_i$ (let $i$ take values from 2 to 1000000) that it will not win. I am not, however, justified in believing the giant conjunctive proposition that $t_1$ will not win & $t_2$ will not win & $\ldots$ & $t_{1,000,000}$ will not win. On the contrary, I am justified in believing that some ticket will win, hence that one of those conjuncts is false. Suggested solution: justified belief is not closed under conjunction. It does not follow from the fact that I am justified in believing $p$ and justified in believing $q$ that I am justified in believing $p$ & $q$.

Many books are written sufficiently meticulously that the author is justified in everything he says and in the beliefs that his statements express; *a fortiori,* this is possible. Still, all but the most arrogant meticulous authors will believe that they have made some errors—and they will be justified in believing that at least one of their statements (and their beliefs so expressed) is false. Such a meticulous author might express this belief that there are errors in his book in the preface rendering his book as a whole inconsistent. So, the story goes, the meticulous author justifiably believes $p_i$ where '$i$' indexes in succession each of the propositions that he expresses in his book and yet he justifiably believes not-($p_1$& $\ldots$ &$p_n$)—

*a fortiori,* he cannot justifiably believe ($p_1 \& \ldots \& p_n$). Once again, it appears that we should conclude that justified belief is not closed under conjunction.

I will present an argument that appearances are deceptive in this case. The two paradoxes do not demand a uniform solution. It appears that way because of an often unobjectionable but *loose* manner in which we apply the term 'belief'. In fact, the lottery paradox is to be solved by denying that we are justified in believing of any ticket that it will not win, as Dana Nelkin and others have argued (Nelkin, 2000)—*strictly speaking,* we are merely justified in believing of any ticket that it is very *likely* not to win.[1] If we speak strictly, our solution to the paradox of the preface should be quite different from our solution to the lottery paradox.

Utterances of the form 'I believe that $p$' and similar forms ('I think that $p$', '$p$, I believe', 'I think so', etc.) often, I suggest, do not express belief in the proposition that $p$. They express, rather, a belief that $p$ is probable (more likely than not, we might say). Consider a stranger who asks one where the post office is. One does not know, but one has a vague idea that it is a mile to the right. Consequently, one says 'I believe that it is a mile down that way'. This is a perfectly proper utterance; I suggest that it is also not literally true. One does not believe categorically that the post office is a mile to the right; one believes that it is more likely there than not. Ironically, a sentence of the form 'I believe that $p$' is sometimes used to convey precisely that one does *not* believe that $p$, strictly speaking.[2] Even if I make an explicitly probabilistic self-ascription of belief, such as 'I believe that it is more likely than not that there are mice in the basement', it is entirely proper to report my belief without a probabilistic modifier—'He thinks that there are mice in the basement'. (Contrast with the

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[1]There might well be reasons for denying that justified belief is closed under conjunction in cases other than those presented by lotteries and humble prefaces. Weiner (2004) provides such cases; although his explicit conclusion is that *knowledge* is not closed under conjunction, his reasons arguably apply *mutatis mutandis* to justified belief.

I will not repeat Nelkin's arguments for her solution to the lottery paradox. My discussion provides additional, independent support for her conclusion.

[2]Another view is that the belief sentence that one utters *is* true, rather than being a falsehood used to communicate a truth. It is just that utterances of sentences apparently concerning belief in one proposition are sometimes made true by belief in a different but related proposition. It is not necessary to decide between these (and other) options in the philosophy of language for our purposes.

unacceptability of reporting my assertion 'There are mice in the basement, more likely than not' with 'He says that there are mice in the basement'.) Third-person belief ascriptions need not attribute belief in the proposition that an overly literal interpretation would suggest any more than their first-person counterparts.

What goes for the term 'belief' goes for any qualification of it—'justified belief', for instance. Just as 'belief that $p$' is loosely used to denote both belief that $p$ *strictly speaking* and mere belief that *probably* $p$, 'justified belief that $p$' is loosely used to denote both justified belief that $p$ strictly speaking and mere justified belief that probably $p$. This is a usage beyond reproach on many occasions for philosophers and non-philosophers alike. It would often be pedantic to explicitly draw the distinction between, for example, believing that it will rain and believing that it probably will rain. It is far from pedantic to draw that distinction in discussing paradoxes whose formulation involves belief ascription.

If one draws no distinction between categorical beliefs and their probabilistic counterparts, *of course* I am justified in believing that my lottery ticket will not win. And *of course* if I am justified in believing that of my own ticket $t_1$, I am justified in believing of any other ticket $t_i$ that it will not win. And yet I am clearly not justified in believing that $t_1$ will not win & $t_2$ will not win & ... & $t_{1,000,000}$ will not win. Continuing to use 'justified belief' loosely, the paradox of the preface looks very similar indeed to the lottery paradox. A meticulous author might well be justified in every belief that his book expresses, and also be justified in believing the negation of the giant conjunction of all the propositions expressed. And so the obvious solution to each paradox appears to be that justified belief is not closed under conjunction.

And this all really is obvious if one uses 'justified belief' loosely—it is obvious that "justified belief" is not closed under conjunction. But if one uses the phrase strictly, it is not at all obvious. It is clear that "justified belief" is not closed under conjunction, and that is the solution to "the lottery paradox" outlined above—which *strictly speaking* is not the lottery paradox at all even though *loosely speaking,* it might be expressed in exactly the same terms.

Strictly speaking, we are not justified in believing of any ticket that it will lose, as Nelkin and others have argued. We are justified in believing of any ticket that it will probably lose—and, loosely speaking, *that is to say* that we are justified in believing that it will lose.

If one has a justified belief that probably $p$ and a justified belief that probably $q$, then it does not follow that one has a justified belief that probably $p \& q$. One *does* have a justified belief that probably $p$ and probably $q$ since it is not justification that is not closed under conjunction—it is *probability.* It does not follow from the fact that each ticket will probably not win that all the tickets will probably not win—of course! However, there is a giant conjunctive propositions that *is* entailed by the set of propositions that $t_i$ will probably not win, $i$ taking values from 1 to 1000000. That giant proposition is that $t_1$ will probably not win & $t_2$ will probably not win & $\ldots$ & $t_{1000000}$ will probably not win. One is justified in believing that giant conjunctive proposition, just as, when rolling a die, one is justified in believing the conjunctive proposition that one will probably not roll a 1 or a 2 and one will probably not roll a 3 or a 4, each of whose conjuncts one justifiably believes, although one is not justified in believing that one will probably not roll a 1, 2, 3 or 4. That it is probability and not justification that is not closed under conjunction is completely obscured if one uses 'justified belief that $p$' indiscriminately to refer to justified belief that $p$ and mere justified belief that probably $p$.

Back to the preface. A meticulous author might very well be justified in believing that each statement in his book is probably true (loosely speaking, we can hence say that he is justified in believing each proposition that he expresses). The author is not justified in believing that probably all the statements in the book are true—indeed, he might very well be justified in believing that probably not-$(p_1 \& p_2 \& \ldots \& p_n)$, or even the stronger categorical counterpart of that proposition. If one uses 'justified belief that $p$' indiscriminately to refer to justified belief that $p$ and mere justified belief that probably $p$, again, the *obvious* solution to *this version* of the paradox of the preface is that "justified belief" is not closed under conjunction. But it is not because justified belief is not closed under conjunction, it is because probability is not closed under conjunction. Our author is most certainly justified in believing probably $p_1$ and probably $p_2$ and... and probably $p_n$; he is not justified in believing probably $(p_1 \& p_2 \& \ldots \& p_n)$.

What, then, of a version of the paradox of the preface that does not trade on a loose usage of 'justified belief'? We suppose that there is a meticulous author who is *really* justified in believing each and every one of the statements that he makes, and yet is really justified in believing that at least one of them is false. I am not sure that we have the materials for a genuine paradox of the preface—it is not at all clear that the hypothetical situation is possible. Often, such a meticulous author will not be justified in believing each and every statement in his book. At least one of them expresses an unjustified belief—he just does not know which one. Call the proposition in which he has an unjustified belief '$p$'. Since he is a meticulous author, he does have a justified belief that *probably* $p$...

If there is no such statement—if the author really is justified in believing each and every proposition that he expresses—then he would also be justified in believing their conjunction. That fact is arguably consistent with him also being justified in believing that *probably* at least one of his claims is false. Perhaps that has turned out to be the case, despite his meticulous nature, in all his previous books, although this time he has hit the jackpot of exceptionless truth.

## References


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