Ground detection by a single electromagnetic far-field measurement

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\textbf{A B S T R A C T}

We consider detecting objects on a flat ground by using the electromagnetic (EM) measurement made from a height. Our study is conducted in a very general and practical setting. The number of the target scatterers is not required to be known in advance, and each scatterer could be either an inhomogeneous medium or an impenetrably perfectly conducting (PEC) obstacle. Moreover, there might be multiscale components of small-size and extended-size (compared to the detecting wavelength) presented simultaneously. Some a priori information is required on scatterers of extended-size. The inverse problem is nonlinear and ill-conditioned. We propose a “direct” locating method by using a single EM far-field measurement. The results extend those obtained in [17,18] for locating multiscale EM scatterers located in a homogeneous space.

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1. Introduction

This paper concerns locating objects on a ground by using the electromagnetic (EM) scattering measurement made from a height. In Fig. 1, we give a schematic illustration of our study, where one wants to detect the multiple objects on the ground $\mathcal{G}$. To that end, one sends certain detecting wave fields and then measures the scattered wave fields from a height, from which to infer knowledge about the target objects. A practical scenario for our study is the scoutplane detection in the battlefield.

In what follows, we present the mathematical formulation for the current study. The detecting waves are chosen to be the time-harmonic electromagnetic plane waves of the following form

$$E^i(x) = p e^{i\omega x \cdot d}, \quad H^i(x) = \frac{1}{i\omega} \nabla \times E^i(x), \quad x \in \mathbb{R}^3$$ (1.1)

where $i = \sqrt{-1}$, $\omega \in \mathbb{R}_+$ denotes the frequency, $d \in \mathbb{S}^2 := \{ x \in \mathbb{R}^3 \mid |x| = 1 \}$ denotes the impinging direction, and $p \in \mathbb{R}^3$ denotes the polarization with $p \cdot d = 0$. $E^i$ and $H^i$ are entire solutions to the Maxwell equations in the free space

$$\nabla \times E^i - i\omega H^i = 0, \quad \nabla \times H^i + i\omega E^i = 0.$$
The ground $\mathcal{G}$ is assumed to be perfectly electric conducting (PEC). The EM waves cannot penetrate inside the ground and propagate only in the space above the ground. If there is no object presented on the ground, one would have a reflected wave field $E_{0\mathcal{G}}^i$ such that the total wave field $E = E^i - E_{0\mathcal{G}}^i$ satisfies the following PEC boundary condition
\[
\nu \wedge E = \nu \wedge (E^i - E_{0\mathcal{G}}^i) = 0 \quad \text{on } \mathcal{G},
\]
(1.2)
where $\nu$ is the unit upward normal vector to $\mathcal{G}$. If $\mathcal{G}$ is flat, the reflected wave field $E_{0\mathcal{G}}^i$ is well-understood through the work [22,23], and if $\mathcal{G}$ is non-flat/rough, the reflection would be much more complex. Throughout the present work, we assume that $\mathcal{G}$ is flat and shall leave the rough case for a future study. Furthermore, without loss of generality, we assume that $\mathcal{G} := \{x := (x_1, x_2, x_3) \in \mathbb{R}^3 \}$.

Denote $\mathbb{R}^3_+ := \{x := (x_1, x_2, x_3) \in \mathbb{R}^3; x_3 > 0\}$, and $\mathbb{R}^3 := \mathbb{R}^3_+ \cap \mathbb{S}^2$. Moreover, we let $\Pi$ denote the usual reflection with respect to $\mathcal{G}$, i.e., $\Pi v = (v_1, v_2, -v_3)$ for a generic 3-vector $v = (v_1, v_2, v_3)$. Then, we have that (cf. [22,23])
\[
E_{0\mathcal{G}}^i = \Pi \circ E^i \circ \Pi.
\]
(1.3)

Next, we consider that there are EM objects presented on the ground. Let $\psi(x')$, $x' \in \mathbb{R}^2$, be a non-negative Lipschitz continuous function such that $\psi(x') = 0$ for $|x'| > R$, where $R$ is a large enough positive constant. Let $\Sigma' := \{x' \in \mathbb{R}^2; \psi(x') > 0\}' := \bigcup_{j=1}^{j_{\text{total}}} \Sigma_j'$, where $\Sigma_j'$, $j = 1, 2, \ldots, l$ denote the simply connected components of $\Sigma'$. Define
\[
\Sigma_j^+ := \{(x', x_3) \in \mathbb{R}^3_+; x' \in \Sigma_j', \ 0 < x_3 < \psi(x')\}, \quad 1 \leq j \leq l; \quad \Sigma^+ := \bigcup_{j=1}^{l} \Sigma_j^+.
\]
(1.4)
Each $\Sigma_j^+$, $1 \leq j \leq l$, represents an EM object on the ground, and will be referred to as a scatterer in the sequel. Let $\varepsilon_j$, $\mu_j$, and $\Sigma_j$ be the EM parameters for the object supported in $\Sigma_j$, respectively, representing the electric permittivity, magnetic permeability and electric conductivity. It is assumed that $\varepsilon_j$, $\mu_j$, and $\Sigma_j$ are all constants, satisfying $0 < \varepsilon_j < +\infty$, $0 < \mu_j < +\infty$, and $0 \leq \Sigma_j \leq +\infty$. Furthermore, it is assumed that $|\varepsilon_j - 1| + |\mu_j - 1| + |\Sigma_j| > 0$ for $j = 1, 2, \ldots, l$. If $\Sigma_j = +\infty$, then $\Sigma_j^+$ is taken to be a PEC obstacle, disregarding $\varepsilon_j$ and $\mu_j$. In the free space, $\varepsilon = \mu = 1$ and $\Sigma = 0$. We set
\[
(\varepsilon(x), \mu(x), \Sigma(x)) := \begin{cases} 
(\varepsilon_j, \mu_j, \Sigma_j) & \text{when } x \in \Sigma_j^+, \ j = 1, 2, \ldots, l; \\
(1, 1, 0) & \text{when } x \in \mathbb{R}^3_+ \setminus \Sigma^+.
\end{cases}
\]
(1.5)

The presence of the scatterer $(\Sigma; \varepsilon, \mu, \Sigma)$ on the ground would further perturb the propagation of the EM field $E^i - E_{0\mathcal{G}}^i$, inducing the so-called scattered wave field $E^s$ in $\mathbb{R}^3_+$. The scattered wave field is radiating in nature, characterized by the Silver–Müller radiation condition
\[
\lim_{|x| \to +\infty} |x| \left| (\nabla \wedge E^s)(x) \wedge \frac{x}{|x|} - i\omega E^s(x) \right| = 0,
\]
(1.6)
which holds uniformly for all directions $\hat{x} := x/|x| \in \mathbb{S}^2$. The total electric wave field $E := E^i - E_{0\mathcal{G}}^i + E^s$ is governed by the following Maxwell system
\[
\nabla \wedge E - i\omega \varepsilon \mu H = 0, \quad \nabla \wedge H + i\omega (\varepsilon + i\Sigma/\omega)E = 0 \quad \text{in } \mathbb{R}^3_+.
\]
(1.7)
where $\varepsilon$, $\mu$, and $\Sigma$ are given in (1.5). Similar to (1.2), we have that
\[
\nu \wedge E = \nu \wedge (E^i - E_{0\mathcal{G}}^i + E^s) = 0 \quad \text{on } \mathcal{G}.
\]
(1.8)
We seek a pair of solutions \((E, H) \in H_{\text{loc}}(\text{curl}, \mathbb{R}^2) \land H_{\text{loc}}(\text{curl}, \mathbb{R}^2)\) to the scattering system (1.6)-(1.8). Particularly, the radiating wave field \(E^s(x)\) has the following asymptotic expansion as \(|x| \to +\infty\),

\[
E^s(x) = \frac{e^{i\omega|x|}}{|x|} A\left(\frac{x}{|x|}; d, p, \omega\right) + \mathcal{O}\left(\frac{1}{|x|^2}\right),
\]

(1.9)

\(A(\hat{x}; d, p, \omega)\) with \(\hat{x} := x/|x| \in S^3_+\) is known as the electric far-field pattern, which encodes the scattering measurement illustrated in Fig. 1.

The ground detection problem can be abstractly formulated as

\[
\mathcal{F}\left(\Sigma^+; \varepsilon, \mu, \Sigma\right) = A(\hat{x}; d, p, \omega),
\]

(1.10)

where \(\mathcal{F}\) is the operator sending the scatterer to the corresponding far-field pattern, defined by the Maxwell system (1.6)-(1.8). It is easily verified that \(\mathcal{F}\) is nonlinear and moreover it is ill-conditioned since \(\mathcal{F}\) is completely continuous (cf. [12]). In what follows, \(A(\hat{x}; d, p, \omega)\) is always assumed to be given with all \(\hat{x} \in S^3_+\). Furthermore, if \(d \in S^3_+\), \(p \in \mathbb{R}^3\) and \(\omega \in \mathbb{R}^+\) are all fixed, then \(A(\hat{x}; d, p, \omega)\) is called a single EM measurement; otherwise, it is called multiple EM measurements.

In practice, a single EM measurement can be obtained by sending a single incident plane wave, and then collecting the scattered electric wave in all the observation angles. Throughout the present study, we shall take a single EM measurement for the ground detection. Moreover, our study shall be conducted in a very general and practical setting. The number of the target scatterers is not required to be known in advance, and each scatterer could be either an inhomogeneous medium or an impenetrable perfectly conducting (PEC) obstacle. Furthermore, there might be multiscale components of small-size and extended-size (compared to the detecting wavelength) presented simultaneously. Some realistic a priori information is required on scatterers of extended-size. We propose a “direct” locating method without any inversion involved. To our best knowledge, both the direct scattering model and the inverse scattering schemes are new to the literature. The results extend those obtained in [17,18] for locating multiscale EM scatterers located in a homogeneous space. The present study is closely related to the inverse electromagnetic scattering problems from rough surfaces; see, e.g., [6–11,16]. We also refer to [3–6,12–14,19–21,25–27] for the recent progress on the inverse scattering theory and numerical study. It remarked that for highly conductive objects with large \(\Sigma\) (1.7) reduces to the eddy current model, which is justified in [1]. A similar asymptotic formalism for detecting and characterizing small conductive inclusions from electromagnetic induction data is developed in a recent work [2]. The mechanisms proposed in this work could be migrated to the eddy current model to detect complicated settings, e.g., multiple regular-size or even multiscale conductive inclusions.

The rest of the paper is organized as follows. In Section 2, we present some results concerning the direct scattering problem for our subsequent use. Section 3 is devoted to the inverse scattering scheme. Numerical results and discussion are presented in Section 4.

2. Scattering from multiscale ground objects

In this section, we consider the scattering from multiscale ground objects. In order to ease the exposition, throughout the rest of the paper, we assume that \(\omega \sim 1\), and hence the size of an EM object can be interpreted in terms of its Euclidean diameter.

Let \(\Sigma^+\) and \(\bigcup_{j=1}^l (\Sigma^+_j; \varepsilon_j, \mu_j, \Sigma_j)\) be as introduced in (1.4) and (1.5). We further assume that there exists \(l_s, l_r \in \mathbb{N} \cup \{0\}\) such that \(l_r + l_s = l\). Let

\[
(\Sigma^+; \varepsilon, \mu, \Sigma) = (\Sigma^+_l; \varepsilon, \mu, \Sigma) \cup (\Sigma^+_s; \varepsilon, \mu, \Sigma),
\]

(2.1)

where by reordering if necessary, we assume that

\[
(\Sigma^+_l; \varepsilon, \mu, \Sigma) = \bigcup_{j=1}^{l_r} (\Sigma^+_j; \varepsilon_j, \mu_j, \Sigma_j) \quad \text{and} \quad (\Sigma^+_s; \varepsilon, \mu, \Sigma) = \bigcup_{j=l_r+1}^l (\Sigma^+_j; \varepsilon_j, \mu_j, \Sigma_j).
\]

(2.2)

Furthermore, we assume that \(\text{diam}(\Sigma^+_j) \sim 1\) for \(1 \leq j \leq l_r\), whereas \(\Sigma^+_l \sim \rho \ll 1\) for \(l_r + 1 \leq j \leq l\). \(\Sigma^+_l\) contains all the extended-size scatterer components, whereas \(\Sigma^+_s\) contains all the small-size scatterer components. For the subsequent discussion, we also classify the target scatterer \((\Sigma^+_s; \varepsilon, \mu, \Sigma)\) in terms of physical properties as follows

\[
(\Sigma^+; \varepsilon, \mu, \Sigma) = (\Sigma^+_M; \varepsilon, \mu, \Sigma) \cup (\Sigma^+_p; \varepsilon, \mu, \Sigma),
\]

(2.3)

where \(\Sigma(x) < +\infty\) when \(x \in \Sigma^+_M\), and \(\Sigma(x) = +\infty\) when \(x \in \Sigma^+_p\). That is, \(\Sigma^+_M\) contains all the inhomogeneous medium components, whereas \(\Sigma^+_p\) contains all the PEC obstacle components of the target scatterer. We remark that it may happen
that \( \Sigma_M^+ = \emptyset \) or \( \Sigma_P^+ = \emptyset \). According to our discussion in Section 1, the EM scattering corresponding to the target scatterer described in (2.3) is governed by the following Maxwell system

\[
\begin{align*}
\nabla \times E - i \omega \mu H &= 0, \quad \nabla \times H + i \omega (\varepsilon + \frac{1}{\omega} \Sigma) E &= 0 \quad \text{in} \ \mathbb{R}^3 \setminus \overline{\Sigma_P^+},
\nE(x) &= E^i(x) - E^o(x) + E^\delta(x) \quad x \in \mathbb{R}^3 \setminus \overline{\Sigma_P^+},
\nu \times E &= 0 \quad \text{on} \ \partial(\mathbb{R}^3 \setminus \overline{\Sigma_P^+}),
\lim_{|x| \to +\infty} \left| \left( \nabla \times E^\delta(x) \right) \times \frac{x}{|x|} - i \omega E^\delta(x) \right| &= 0.
\end{align*}
\]

Next, we introduce an auxiliary scattering system defined over the whole space \( \mathbb{R}^3 \). Starting from now on, for an EM scatterer \((O^+; \varepsilon_0, \mu_0, \Sigma_0)\), where \( O^+ \subset \mathbb{R}^3, \varepsilon_0, \mu_0 \), and \( \Sigma_0 \) are all constants with \( \Sigma_0 \) possibly being \( +\infty \), we define \((O^-; \varepsilon_0, \mu_0, \Sigma_0) = (\Pi O^+; \varepsilon_0, \mu_0, \Sigma_0)\) and \((O; \varepsilon_0, \mu_0, \Sigma_0) = (O^+; \varepsilon_0, \mu_0, \Sigma_0) \cup (O^-; \varepsilon_0, \mu_0, \Sigma_0)\). Here, \( \Pi O^+ \) denotes the reflected domain of \( O^+ \) with respect to \( \mathcal{H} \). For the ground scatterer \((\Sigma^+; \varepsilon, \mu, \Sigma)\) in (2.3), we consider the following scattering system in the whole space \( \mathbb{R}^3 \) corresponding to the scatterer \((\Sigma; \varepsilon, \mu, \Sigma) = (\Sigma_M; \varepsilon, \mu, \Sigma) \cup (\Sigma_P; \varepsilon, \mu, \Sigma)\)

\[
\begin{align*}
\nabla \times \mathcal{E} - i \omega \mu \mathcal{H} &= 0, \quad \nabla \times \mathcal{H} + i \omega (\varepsilon + \frac{1}{\omega} \Sigma) \mathcal{E} &= 0 \quad \text{in} \ \mathbb{R}^3 \setminus \overline{\Sigma_P},
\n\mathcal{E}(x) &= E^i(x) - E^o(x) + \mathcal{E}^\delta(x) \quad x \in \mathbb{R}^3 \setminus \overline{\Sigma_P},
\nu \times \mathcal{E} &= 0 \quad \text{on} \ \partial \Sigma_P,
\lim_{|x| \to +\infty} \left| \left( \nabla \times \mathcal{E}^\delta(x) \right) \times \frac{x}{|x|} - i \omega \mathcal{E}^\delta(x) \right| &= 0.
\end{align*}
\]

We refer to [24] for the well-posedness of the Maxwell system (2.5).

We are in a position to establish an important relationship between the two scattering systems (2.4) and (2.5).

**Theorem 2.1.** Let \((E, H) \in H_{\text{loc}}(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P^+}) \cap H_{\text{loc}}(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}),\) and \((\mathcal{E}, \mathcal{H}) \in H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}) \cap H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P})\) be, respectively, solutions to the scattering systems (2.4) and (2.5). Then we have that

\[
\mathcal{E} = -\Pi \circ \mathcal{E} \circ \Pi, \quad \mathcal{H} = \Pi \circ \mathcal{H} \circ \Pi,
\]

and

\[
E = \mathcal{E}|_{\mathbb{R}^3}, \quad H = \mathcal{H}|_{\mathbb{R}^3}.
\]

**Proof.** Set

\[
\mathcal{E} = -\Pi \circ \mathcal{E} \circ \Pi, \quad \mathcal{H} = \Pi \circ \mathcal{H} \circ \Pi.
\]

Since \((\mathcal{E}, \mathcal{H}) \in H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}) \cap H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}),\) it is easily seen that \((\mathcal{E}, \mathcal{H}) \in H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}) \cap H(\text{curl}; \mathbb{R}^3 \setminus \overline{\Sigma_P}).\) Moreover, since \(\mathcal{E}\) and \(\mathcal{H}\) satisfy the Maxwell equations,

\[
\nabla \times \mathcal{E} - i \omega \mu \mathcal{H} = 0, \quad \nabla \times \mathcal{H} + i \omega (\varepsilon + \frac{1}{\omega} \Sigma) \mathcal{E} = 0 \quad \text{in} \ \mathbb{R}^3 \setminus \overline{\Sigma_P},
\]

one can verify by straightforward computations that

\[
\nabla \times \mathcal{E} - i \omega \mu \mathcal{H} = 0, \quad \nabla \times \mathcal{H} + i \omega (\varepsilon + \frac{1}{\omega} \Sigma) \mathcal{E} = 0 \quad \text{in} \ \mathbb{R}^3 \setminus \overline{\Sigma_P}.
\]

Next, by the geometric interpretation of vector product, we note the following fact for any two vectors \(a\) and \(b\),

\[
a \wedge b = (\Pi a) \wedge (\Pi b).
\]

Hence, for any \(x \in \partial \Sigma_P\) and the corresponding normal vector \(n(x)\), by using (2.10), we have

\[
\nu(x) \wedge (\Pi \circ \mathcal{E} \circ \Pi)(x) = (\Pi^{-1} \nu)(x) \wedge (\mathcal{E} \circ \Pi)(x).
\]

On the other hand, by noting \(\Pi \Sigma_P = \Sigma_P\), we see that \(\Pi^{-1} \nu(x) = (\nu \circ \Pi)(x)\), which together with (2.11) further implies that

\[
(\nu \wedge \mathcal{E})(x) = -\nu(x) \wedge (\Pi \circ \mathcal{E} \circ \Pi)(x) = -(\nu \wedge \mathcal{E})(\Pi x) = 0, \quad x \in \partial \Sigma_P.
\]

Now, we set

\[
\mathbf{E} := \mathcal{E} + \Pi \circ \mathcal{E} \circ \Pi \quad \text{and} \quad \mathbf{H} := \mathcal{H} - \Pi \circ \mathcal{H} \circ \Pi.
\]
By \((2.9)-(2.13)\), we readily see that
\[
\nabla \wedge E - i\omega \mu H = 0, \quad \nabla \wedge H + i\omega \left(\varepsilon + i \frac{\Sigma}{\omega}\right)E = 0 \quad \text{in } \mathbb{R}^3 \setminus \Sigma_p, \quad \nu \wedge E = 0 \quad \text{on } \partial \Sigma_p.
\]
(2.14)

On the other hand, by \((2.4)\),
\[
\epsilon'(x) = E'(x) - E'_{\parallel} + \epsilon'_{\parallel}(x), \quad x \in \mathbb{R}^3 \setminus \Sigma_p.
\]
(2.15)

It is directly verified that
\[
E = \epsilon' + \Pi \circ \epsilon' \circ \Pi = \epsilon'_{\parallel} + \Pi \circ \epsilon'_{\parallel} \circ \Pi.
\]
(2.16)

Since \(\epsilon'_{\parallel}\) satisfies the Silver–Müller radiation condition, \((2.16)\) clearly implies that \(E\) satisfies the Silver–Müller radiation condition, namely,
\[
\lim_{|x| \to +\infty} \left| \left( \nabla \wedge E \right)(x) \wedge \frac{x}{|x|} - i\omega E(x) \right| = 0.
\]
(2.17)

By the well-posedness of the Maxwell system \((2.16)-(2.17)\), we immediately have that
\[
E = H = 0,
\]
(2.18)

which in turn implies \((2.6)\).

Finally, let \(v_0 = (0, 0, 1)\) be the unit upward normal vector to \(\mathcal{G} \setminus \Sigma_p\). By using \((2.6)\) and straightforward calculation, one can show that
\[
v_0 \wedge \epsilon' = 0 \quad \text{on } \mathcal{G} \setminus \Sigma_p,
\]
(2.19)

which together with the fact that \(\nu \wedge \mathcal{G} = 0\) on \(\partial \Sigma_p\) in \((2.5)\) that
\[
v \wedge \epsilon' = 0 \quad \text{on } \partial \left(\mathbb{R}^3 \setminus \Sigma_p\right).
\]
(2.20)

By using \((2.20)\), and comparing the Maxwell systems \((2.4)\) and \((2.5)\), one easily seen that \(E\) and \(H\) in \((2.4)\) are actually the restrictions of \(\epsilon'\) and \(\mathcal{H}\) in \((2.5)\) in the upper half space.

The proof is completed. \(\Box\)

By Theorem 2.1, one readily has

**Corollary 2.1.** Let \(A(\hat{x}; d, p, \omega), \hat{x} \in S^2_+\), and \(\mathcal{A}(\hat{x}; d, p, \omega), \hat{x} \in S^2\), be the scattering amplitudes corresponding to the Maxwell systems \((2.4)\) and \((2.5)\), respectively. Then we have
\[
\mathcal{A} = -\Pi \circ \mathcal{A} \circ \Pi \quad \text{and} \quad A = \mathcal{A}|_{S^2_+}.
\]
(2.21)

Next, we consider the scattering from multiple small and extended scatterers, respectively. We first consider the scattering from multiple small scatterers of the form \((\Sigma^+_j; \varepsilon, \mu, \Sigma)\) in \((2.2)\) by taking \(l_s = 0\). Let the small scatterer components be given as
\[
\Sigma^+_j = z_j + \rho D^+_j, \quad j = 1, 2, \ldots, l_s,
\]
(2.22)

where \(z_j \in \mathcal{G}\), and \(D^+_j, \ j = 1, 2, \ldots, l_s\), are simply-connected \(C^2\) domains in \(\mathbb{R}^3_+\) that contain the origin and
\[
L_s := \min_{1 \leq j, j' \leq l_s, j \neq j'} \text{dist}(z_j, z_j') \gg 1.
\]
(2.23)

We have

**Lemma 2.1.** Let \((\Sigma^+_j; \varepsilon, \mu, \Sigma)\) be described as above and \(A(\hat{x}) := A(\hat{x}; d, p, \omega, (\Sigma^+_j; \varepsilon, \mu, \Sigma)), \hat{x} \in S^2_+\). Set
\[
A(\hat{x}) = \begin{cases} 
A(\hat{x}) & \text{when } \hat{x} \in S^2_+ \\
-\Pi A(\Pi \hat{x}) & \text{when } \hat{x} \in S^2_-.
\end{cases}
\]
(2.24)

Then we have
\[
A(\hat{x}) = \sum_{j=1}^{l_s} e^{i\omega(d-\hat{x})-z_j} \left[ (\omega \rho)^2 \left( \sum_{m=-1, 0, 1} a_{1,m} U^m_1(\hat{x}) + b_{1,m} V^m_1(\hat{x}) \right) + O((\omega \rho)^4) \right] + O(L_s^{-1}),
\]
(2.25)
where $U_{1}^{m}$ and $V_{1}^{m}$ are the vectorial spherical harmonics (cf. [12])

\[
\begin{cases}
U_{n}^{m}(\hat{x}) := \frac{1}{\sqrt{n(n+1)}} \text{Grad} Y_{n}^{m}(\hat{x}) & n \in \mathbb{N}, \ m = -n, \ldots, n, \\
V_{n}^{m}(\hat{x}) := \hat{\theta} \wedge U_{n}^{m}(\hat{x}),
\end{cases}
\tag{2.26}
\]

and $a_{1,m}^{j}$ and $b_{1,m}^{j}$ are constants independent of $\omega \varphi$, $z_{j}$ and $l_{s}$.

**Proof.** By Corollary 2.1, we clearly have that

\[
A(\hat{x}) = \mathcal{A}(\hat{x}; d, p, \omega, (\Sigma_{2}^{\ast}; \varepsilon, \mu, \Sigma)), \quad \hat{x} \in \mathbb{S}^{2}.
\]

Hence, Lemma 2.1 follows directly from Lemmas 3.1 and 3.3 in [17]. \( \square \)

In the rest of this section, we consider the scattering from multiple extended scatterers of the form $(\Sigma_{1}^{+}; \varepsilon, \mu, \Sigma)$ in (2.2) by taking $l_{s} = 0$. Let $\Gamma^{+}$ be a simply-connected domain in $\mathbb{R}^{3}_{+}$ that touches $\mathcal{G}$ at the origin. Henceforth, we denote by $\mathcal{R}^{+} := \mathcal{R}(\theta) \in SO(3)$ the 3D rotation matrix around the $x_{3}$-axis. Here, $\theta \in [0, 2\pi]$ denotes the corresponding Euler angle. Moreover, we define a dilation/scaling operator as follows

\[
A_{r} \Gamma^{+} := \{rx; x \in \Gamma^{+}\}, \quad r \in \mathbb{R}_{+}.
\]

Next, we introduce

\[
\mathcal{D} := \{\Gamma_{j}^{+}\}_{j=1}^{l'}, \quad l' \in \mathbb{N}
\tag{2.27}
\]

where $\Gamma_{j}^{+} \subset \mathbb{R}^{3}_{+}$ and $\mathcal{F}_{j}^{+}$ is a bounded simply-connected Lipschitz domain that contains the origin. $\mathcal{D}$ is called a base scatterer class, and each base scatterer $\Gamma_{j}^{+}$, $1 \leq j \leq l'$, could be an inhomogeneous medium or a PEC obstacle. Next, we introduce the multiple extended scatterers for our study via the base class $\mathcal{D}$ in (2.27). Set $r_{j} \in \mathbb{R}_{+}$ such that

\[
r_{j} \in [R_{0}, R_{1}], \quad 0 < R_{0} < R_{1} < +\infty, \quad R_{0}, R_{1} \sim O(1),
\]

and moreover, let $\theta_{j} \in [0, 2\pi]$, $j = 1, 2, \ldots, l_{s}$, be $l_{s}$ Euler angles. For $z_{j} \in \mathbb{R}^{3}$, we let

\[
\Sigma_{r}^{+} = \bigcup_{j=1}^{l_{s}} \Sigma_{j}^{+}, \quad \Sigma_{j}^{+} := z_{j} + \mathcal{R}_{j}^{+} A_{r_{j}} \Gamma_{j}^{+}, \quad \Gamma_{j}^{+} \in \mathcal{D}, \quad \mathcal{R}_{j}^{+} := \mathcal{R}(\theta_{j}).
\tag{2.28}
\]

The EM parameters of $\Sigma_{j}^{+}$ is inherited from those of the base scatterer $\Gamma_{j}^{+}$. For technical purpose, we impose the following sparsity assumption on the extended scatterer $\Sigma_{r}^{+}$ introduced in (2.28),

\[
L_{e} = \min_{j \neq f, 1 \leq j, f \leq l_{s}} \text{dist}(\Sigma_{j}^{+}, \Sigma_{f}^{+}) \gg 1.
\tag{2.29}
\]

**Lemma 2.2.** Let $(\Sigma_{1}^{+}; \varepsilon, \mu, \Sigma)$ be described as above. Then we have

\[
\begin{align*}
A_{r}(\hat{x}; d, p, \omega, \bigcup_{j=1}^{l_{s}} (\Sigma_{j}^{+}; \varepsilon_{j}, \mu_{j}, \Sigma_{j}))
&= \sum_{j=1}^{l_{s}} e^{i \omega d(\hat{x}) z_{j}} r_{j} A_{r}(\mathcal{R}_{j}^{T} \hat{x}; (\mathcal{R}_{j}^{T})^{T} d, (\mathcal{R}_{j}^{T})^{T} p, r_{j} \omega, \bigcup_{j=1}^{l_{s}} (\Gamma_{j}^{+}; \varepsilon_{j}, \mu_{j}, \Sigma_{j})).
\end{align*}
\tag{2.30}
\]

**Proof.** The proof follows from a similar argument to that for Proposition 3.1 in [18] by using change of variables for the corresponding Maxwell equations. The only point one should note with attention is that for $z \in \mathcal{G}$, one clearly has $z = \Pi z$. \( \square \)

### 3. Locating multiscale ground objects

With the preparations made earlier, we are ready to develop the inverse scattering scheme of detecting the multiscale ground objects introduced in (2.1). Our result extends those developed in [17,18] for locating multiscale space objects to this interesting case of ground detection.

We first consider locating multiple small ground objects of $(\Sigma_{1}^{+}; \varepsilon, \mu, \Sigma)$ as described in (2.22)-(2.23). Let $A(\hat{x}) := A(\hat{x}; d, p, \omega, (\Sigma_{1}^{+}; \varepsilon, \mu, \Sigma))$, $\hat{x} \in \mathbb{S}^{2}_{+}$. The next theorem underlies the foundation for our first locating scheme.
Theorem 3.1. For \( z \in \mathcal{G} \), we define
\[
I_s(z) := \frac{1}{\|A(\hat{x})\|_{L^2(S^2)}^2} \sum_{T=U,V} \sum_{m=-1,0,1} \left( |\langle A(\hat{x}), e^{i\omega(t \cdot \hat{x}) z T^m_1(x)} \rangle_{L^2(S^2)}|^2 + |\langle A(\Pi \hat{x}), e^{i\omega(t \cdot \hat{x} + z T^m_1(x))} \rangle_{L^2(S^2)}|^2 \right),
\]
where \( A(\hat{x}) \) is given in (2.24). Then, \( z_j, j = 1, \ldots, l_s \), are local maximizers for \( I_s(z) \).

Proof. By using Lemma 2.1, the proof follows from a similar argument to that of Theorem 2.1 in [17]. \( \square \)

Based on Theorem 3.1 we can formulate Scheme S to locate the multiple small scatterer components of \((\Sigma^+_v; \varepsilon, \mu, \Sigma)\) in (2.22)-(2.23).

### Scheme S

| Step 1 | For an unknown scatterer \((\Sigma^+_v; \varepsilon, \mu, \Sigma)\), collect the far-field pattern \( A(\hat{x}) \) by sending a single pair of detecting plane waves (1.1). |
| Step 2 | Select a sampling region with a mesh \( \mathcal{T}_s \) containing \( \mathcal{G} \cap \Sigma^+_v \). |
| Step 3 | For each sampling point \( z \in \mathcal{T}_s \), calculate \( I_s(z) \) in (3.1). |
| Step 4 | Locate all the local maximizers of \( I_s(z) \) on \( \mathcal{T}_s \), which represent the locations of the scatterer components. |

Next, we consider the locating of multiple extended scatterers of \((\Sigma^+_v; \varepsilon, \mu, \Sigma)\) in (2.28)-(2.29). To that end, we need assume that the base scatterer class \( \mathcal{D} \) in (2.27) is known in advance. Let \( \mathcal{D} \) denote the set consisting of the scatterers of the form \( \mathcal{D}^+ \Sigma^+_v \) with all possible \( \mathcal{D}^+ \in SO(3) \) around the \( x_3 \)-axis, \( r \in [R_0, R_1] \) and \( \Sigma^+_v \in \mathcal{D} \). Then, we introduce \( \mathcal{D} \) with
\[
\mathcal{D} := \{ \tilde{F}_j^+ \}_{j=1}^N, \quad l'' \in \mathbb{N}
\]
such that \( \mathcal{D} \) is an \( \varepsilon \)-net of \( \mathcal{D} \), where \( \varepsilon \in \mathbb{R}_+ \) is sufficiently small. Moreover, it is assumed that \( \mathcal{D} \) is admissible in the following sense:

(a) \( \|A(\hat{x}; d, p, \omega, \tilde{F}_j^+)\|_{L^2(S^2)} \geq \|A(\hat{x}; d, p, \omega, \tilde{F}_j^+)\|_{L^2(S^2)} \), \( 1 \leq j \leq l'' - 1 \);

(b) \( A(\hat{x}; d, p, \omega, \tilde{F}_j^+) \neq A(\hat{x}; d, p, \omega, \tilde{F}_{j'}^+) \), \( 1 \leq j, j' \leq l'' \).

The first condition above can be fulfilled by reordering if necessary, whereas the second condition is the related to the uniqueness in the corresponding inverse problem. However, the uniqueness result is not available in the literature. Nevertheless, even if two base scatterers \( \tilde{F}_j^+ \) and \( \tilde{F}_{j'}^+ \) produce the same the far-field pattern, the locating scheme developed in the sequel would still work, and the only problem one would encounter is that if both of them are presented as the target ground objects, then the locating scheme cannot distinguish them.

Theorem 3.2. For \( z \in \mathcal{G} \), we define
\[
I'_s(z) := \frac{1}{\|A(\hat{x}; \tilde{F}_j^+\|_{L^2(S^2)}} |\langle A(\hat{x}; \Sigma^+_v), e^{i\omega(t \cdot \hat{x}) z A(\hat{x}; \tilde{F}_j^+)} \rangle_{L^2(S^2)}|, \quad j = 1, 2, \ldots, l''.
\]
Consider the indicator function \( I'_s \) introduced above. If for some scatterer component, say \( \Sigma^+_j^+ \), \( 1 \leq j \leq l, \) of \((\Sigma^+_v; \varepsilon, \mu, \Sigma)\), one has that
\[
d_H(\Sigma^+_j^+, \tilde{F}_j^+) \leq \varepsilon,
\]
where \( d_H \) denotes the Hausdorff distance. Then
\[
I'_s(z_{j_0}) = 1 + O(\varepsilon + l^{-1}_s),
\]
and moreover, \( z_{j_0} \) is a local maximizer for \( I'_s(z) \).

Proof. By using Lemma 2.2, the proof follows from a similar argument to that for Theorem 3.1 in [18]. \( \square \)

Based on Theorem 3.2 we can formulate Scheme R to locate the multiple extended scatterer components of \((\Sigma^+_v; \varepsilon, \mu, \Sigma)\) in (2.28)-(2.29).
Finally, we consider the locating of the general multiscale scatterers of \((\Sigma^+, \varepsilon, \mu, \Sigma)\) in (2.1), by concatenating Schemes S and R through the local tuning technique introduced in [18], which we briefly describe in the following. Let \(A(\tilde{x}, \Sigma^+)\) be the far-field pattern of \((\Sigma^+; \varepsilon, \mu, \Sigma)\) collected corresponding to a single pair of incident plane waves (1.1). Then, we apply \(A(\tilde{x}, \Sigma^+)\) as the far-field pattern to Scheme R to approximately locate the positions of the extended scatterer components of \((\Sigma^+, \varepsilon, \mu, \Sigma)\). Suppose that \(\tilde{z}_j + \tilde{r}_j^+, j = 1, 2, ..., l_j\), are the approximate scatterers found above. We next refine the mesh \(\mathcal{T}_h\) around each \(\tilde{z}_j\), and the \(\varepsilon\)-net \(\mathcal{G}\) around each \(\tilde{r}_j^+\). After that, for each set of sampling points from the refined mesh made above, say \(\tilde{z}_j\), \(j = 1, 2, ..., l_j\), and each set of base scatterers from the refined \(\varepsilon\)-net, say \(\tilde{r}_j^+, j = 1, 2, ..., l_j\), one calculates

\[
\tilde{A}(\tilde{x}) := A(\tilde{x}, \Sigma^+_r) - \sum_{j=1}^{l_j} e^{i\omega(d_{\varepsilon}(\tilde{x}, \tilde{z}_j))} A(\tilde{x}, \tilde{r}_j^+). \tag{3.6}
\]

By using \(\tilde{A}(\tilde{x})\) as the far-field pattern for Scheme S, and by running through all the possible sampling points and base scatterers from the refined sampling mesh and \(\varepsilon\)-net, one can locate the clustered local maximizers, which represent the locations of the small scatterer components of \((\Sigma^+, \varepsilon, \mu, \Sigma)\). We refer to the locating procedure sketched above as Scheme M.

4. Numerical experiments and discussions

We present extensive numerical experiments in this section to illustrate the salient features of our new locating schemes (Scheme S, R and M) for the inverse EM scattering problem with locally perturbed ground objects in three dimensions. An oblique EM plane wave with the incident direction of polar angle \(\pi/6\) radian and azimuthal angle \(\pi/3\) radian is employed as the detecting wave field incident on the ground objects and it yields a perturbed EM wave scattering off from ground objects to the infinity. In the examples below, we set \(\varepsilon_0 = \mu_0 = 1\) and \(\Sigma_0 = 0\) outside the scatterer and above the ground, and hence the wavelength is unitary (namely \(\lambda = 1\)) in the homogeneous background. Unless otherwise specified, all the ground objects are either PEC conductors or inhomogeneous media with all other parameters the same as those in the homogeneous background except \(\varepsilon = 4\).

In the sequel, the exact EM far field data are synthesized in the following way. The scattered field is obtained by solving the Maxwell system (2.4) by a forward solver using \(H(\text{curl})\)-conforming quadratic edge finite elements of Nedélec’s first kind on a truncated domain. The computational domain is an upper semi-sphere centered at the origin, and enclosed by a semi-spherical PML layer to damp the reflection and a PEC boundary on the ground, namely the \(x-y\) plane. Local adaptive refinement scheme within the inhomogeneous scatterer or around the PEC components is adopted to enhance the accuracy of the scattered wave. The far-field data are approximated by the Stratton-Chu integral equation representation using the spherical Lebedev quadrature (cf. [15,25]). The computation is carried out on a sequence of successively refined meshes till the relative maximum error of successive groups of far-field data is below 0.1\%. The synthetic far field data on the finest mesh are taken as the exact one.

The electric far-field patterns \(A(\tilde{x}, \Omega), \Omega = \Omega^{(r)}\) or \(\Omega^{(m)}\), are observed at Lebedev quadrature points distributed on the unit upper half-sphere \(S^2_+\) with sufficient order of accuracy (cf. [15] and references therein). The exact far-field data \(A(\tilde{x}, \Omega)\) are corrupted point-wise by the formula

\[
A_\delta(\tilde{x}, \Omega) = A(\tilde{x}, \Omega) + \delta\zeta_1 \max_\tilde{x} |A(\tilde{x}, \Omega)| \exp(i2\pi\zeta_2). \tag{4.1}
\]

where \(\delta\) refers to the relative noise level, and both \(\zeta_1\) and \(\zeta_2\) follow the uniform distribution ranging from \(-1\) to \(1\). The values of the indicator functions have been normalized between 0 and 1 to highlight the positions identified.

In the following, we demonstrate the effectiveness and efficiency of Schemes S, R and M by three groups of experiments. The first group of experiments is on locating small-size, or partially-small ground objects in various scenarios by Scheme S, and the second group of experiments on testing Scheme R for locating extended-size ground objects. In the third group of experiments, we shall test the performance of Scheme M on locating multi-scale multiple ground objects.
4.1. Scheme S

**Example 1 (A mini-rocket on the ground).** In this example, we consider a mini-rocket of height $0.1\lambda$ and located at the origin, with the EM parameters given by $\varepsilon = 4$, $\mu = 1$ and $\Sigma = 0$, see Fig. 2(a) by zooming-in around the origin. The orthogonal slices of the contours of the indicator function $I(z)$ for Scheme S are given in Fig. 2(b). It can be seen that the position of the mini-rocket is highlighted as predicted. Scheme S can locate the small scatterer in a very accurate and stable manner even if 20% random noise is attached to the measurement data.

**Example 2 (A mini-rocket and a mini-tank on the ground).** In this example, we consider a scatterer of multiple components: a mini-rocket of height $0.1\lambda$ and located at $(-2\lambda, -2\lambda, 0)$ with the EM parameters given by $\varepsilon = 4$, $\mu = 1$ and $\Sigma = 0$, and a PEC mini-tank of height $0.075\lambda$ located at $(2\lambda, 2\lambda, 0)$, see Fig. 3(a)–(b) by zooming in around the respective ground object. For this example, the orthogonal slices of the contours of the indicator function $I(z)$ for Scheme S are shown in Fig. 3(c). Scheme S yields a very accurate identification of the location of both scatterers even if the measurement data is significantly perturbed to a high-level noise of 20%. This example also demonstrates that Scheme S can locate the multiple scatterer components without knowing the physical property of each component in advance.

**Example 3 (Two close small half-balls on the ground).** The scatterer consists of two small PEC half-balls of radius 0.1, located at $(-0.35, 0, 0)$ and $(0.35, 0, 0)$, respectively, as shown in Fig. 4(a). We shall investigate the lower resolution limit between the ground objects for Scheme S. The results are shown in Fig. 4(b). It can be seen in this case, namely the distance between the two components is of a half wavelength, Scheme S can locate both scatterer components and separate them very well. If we further reduce the distance between the two components (less than a half wavelength), Scheme S can no longer separate the two scatterer components, although it can still roughly locate their average position.

In the next three examples, we test some partially-small ground objects, which are only small in certain but not all dimensions.

**Example 4 (A half-cylinder crack bar).** The scatterer consists of a half-cylinder crack bar over the $x$–$y$ plane with axial direction along the $x$-axis, located at origin with height $2\lambda$ and bottom radius $0.1\lambda$, respectively, see Fig. 5(a). A single impinging EM wave is enough to provides us all important information of the scatterer, e.g., the rough position and approximate length as shown in Fig. 5(b).

**Example 5 (A half Torus on the ground).** The ground scatterer consists of a torus with major radius $1\lambda$ and minor radius $0.2\lambda$ shown in Fig. 6(a). Its geometric information can be extracted by sending EM incident wave fields with a fixed polar angle $\pi/6$ and eight azimuthal angle $0, \pi/4, \ldots, 7\pi/4$. We take the maximum value of eight indicator functions associated with eight incident angles using Scheme S and plot the contour plot in Fig. 6(b), which clearly shows the trace of the thin torus on the ground.
Fig. 3. Example 2. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

(c) Slice plots of the indicator function of two ground components.

(a) Mini-rocket at \((-2, -2, 0)\) (zoomed in).
(b) Mini-tank at \((2, 2, 0)\) (zoomed in).

Fig. 4. Example 3. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

(a) Two half-balls.
(b) Slice plots of indicator function.
Example 6 (A thin-aircraft). In this example we consider a PEC thin aircraft of height 0.1, see Fig. 7(a). Following the same multi-static data of up to eight incident angles, the position and its geometric information can be extracted from the indicating behavior in Fig. 7(b).

4.2. Scheme R

In this subsection, we shall consider two examples to demonstrate the capability of Scheme R for locating extended-size ground objects. The synthesized far field data are corrupted by a noise level of 5%.

Example 7 (An extended-size tank on the ground). As shown in Fig. 8(a), we consider an extended-size PEC tank of length 1.5, width 1, height 0.5 displaced at (0, 0, 0). The far field data are collected in advance associated with the augmented reference set, namely four different orientations with azimuthal angles 0, π/2, π and 3π/2 (see Fig. 8(b)–(e)). Reconstruction results using Scheme R are illustrated in Fig. 8(f)–(i). It is found that only Fig. 8(i) gives a significantly highlighted region (achieving the maximum magnitude of unit) around the exact position (0, 0, 0), it further tells us what orientation the tank takes with the information carried in the augmented shape set.

Example 8 (Extended-size ellipsoid and peanut). In this example, we consider a ground scatterer consists of two PEC components as shown in Fig. 9: (1) an extended semi-ellipsoid with the ground center chosen at (−2.5, −2.5, 0) with semi-axis
Fig. 7. Example 6. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

(a) A thin aircraft.  
(b) Slice plots of indicator function.

Fig. 8. Example 7. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

(a) An extended-size PEC tank.  

(b) 0  
(c) $\pi/2$  
(d) $\pi$  
(e) $3\pi/2$

Basis ground objects with different orientations.

(f) 0  
(g) $\pi/2$  
(h) $\pi$  
(i) $3\pi/2$

Slice results associated with different orientations.
Fig. 9. Example 8. A two-component ground scatterer.

Fig. 10. Augmented reference shape set in Example 8.

Fig. 11. Example 8: Locating the first component. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

Fig. 12. Example 8: Locating the second component. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)
radius (2, 0, 0) along x-, y- and z-axes, which is further rotated by $\pi/2$; and (2) an extended peanut parametrized by $x(t) = \sqrt{3}\cos^2(t) + 1\cos(t)$, $y(t) = \sqrt{3}\cos^2(t) + 1\sin(t)$ in the x–y plane and revolved around x-axis, which is then displaced at (2.5$\lambda$, 2.5$\lambda$, 0). The reference shape set is further augmented by four different orientations as shown in Fig. 10. Using the far field data associated with those a priori augmented admissible shapes, Scheme R is then implemented for imaging purpose.

By the magnitude of the far field pattern, those associated with the half peanut are first examined. The slice plots of the indicator function using Scheme R are shown in Fig. 11(a)–(d). For noisy far-field data, Scheme R can successfully determine the location of the peanut with the right configuration by the dark red part as shown in Fig. 11(a). After the peanut is determined and trimmed from the sampling domain, if one continues Scheme R by a dictionary match of the augmented
far field data associated with the reference half ellipsoid component, the significant peak value in Fig. 12(c) indicates the position and orientation of the second component.

4.3. Scheme M

Example 9 (A small kite and an extended peanut). This example is the most challenging one. The true scatterer consists of two components of different scales in terms of the detecting wavelength: a small half kite and an extended half peanut on the ground (see Fig. 13).

The small half kite component is parametrized by $x(t) = 0.2(\cos(t) + 0.65 \cos(2t) - 0.65)$, $y(t) = 0.2(1.5 \sin(t))$, revolved along the $x$-axis, displaced at $(-2.5\lambda, -2.5\lambda, 0)$, and further rotated by $\pi/2$ on the ground. While the extended half peanut is specified as in Example 8, and then displaced at $(2.5\lambda, 2.5\lambda, 0)$.

The reference shape of the extended scatterer (half-peanut) in the admissible set is augmented by four different orientations as shown in Fig. 10(a)–(d), and then the far field data set are collected with respect to the augmented admissible set for a dictionary search of the extended components.

Firstly, we employ the reference far-field data in the augmented admissible data set to find the location and shape of the half-peanut using Scheme R. Up to this stage, we obtain an initial guess that the rough position of the extended component is distributed on a region around $(2.5\lambda, 2.5\lambda, 0)$ as indicated in Fig. 14(a). However, the orientation angle can be correctly identified to be $0$ by comparing the peak of the indicator function in Fig. 14 with the reference set in Fig. 10(a)–(d).

Next, we define a locally finer sampling mesh around $(2.5, 2.5, 0)$ shown in Fig. 15. For every sampling point $z_j$ on this local fine mesh, we subtract the corresponding far-field pattern associated with the reference half peanut component by the updating formula (3.6) from the total far field data, and then use Scheme S to detect the small-size components. It can be seen from Fig. 16(a)–(c) that, as the sampling point of the half peanut moves from $(2.3, 2.5, 0)$ to $(2.5, 2.5, 0)$, the peak value and focused highlighted region are centered at $(-2.5, -2.5, 0)$, which indicates the existence of a small-size component lying at the position $(-2.5, -2.5, 0)$.

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References


