# 52nd Spring Topology and Dynamical Systems Conference 

Auburn University<br>Auburn, Alabama, USA

March 14-17, 2018

## Contents

Invited Speakers (Plenary, Semi-plenary) ..... 1
Mladen Bestvina ..... 1
John Bryant ..... 1
Suddhasattwa Das ..... 2
Albert Fathi ..... 2
Steve Ferry ..... 3
Paul Gartside ..... 3
Boris Goldfarb ..... 3
Kazuhiro Kawamura ..... 4
Danuta Kołodziejczyk ..... 4
David Lipham ..... 5
Sara Maloni ..... 5
Michat Misiurewicz ..... 5
Janusz R. Prajs ..... 6
Kevin Schreve ..... 6
Jennifer Schultens ..... 6
Slawomir Solecki ..... 7
Balzs Strenner ..... 7
Bena Tshishiku ..... 7
Michael Usher. ..... 8
Benjamin Vejnar ..... 8
Susan Williams. ..... 8
Andy Zucker ..... 9
Workshops ..... 10
Mladen Bestvina ..... 10
Jerzy Dydak ..... 10
Algebraic Topology and Applications ..... 11
Jan P. Boronski ..... 11
Jeremy Brazas ..... 11
Alex Clark. ..... 11
Jerzy Dydak ..... 12
Robert D Edwards ..... 12
Paul Fabel. ..... 13
Steven Hurder ..... 13
Daniel Ingebretson ..... 13
James Keesling ..... 14
Olga Lukina ..... 14
Matthew Lynam ..... 14
Aura Lucina Kantn Montiel ..... 15
Oleg R Musin ..... 15
Piotr Oprocha ..... 15
Leonard R. Rubin ..... 16
Petra Staynova ..... 16
Marilyn Vazquez ..... 16
Thomas Weighill ..... 17
Continuum Theory (dedicated to Janusz Prajs) ..... 18
Hussam Abobaker ..... 18
Roshan Adikari ..... 18
Maria Elena Aguilera ..... 18
Ameen A Alhassan ..... 18
Ana Anušić ..... 19
David P Bellamy ..... 19
Jan P. Boroński . ..... 19
Włodzimierz J. Charatonik ..... 20
Włodzimierz J. Charatonik ..... 20
Alejandro Illanes ..... 20
Matt Insall ..... 21
James Kelly ..... 21
Judy Kennedy ..... 21
Bostjan Lemez ..... 22
Wayne Lewis ..... 22
Sergio Macias ..... 22
Marcus Marsh ..... 23
Verónica Martínez de la Vega ..... 23
Faruq Mena ..... 23
Daria Michalik ..... 24
Van Nall ..... 24
Jeffrey Norden ..... 24
Yaziel Pacheco Juárez ..... 25
Şahika Sahan ..... 25
Scott Varagona ..... 25
Hugo Villanueva ..... 25
Dynamical Systems ..... 26
Lori Alvin ..... 26
Alexander Blokh ..... 26
Jan P. Boroński ..... 27
Will Brian ..... 27
Henk Bruin ..... 28
Jernej Cinc ..... 28
Joanna Furno ..... 28
Sharan Gopal ..... 29
Sina Greenwood ..... 29
Hayato Imamura ..... 30
James Keesling ..... 30
Olga Lukina ..... 30
Alica Miller ..... 31
C.A. Morales ..... 31
Christopher Mouron ..... 31
Piotr Oprocha ..... 32
Lex Oversteegen ..... 32
Michael Sullivan ..... 32
Christian Wolf ..... 33
Tomoo Yokoyama ..... 33
Geometric Group Theory ..... 34
Aaron Abrams ..... 34
Jim Belk ..... 34
Maxime Bergeron ..... 34
David Bruce Cohen ..... 35
Pallavi Dani . ..... 35
Spencer Dowdall ..... 36
Eduard Einstein ..... 36
Jonah Gaster ..... 36
Michael Hull ..... 37
Edgar A. Bering IV ..... 37
Kevin Kordek ..... 37
Robert Kropholler ..... 38
Justin Malestein ..... 38
Yulan Qing ..... 38
Kim Ruane ..... 39
Nick Salter ..... 39
Bakul Sathaye ..... 39
Rachel Skipper ..... 40
Ignat Soroko ..... 40
Balazs Strenner ..... 40
Daniel Studenmund ..... 41
Amanda Taylor ..... 41
Phillip Wesolek ..... 41
Geometric Topology ..... 42
Igor Belegradek ..... 42
Greg Bell ..... 42
Vladimir Chernov ..... 42
Daniel C. Cohen ..... 43
Trevor Davila ..... 43
Boris Goldfarb ..... 43
Shijie Gu ..... 44
Craig Guilbault ..... 44
Greg Kuperberg ..... 44
Jay Leach ..... 45
Atish J. Mitra ..... 45
Oleg R Musin ..... 45
Hoang Nguyen ..... 46
Boris Okun ..... 46
Daniel Ramras ..... 46
David Rosenthal ..... 47
Rustam Sadykov ..... 47
Steven Scheirer ..... 47
Daniel S. Silver ..... 48
Hung Cong Tran ..... 48
Victor Turchin ..... 48
Nicol Zava ..... 49
Set-Theoretic Topology ..... 50
Samer Assaf. ..... 50
Jocelyn Bell ..... 50
Christopher Caruvana ..... 50
Steven Clontz ..... 51
Natasha Dobrinen ..... 51
Alan Dow ..... 51
Ziqin Feng ..... 52
Ivan S. Gotchev ..... 52
Hongfeng Guo ..... 52
Joan Hart ..... 53
Neil Hindman ..... 53
Jared Holshouser ..... 53
Wanjun Hu ..... 54
Akira Iwasa ..... 54
Mikolaj Krupski ..... 54
Paul Larson ..... 54
Arkady Leiderman ..... 55
David Milovich ..... 55
Peter Nyikos ..... 56
Yasser Ferman Ortiz-Castillo ..... 56
Strashimir G. Popvassilev ..... 57
Ted Porter. ..... 57
Alexander Shibakov ..... 57
Ihor Stasyuk ..... 58
Paul Szeptycki ..... 58
Vladimir Tkachuk ..... 58
Keith Whittington ..... 59
Lynne Yengulalp ..... 59

# Invited Speakers (Plenary, Semi-plenary) 

PL Morse theory and applications<br>Mladen Bestvina<br>University of Utah<br>bestvina@math.utah.edu

The goal of the talk is to introduce PL Morse theory. This theory is concerned with describing the homotopy type of a simplicial complex equipped with a real valued PL function, based on the local information at the vertices. I will also give some applications, coming from my paper with Noel Brady (published some time ago). For example, I intend to explain why the kernel of the homomorphism $F_{2} \times F_{2} \rightarrow Z$ that sends all 4 standard generators of the product of two free groups of rank 2 to a generator of Z is finitely generated but not finitely presented. The talk will also serve as a starting point for a workshop, where further applications by others in geometric group theory will be given.

Some Counterexamples to the Bing-Borsuk Conjecture John Bryant<br>Department of Mathematics, Florida State University<br>jbry98@comcast.net<br>Coauthors: Steve Ferry, Department of Mathematics, Rutgers University

An $n$-dimensional ANR homology manifold $X$ is resolvable if there is a topological $n$-manifold $M$ and a cell-like map $f: M \rightarrow X$. A space $X$ has the disjoint disks property, or DDP, if every two maps of a 2-cell into $X$ can be approximated by maps with disjoint images. A space $X$ is topologically homogeneous, if, for every pair of points $x, y \in X$, there is a homeomorphism $h: X \rightarrow X$ such that $h(x)=y$. We present a variant of the construction of non-resolvable homology $n$-manifolds, $n \geq 6$, by Bryant-Ferry-Mio-Weinberger (Topology of homology manifolds, Ann. of Math. 143 (1996)) that produces a class of homology manifolds with the DDP that are topologically homogeneous. We also prove that every $n$ dimensional ANR homology manifold, $n \geq 6$, is the cell-like image of a space in this class. In particular, for every $n \geq 6$ and every integer $k \in 1+8 \mathbb{Z}$ there is a topologically homogeneous homology $n$-manifold $X$ with the DDP such that $X$ is homotopy equivalent to the $n$-sphere $S^{n}$ and the Quinn resolution obstruction $\sigma(X)=k$. If $k \neq 1$, then $X$ is not resolvable; hence, is a counterexample to a conjecture of R.H. Bing and K. Borsuk (Some remarks concerning topologically homogeneous spaces, Ann. of Math. 81 (2) (1965)) that a topologically homogeneous euclidean neighborhood retract is a topological manifold. We do not know whether every connected homology $n$-manifold, $n \geq 6$, with the DDP is topologically homogeneous.

Delay-coordinate maps and the spectra of Koopman operators<br>Suddhasattwa Das<br>Courant Institute of Mathematical Sciences, New York University<br>dass@cims.nyu.edu<br>Coauthors: Dimitrios Giannakis

The Koopman operator associated to every invertible dynamical system is an unitary operator acting on the space of $L^{2}$ functions. It provides an alternate method of studying many properties of the dynamics, like mixing, ergodicity, forecasting etc, as a linear operator. Koopman eigenfunctions represent the non-mixing component of the dynamics. They factor the dynamics, which can be chaotic, into quasiperiodic rotation on tori. Here, we describe a kernel integral operator acting on $L^{2}$ functions, which has the property of commuting with the Koopman operator and thus has common eigenfunctions. This is constructed by incorporating infinitely many delay coordinates in the kernel of this integral operator. As a by product, it also annihilates the continuous spectrum and thus maps into the subspace associated to the pure-point spectrum. This enables efficient approximation of Koopman eigenfunctions from highdimensional data in systems with point or mixed spectra, using just the information provided by a single, generic trajectory.

Recurrence on abelian cover. Application to closed geodesics in manifolds of negative curvature.
Albert Fathi
Georgia Institute of Technology
afathi30@gatech.edu
If h is a homeomorphism on a compact manifold which is chain-recurrent, we will try to understand when the lift of $h$ to an abelian cover (i.e. the covering whose Galois group is the first homology group of the manifold) is also chain-recurrent. This is related to the proof by John Franks of the Poincar-Birkhoff theorem. It has new consequences on density of classes of closed geodesics in a manifold of negative curvature.

Some Counterexamples to the Bing-Borsuk Conjecture<br>Steve Ferry<br>Rutgers University<br>steveferry@gmail.com<br>Coauthors: John Bryant

This will be a lead-in to John Bryant's talk. Topics will include as many as possible of the following:

1. Classical characterizations of the 1- and 2-spheres.
2. Sobering examples in dimension 3. $S^{3} /$ Fox-Artin arc.
3. Definition and properties of ANR homology manifolds.
4. Cannon's Conjecture and Edwards' Theorem.
5. Definition of the Quinn invariant and introduction to controlled topology.
6. Controlled surgery exact sequence.
7. Bryant-Ferry-Mio-Weinberger construction of counterexamples to Cannon's Conjecture in the special case of the sphere (or torus).
8. Statement of new results.

## Compact sets - order, topology and mirrors

Paul Gartside
University of Pittsburgh
gartside@math.pitt.edu
Coauthors: Ana Mamatelashvili, Andrea Medini, Lyubomyr Zdomskyy
We investigate the order and topological structure of compact subsets of separable metric spaces.

## Topology in Data Science

## Boris Goldfarb

University at Albany, SUNY
bgoldfarb@albany.edu
This is a survey of some established applications of algebraic topology to data science. Persistent homology leverages linear algebra to compute geometric features of discrete, even if large, data sets by the use of Vietoris-Rips complexes associated to a range of parameter values. I will draw some parallels with the subject of Jerzy Dydak's workshop at the conference. Another application of topology is based on the Mapper algorithm. These are some methods that came from topology and are finding a rapidly increasing number of practical applications. I will make a point that behind these accomplishments and beyond them there is a valuable collection of instincts and habits that allow topologists to generate new ideas in statistical data science.

# Derivations and cohomologies of Lipschitz algebras over compact Riemannian manifolds 

Kazuhiro Kawamura
Institute of Mathematics, University of Tsukuba
kawamura@math.tsukuba.ac.jp
For a compact Riemannian manifold $M$ with the induced metric (or more generally a compact metric space with the local-unique-geodesics-property), $\operatorname{Lip}(M)$ denotes the commutative Banach algebra of complex-valued Lipschitz functions on $M$. Motivated by a classical work due to de Leeuw, we introduce a compact (non-metrizable) Hausdorff space $\hat{M}$, an analogue of the space of directions and show that, for each $n \geq 1$, the Hochschild cohomology $\mathrm{H}^{n}(\operatorname{Lip}(M), C(\hat{M}))$ (in the sense of B.E. Johnson and A.Y. Helemskii) has the infinite rank as a $\operatorname{Lip}(\mathrm{M})$-module, where $C(\hat{M})$ is the $\operatorname{Lip}(\mathrm{M})$-module of all complexvalued continuous functions on $\hat{M}$. In particular, the global dimension of $\operatorname{Lip}(\mathrm{M})$ is infinite, an analogue of a result on $C^{1}$-function algebras over smooth manifolds due to Pugach and Kleshchev. The coefficient module $C(\hat{M})$ is rather big and a more natural module for the study would be $C(M)$, for which we have $\mathrm{H}^{1}(\operatorname{Lip}(M), C(M))=0$.

## Polyhedra with finite depth

Danuta Kołodziejczyk
Warsaw University of Technology
dakolodz@gmail.com
In this talk every polyhedron is finite and every $A N R$ is compact. Recall that a map $f: X \rightarrow Y$ is a homotopy domination if there exists a map $g: Y \rightarrow X$ such that $f g$ is homotopic to $i d_{Y}$. Then we say that $Y$ is homotopy dominated by $X$, and we write $X \geq Y$.

Given a polyhedron $P$, one may ask, is it true that each sequence $P \geq X_{1} \geq X_{2} \geq \ldots$ contains only finitely many different homotopy types of $X_{i}$ or, does there exist an integer $l_{P}$ (depending only on $P$ ) such that each sequence of this kind contains only $\leq l_{P}$ different homotopy types of $X_{i}$ ? In the second case, $P$ have finite depth. These questions are closely related to a famous problem of K. Borsuk (1967): Is it true that two $A N R^{\prime} s, X$ and $Y$, homotopy dominating each other have the same homotopy type?

The answer is known to be positive for all the polyhedra $\left(A N R^{\prime} s\right)$ with virtually-polycyclic fundamental groups [DK, Top. Appl. 153, 2005; Fund. Math. 197, 2007], and clearly, for 1-dimensional polyhedra. On the other hand, as we showed earlier, there exist polyhedra homotopy dominating infinitely many different homotopy types [DK, Fund. Math. 96; Proc. Amer. Math. Soc. 2002].

We proved that for some classes of polyhedra $P$ these questions can be reduced to the same questions for the fundamental group $\pi_{1}(P)$ with retractions. We will present some positive results and interesting related problems (also on finitely presented groups) which remain unsolved.

For similar classes of $A N R^{\prime} s$ we get a positive answer to an other open question of K. Borsuk: Are the homotopy types of two quasi-homeomorphic $A N R^{\prime} s$ equal?

## Compactifying connected spaces: 2 problems

David Lipham
Auburn University
dsl0003@auburn.edu
This talk will address dense embeddings of two types of connected spaces into compact Hausdorff spaces. Through examples and partial results, we will explore dual problems on preserving irreducibility and destroying cut points:

Let $X$ be a connected Tychonoff space.
Problem 1. If $X$ is irreducible between every two of its points, is there a compact Hausdorff space $Y=\bar{X}$ that is irreducible between some two of its points?

Problem 2. If every point of $X$ is a cut point, is there some point $x \in X$ and a compact Hausdorff space $Y=\bar{X}$ such that $Y \backslash\{x\}$ is connected?

Polyhedra inscribed in quadrics and their geometry<br>Sara Maloni<br>University of Virginia<br>sm4cw@virginia.edu<br>Coauthors: J. Danciger, J.M. Schlenker

In 1832 Steiner asked for a characterization of polyhedra which can be inscribed in quadrics. In 1992 Rivin answered in the case of the sphere, using hyperbolic geometry. In this talk, I will describe the complete answer to Steiner's question, which involves the study of interesting analogues of hyperbolic geometry including anti de Sitter geometry. Time permitting, we will also discuss future directions in the study of convex hyperbolic and anti de Sitter manifolds.

## Lozi-like maps

Michał Misiurewicz
Indiana University-Purdue University Indianapolis
mmisiure@math.iupui.edu
Coauthors: Sonja Štimac
We define a broad class of piecewise smooth plane homeomorphisms which have properties similar to the properties of Lozi maps, including the existence of a hyperbolic attractor. We call those maps Lozi-like. The basic structure of such a map is determined by the set of kneading sequences, or each of the two equivalent objects: the folding pattern and the folding tree.

# Open Problems in the Study of Homogeneous Continua 

Janusz R. Prajs
California State University, Sacramento
prajs@csus.edu
In this talk, I will present and discuss published and unpublished open problems and partial results I have encountered in my study. Most of these problems are in the area of homogeneous continua.

## Action dimension of simple complexes of groups

Kevin Schreve
University of Michigan
schreve@umich.edu
Coauthors: Mike Davis and Giang Le
The geometric dimension of a discrete group $G$ is the minimal dimension of a model for the classifying space BG. The action dimension of G is the minimal dimension of a manifold model. I will talk about some computations of the action dimensions for certain complexes of groups, including Artin groups, graph products, and fundamental groups of complex hyperplane complements.

## Surfaces in Seifert fibered spaces

Jennifer Schultens
UC Davis
jcs@math.ucdavis.edu
Coauthors: Yoav Moriah
Seifert fibered spaces are a family of 3 -manifolds whose members are classified by a finite set of invariants. The structure of these manifolds allows us to concretely describe surfaces embedded in them. We will discuss two types of surfaces in Seifert fibered spaces (incompressible surfaces and Heegaard surfaces), their interaction with each other, and new results concerning isotopy of these types of surfaces.

## Compact Spaces and Logic

Slawomir Solecki
Cornell University
ssolecki@cornell.edu
Fraïssé theory is a method in classical Model Theory of producing canonical limits of certain families of finite structures. For example, the random graph is the Fraïssé limit of the family of all finite graphs. It turns out that this method can be dualized, with the dualization producing projective Fraíssé limits, and applied to the study of compact metric spaces. I will describe recent results, due to several people, on connections between projective Fraïssé limits and the structure of some canonical compact spaces and their homeomorphism groups (the pseudoarc, the Menger curve, the Lelek fan, simplexes with the goal of developing a projective Fraïssé homology theory).

## Number-theoretic and algorithmic aspects of surface homeomorphisms <br> Balzs Strenner <br> Georgia Tech <br> bstrenner7@gatech.edu

The first half of the talk will be a survey of results connecting surface homeomorphisms with number theory. The starting point of these connections is the fact that many surface homeomorphisms have an associated stretch factor which is an algebraic integer. In the second half, we will discuss algorithms to compute the stretch factor (and other data corresponding to surface homeomorphisms). The talk will not assume much background in either topology or number theory, so it should be accessible to a broad audience.

## Cohomology of arithmetic groups and characteristic classes of manifold bundles <br> Bena Tshishiku <br> Harvard University <br> bena@math.harvard.edu

A basic problem in the study of fiber bundles is to compute the ring $\mathrm{H}^{*}(\operatorname{BDiff}(\mathrm{M}))$ of characteristic classes of bundles with fiber a smooth manifold M . When M is a surface, this problem has ties to algebraic topology, geometric group theory, and algebraic geometry. Currently, we know only a very small percentage of the total cohomology. In this talk I will explain some of what is known and discuss some new characteristic classes (in the case dim $\mathrm{M} \gg 0$ ) that come from the unstable cohomology of arithmetic groups.

# Symplectic embeddings of ellipsoids into polydisks 

Michael Usher
University of Georgia
usher@uga.edu
Symplectomorphisms describe the possible evolutions of the phase space of a conservative physical system according to the laws of classical mechanics. Gromov's non-squeezing theorem from 1985 showed, surprisingly, that a ball in $R^{2 n}$ can be embedded into a cylinder by a symplectomorphism only if the radius of the cylinder is at least the radius of the ball, and over time it has become clear that the question of when one region in $R^{2 n}$ embeds into another is extremely delicate, with connections to complex algebraic geometry and elementary number theory. I will discuss some what is known about which four-dimensional ellipsoids symplectically embed into which products of disks, including some new results showing that qualitative features of the answer depend sensitively on the ratio of the areas of the disks.

## Fixed points of continuous group actions on compact metrizable spaces <br> Benjamin Vejnar <br> Charles University, Prague <br> benvej@gmail.com

In the late 60's Boyce and Huneke independently solved a twenty years old question of Isbell by giving an example of a pair of commuting continuous functions of the closed unit interval into itself which do not have a common fixed point. It follows that the action of the free commutative semigroup with two generators need not to have a fixed point when acting on the closed interval.

In this talk we study the conditions under which every continuous action of a topological (semi)group on a continuum (that is usually one-dimensional in its nature) has a fixed point. We are dealing e.g. with commutative or compact (semi)groups and with the classes of continua including dendrites, dendroids, uniquely arcwise connected continua or tree-like continua.

## Graphs, Links and Mahler Measure

Susan Williams
University of South Alabama
swilliam@southalabama.edu
Coauthors: Daniel Silver
The relationship between plane graphs and knots is both well known and striking. Plane graphs with edge weights $\pm 1$ correspond to arbitrary link diagrams via the medial construction. Combinatorial graph invariants can yield knot invariants. We define the complexity of the graph to be the count of spanning trees, with each tree counted as the product of its edge weights. Then the complexity of a plane graph coincides with the determinant of the associated link.

An infinite graph $G$ is $d$-periodic if it has a free $\mathbb{Z}^{d}$-action by graph automorphisms with finite quotient graph. For these a Laurent polynomial invariant $L\left(x_{1}, \ldots, x_{d}\right)$ can be defined. Its Mahler measure is the exponential growth rate of the complexities of an expanding sequence of finite quotients of $G$. When $G$ is a plane 1-periodic graph, or 2-periodic graph with all edge weights 1 , the Mahler measure is a growth rate of link determinants.

Lehmer's question, an 80-year-old open question about the roots of monic integral polynomials, is equivalent to a question about complexity growth of edge-weighted 1-periodic graphs. We do not know if the graphs can be assumed to be planar.

## A direct solution to the generic point problem

Andy Zucker
Carnegie Mellon University
andrewz@andrew.cmu.edu
We provide a new proof of a recent theorem of Ben-Yaacov, Melleray, and Tsankov. If G is a Polish group and X is a minimal, metrizable G-flow with all orbits meager, then the universal minimal flow $\mathrm{M}(\mathrm{G})$ is non-metrizable. In particular, we show that given X as above, the universal highly proximal extension of X is non-metrizable.

## Workshops

## PL Morse theory and applications

Mladen Bestvina
University of Utah
bestvina@math.utah.edu
The workshop is a followup to the plenary talk with the same title. Further applications of PL Morse theory in geometric group theory will be given. Several classes of groups standard in the subject will be introduced, e.g. Thompson's group, right angled Artin groups. Some of the examples will be presented by Matt Zaremsky and Robert Kropholler.

## Topological aspects of coarse geometry <br> Jerzy Dydak <br> University of Tennessee <br> jdydak@utk.edu

The workshop will be devoted to topics of interest to topologists that arise in coarse geometry:

1. Ends of spaces
2. Compactifications
3. Extension theorems
4. Embedding theorems

Thomas Weighill will present a unified approach via neighborhood operators to extension theorems in three categories: topological, uniform, and coarse. He will also discuss Holloway's criterion for coarse embeddings into Hilbert spaces.

## Algebraic Topology and Applications

Minimal Sets of Torus Homeomorphisms without Minimal Squares
Jan P. Boroński
AGH University of Science and Technology $\mathcal{E I T} 4$ Innovations University of Ostrava
jBoroński@wms.mat.agh.edu.pl
Coauthors: Alex Clark and Piotr Oprocha
In my talk I shall revisit our construction from [BCO], and focus on a particular family of minimal spaces without minimal Cartesian squares, that arise as minimal sets of torus homeomorphisms homotopic to the identity. Namely,
Theorem. There exists a torus homeomorphism $\varphi: \mathbb{T}^{2} \rightarrow \mathbb{T}^{2}$ homotopic to the identity with a minimal set $Y$, such that $Y \times Y$ does not admit a minimal homeomorphism.

One can picture these spaces as modifications of Denjoy minimal continua, that result from a blow-up of a single orbit to a null sequence of pseudo-arcs. Each of the uncountable collection of Denjoy continua gives rise to an uncountable collection of nonconjugate homeomorphisms with the properties as above.

## References

[BCO] Boroński J.P.; Clark A.;Oprocha P., A compact minimal space $Y$ such that its square $Y \times Y$ is not minimal (2017) arXiv:1612.09179

Continuity of the $\pi_{1}$-action
Jeremy Brazas
West Chester University
jbrazas@wcupa.edu
The homotopy groups of a space can be endowed with a variety of topologies that distinguish local properties. When $\pi_{n}(X)$ is equipped with the natural quotient topology, $\pi_{n}(X)$ has the structure of a quasitopological group, however, Paul Fabel has shown the group operation of $\pi_{n}(X)$ can fail to be continuous. For each $n \geq 1$, we show there exists a space $X$ for which the natural action of $\pi_{1}(X)$ on $\pi_{n}(X)$ fails to be continuous. The talk will conclude with a discussion of the analogous question for the coarser $\tau$-topology.

## The Homology Core

Alex Clark
University of Leicester
Alex.Clark@leicester.ac.uk
Coauthors: John Hunton (Durham)
In this talk I will discuss a topological invariant we introduced for spaces that admit especially nice representations as inverse limits. Tiling spaces that arise from aperiodic tilings and similar foliated spaces provide a rich source of examples for which the invariant can be used. We will discuss the general theory and how the homology core can be applied to classify specific classes of spaces.

# Profinite structures on residually finite groups 

Jerzy Dydak
University of Tennessee
jdydak@utk.edu
Coauthors: Joanna Furno (University of Houston) and James Keesling (University of Florida)
Given a countable group $G$ one can put a large scale (coarse) structure on it by selecting an increasing sequence $\left\{F_{n}\right\}_{n \geq 1}$ of finite subsets of $G$ whose union is $G$ and declaring a cover $\mathcal{U}$ of $G$ to be uniformly bounded if and only if there is $n \geq 1$ such that $\mathcal{U}$ refines $\left\{g \cdot F_{n}\right\}_{g \in G}$. It is well-known and easy to show that the coarse structure obtained that way is unique and is equal to the bounded metric structure given by any word metric on $G$ if $G$ is finitely generated.

In this talk we consider a dual situation in case of residually finite countable groups $G$. Given a decreasing sequence $\left\{G_{n}\right\}_{n \geq 1}$ of subgroups of $G$ of finite index whose intersection consists of the neutral element $e_{G}$ only, we define a small scale (uniform) structure on $G$ (which we call a profinite structure on $G)$ by declaring a cover $\mathcal{U}$ of $G$ to be uniform if and only if there is $n \geq 1$ such that $\left\{g \cdot G_{n}\right\}_{g \in G}$ refines $\mathcal{U}$. One can give a characterization of profinite structures on $G$ in terms of topological group structures on $G$.

A natural question is if profinite structures are unique on countable groups. It turns out the answer is no. However, in case of finitely generated residually finite groups $G$ there is only one profinite structure.

This is joint work with Joanna Furno (University of Houston) and James Keesling (University of Florida).

## Two of My Favorite Conjectures

## Robert D Edwards

$U C L A$
rde@math.ucla.edu
Both are related to the Hilbert-Smith Conjecture, which may be stated as: Any locally compact subgroup of the homeomorphism group of a manifold must be a Lie group. This is a fundamental open problem in topology, dating from the late 1930s. But as great as the HSC is, I think it is the "wrong" conjecture, because it puts too much focus on manifolds. Imho a better (and stronger) conjecture is the Free-Set Z-Set Conjecture (google it; I first posed it in 1991), the key special case of which is: A locally compact topological group which acts freely on an ENR must be a Lie group.

My second conjecture, also stronger than the HSC, might be named the Doubly-Small ManifoldHomeomorphism Conjecture: Any homomorphism $Z \rightarrow$ Homeo(Manifold) (or Diffeo(Manifold)) which 1) has sufficiently small image, i.e. has image sufficiently close to Iden(M), and 2) accumulates at Iden(M), must be the trivial homomorphism.

In my talk I will flesh out these conjectures, with explanations and examples.

# Topological R-trees as 'covers' of Peano spaces 

Paul Fabel
Mississippi State University
fabel@math.msstate.edu
Coauthors: Jeremy Brazas
We discuss a fundamental example which is also a plausible counterexample to a question posed by J.Dydak at the summer 2011 topology conference in Strobl, Austria. If $D^{2}$ is the closed unit disk, if $X$ is a connected, locally path connected, metrizable space, and if $\Pi: X \rightarrow D^{2}$ is a map such that all based paths in $D^{2}$ have unique based lifts in $X$, must $\Pi$ be a homeomorphism? We manufacture a topological $R$-tree $X$ satisfying all the mentioned properties with the plausible exception of unique lifting. Unique lifting lives or dies by the answer to a fundamental question concerning dendrites (the 1-dimensional retracts of $D^{2}$ ). Suppose both the loop $\alpha: S^{1} \rightarrow D^{2}$ and the concatenated loop $\alpha * \beta: S^{1} \rightarrow D^{2}$ can be lifted to respective loops in respective dendrites. Can $\beta$; be lifted to a loop in some dendrite?

## Shape and periodic pseudo-orbits for Kuperberg flows

Steven Hurder
University of Illinois at Chicago
hurder@uic.edu
The main result of this talk relates the growth rates for the periodic pseudo-orbits in a Kuperberg aperiodic flow of a compact 3-manifold, with the shape of its minimal set and the slow entropy of the flow. This work is part of the study of the chain recurrent sets for Kuperberg flows, a project joint with Ana Rechtman and Daniel Ingebretson.

## Dimension and differential structures on Cantor sets

Daniel Ingebretson
University of Illinois at Chicago
dingeb2@uic.edu
Each differential structure on a Cantor set is determined by a scaling function on its dual. For Cantor sets that are attractors of smooth iterated function systems, we study the relation between differential structures, scaling functions, and the Hausdorff dimension of the Cantor set.

## The algebraic topology of $\beta X$ with applications

James Keesling
University of Florida
kees@ufl.edu
We present techniques that allow computation of the Cech-cohomology of the Stone-Cech compactification. The approach allows one to discover geometric features of this compactification as well. We give several theorems as examples.

## Manifold-like matchbox manifolds

Olga Lukina
University of Illinois at Chicago
lukina@uic.edu
Coauthors: Alex Clark, Steven Hurder
A matchbox manifold is a compact connected metrizable space where every point has a neighborhood homeomorphic to the product of a Euclidean disk and a Cantor set. A matchbox manifold is manifold-like if for every $\epsilon>0$, there is a surjection onto a compact manifold with preimages of diameter less than $\epsilon$. In the talk, we show that if a matchbox manifold is manifold-like, then it is homeomorphic to a weak solenoid, that is, the inverse limit of finite-to-one coverings of a manifold.

## Approximate Inverse Limits and (m,n)-dimension

Matthew Lynam
East Central University, Ada, OK
mlynam@ecok.edu
Coauthors: Leonard R. Rubin
In this talk we are going to consider $(m, n)$-dimension which was introduced by V. Fedorchuk as a generalization of covering dimension. Approximate inverse systems of metric compacta $X_{a}$ were defined and applied in dimension theory by S. Mardešić and L. Rubin. The limit of such a system is always a compact Hausdorff space, and every compact Hausdorff space $X$ can be represented as the limit of such a system in which each $X_{a}$ is a compact polyhedron with $\operatorname{dim} X_{a} \leq \operatorname{dim} X$. Moreover, if $X$ is the limit of an approximate inverse system of metric compacta $X_{a}$, and $\operatorname{dim} X_{a} \leq k$ for each $a$, then $\operatorname{dim} X \leq k$. Similarly if $G$ is an abelian group and $\operatorname{dim}_{G} X_{a} \leq k$ for each $a$, then $\operatorname{dim}_{G} X \leq k$. We show that the same result is true for Fedorchuk's dimension, that is, if the $(m, n)$-dimension of $X_{a}$ is $\leq k$ for each $a$, then the ( $m, n$ )-dimension of $X$ is also $\leq k$.

# G-fibrations induced by the functor of twisted product via $\alpha$ 

Aura Lucina Kantn Montiel
Papaloapan University, Mexico
alkantun@yahoo.com
By a $G$-fibration, we mean the equivariant version of a Hurewicz fibration: an equivariant map with the right lifting property respect to the $G$-embeddings $X \times\{0\} \hookrightarrow X \times I$.

Given a continuous homomorphism of topological groups $\alpha: G^{\prime} \rightarrow G$, every $G$-space and every $G$-map can be regarded as $G^{\prime}$-space and $G^{\prime}$-map respectively,so we get the restriction functor res : $G$-Top $\rightarrow G^{\prime}$ Top. This functor preserves equivariant fibrations, in other words, every $G$-fibration is also a $G^{\prime}$-fibration via $\alpha$.

The functor of twisted product $G \times_{\alpha}-: G^{\prime}-T o p \rightarrow G$-Top is right adjoint of the restriction functor. We will show that this functor also preserve equivariant fibrations, that is to say, if $p: E \rightarrow B$ is a $G^{\prime}$-fibration, then the induced $G$-map $\tilde{p}: G \times{ }_{\alpha} E \rightarrow G \times{ }_{\alpha} B$ is a $G$-fibration.

## Borsuk-Ulam type theorems for G-spaces

Oleg R Musin
University of Texas Rio Grande Valley
oleg.musin@utrgv.edu
In this talk we consider spaces with free actions of a finite group $G$ for which theorems of Borsuk-Ulam type (BUT) are true. There are several equivalent definitions for BUT-spaces that can be considered as their properties. To study BUT-spaces, one of the main tools is Yang's cohomological index. For manifolds the BUT-property depending on the free equivariant cobordism class of a manifold. In particular, necessary and sufficient conditions will be considered for a manifold with a free involution to be a BorsukUlam type. Tucker and Ky Fan's lemma are combinatorial analogs of the Borsuk-Ulam theorem (BUT). We consider generalizations of these lemmas for BUT-manifolds. Proofs rely on a generalization of the odd mapping theorem and on a lemma about the doubling of manifolds with boundaries that are BUTmanifolds. In this talk we also present Tucker's type lemmas for $G$-simplicial complexes and manifolds.

## Dynamical properties of maps on quasi-graphs

Piotr Oprocha
AGH University, Poland
oprocha@agh.edu.pl
Coauthors: Jian Li and Guohua Zhang
Quasi-graphs are natural generalizations of topological graphs. The simplest example of such space is the Warsaw circle. Even from this simplest example it is clear that the structure of $\omega$-limit set can be richer than is possible in graph maps. On the other hand, some similarities still exist. In this talk we will focus mainly on topological entropy and properties of invariant measures.

# Topological $n$-cells and Hilbert cubes in Inverse Limits 

Leonard R. Rubin
University of Oklahoma
lrubin@ou.edu
It has been shown by S . Mardešić that if a compact metrizable space $X$ has $\operatorname{dim} X \geq 1$ and $X$ is the inverse limit of an inverse sequence of compact triangulated polyhedra with simplicial bonding maps, then $X$ must contain an arc. We prove that if $\mathbf{X}=\left(\left|K_{a}\right|, p_{a}^{b},(A, \leq)\right)$ is an inverse system in set theory of triangulated polyhedra $\left|K_{a}\right|$ with simplicial bonding functions $p_{a}^{b}$ and $X=\lim \mathbf{X}$, then there exists a uniquely determined sub-inverse system $\mathbf{X}_{X}=\left(\left|L_{a}\right|, p_{a}^{b}| | L_{b} \mid,(A, \preceq)\right)$ of $\mathbf{X}$ where for each $a, L_{a}$ is a subcomplex of $K_{a}$, each $p_{a}^{b}| | L_{b}\left|:\left|L_{b}\right| \rightarrow\right| L_{a} \mid$ is surjective, and $\lim \mathbf{X}_{X}=X$. We shall use this to generalize the Mardešić result by characterizing when the inverse limit of an inverse sequence of triangulated polyhedra with simplicial bonding maps must contain a topological $n$-cell and do the same in the case of an inverse system of finite triangulated polyhedra with simplicial bonding maps. We shall also characterize when the inverse limit of an inverse sequence of triangulated polyhedra with simplicial bonding maps must contain an embedded copy of the Hilbert cube. In each of the above settings, all the polyhedra have the weak topology or all have the metric topology (these topologies being identical when the polyhedra are finite).

## The Ellis Semigroup of Generalized Morse Sequences <br> Petra Staynova <br> University of Leicester <br> petra.staynova@gmail.com

In 1997, Haddad and Johnson prove that the Ellis semigroup of any generalised Morse sequence has four minimal idempotents. They base their proof on a proposition stating that any IP cluster point along an integer sequence can be represented as an IP cluster point along either a wholly positive or wholly negative integer IP sequence. In this note, we provide large class of counterexamples to that proposition. We also provide a proof of their main theorem via the algebra of the Ellis semigroup.

## Persistence in Data Clustering

Marilyn Vazquez
George Mason University
mvazque3@masonlive.gmu.edu
Coauthors: Tim Sauer, Tyrus Berry
Data clustering is an important task for discovering patterns in data. In our approach, we assume data lives in a manifold and is sampled according to some probability measure. Clusters are connected components of the superlevel set of a density, but non-uniform sampling of data can cause problems such the presence of artificial clusters. In our work, we use ideas from persistent 0-homology to solve these problems. We discuss applications to image segmentation.

## Warped spaces and the maximal Roe algebra

Thomas Weighill
University of Tennessee
tweighil@vols.utk.edu
Coauthors: Logan Higginbotham
Let G be a finitely generated group acting on a proper metric space X. The warped space, introduced by Roe, can be viewed as a kind of large scale quotient of X by the action of G . For example, warped cones (i.e. where X is a metric cone over a compact space) provide many examples of spaces with exotic large scale behaviour. In this talk we generalize this construction to the setting of large scale spaces and an arbitrary group G, and introduce the notion of a coarsely discontinuous action by coarse equivalences. For such actions, one can recover $G$ as a kind of deck transformation group when $X$ satisfies a large scale connectedness condition. We also give a relation between the maximal Roe algebra of the warped space and the crossed product of the maximal Roe algebra of X with G . This is joint work with Logan Higginbotham.

# Continuum Theory (dedicated to Janusz Prajs on the occasion of his 60th birthday) 

On thin continua<br>Hussam Abobaker<br>Missouri University of Science and Technology<br>haq3f@mst.edu<br>Coauthors: Włodzimierz J. Charatonik

We introduce a notion of a thin continuum. A continuum is thin provided every nondegenerate subcontinum has nonempty interior. We prove that thin continua are hereditarily locally connected, we provide some characterizations, we discuss maps of thin continua, and universal thin dendrites with bounded order of ramification points.

Endpoints of nondegenerate hereditarily decomposable chainable continua.
Roshan Adikari
Texas Tech University
roshan.adikari@ttu.ecu
We use the concept of "pseudo-endpoints" to investigate properties of endpoints of nondegenerate hereditarily decomposable chainable continua.

## Whitney blocks and $m$-mutual aposyndesis

Maria Elena Aguilera
Missouri University of Science and Technology
maria.aguilera78@gmail.com
Let $C(X)$ the hyperspace of subcontinua of a continuum $X$. A Whitney block is a set of the form $\mu^{-1}([s, t])$, where $\mu: C(X) \rightarrow[0,1]$ is a Whitney map and $0 \leq s<t \leq 1$. This presentation reports the following theorem: if $X$ is $m$-mutually aposyndetic, for $m \geq 2$, then each Whitney block also has this property.

## Properties of Endpoints and Opposite Endpoints of Hereditarily Decomposable Chainable Continua

Ameen A Alhassan
ameen9696@gmail.com
Let $X$ be an hereditarily decomposable chainable continuum such that $X$ is the union of two proper subcontinua $H$ and $K$ and their intersection is nondegenerate. Properties of endpoints and opposite endpoints of such continuum $X$ and the subcontinua $H$ and $K$ will be considered.

## Planar embeddings of chainable continua

Ana Anušić
University of Zagreb
ana.anusic@fer.hr
Coauthors: Henk Bruin and Jernej Cinc (Vienna)
We study planar embeddings of chainable continua. Using the theorem of Mazurkiewicz we show that every indecomposable chainable continuum has uncountably many non-equivalent planar embeddings. This answers the question by Mayer from 1982.

## Possible insights into homogeneous indecomposable continua

David P Bellamy
University of Delaware
argenthorn@yahoo.com
J T Rogers, Jr asked more than thirty years ago whether there are homogeneous indecomposable continua of dimension greater than one. I will present two theorems, or perhaps they are more properly called lemmas, which will restrict to some extent what such examples might look like. The results could perhaps be summarized by saying that a higher dimensional homogeneous indecomposable continuum has subcontinua with properties that seem extremely far-fetched.

## Ample Continua in Cartesian Products of Continua

Jan P. Boroński
AGH University of Science and Technology $\mathcal{G}$ IT4Innovations University of Ostrava
jan.Boroński@osu.cz
Coauthors: D.R. Prier, and M. Smith
Recall that the notion of an ample continuum was introduced by Prajs and Whittington in [PW]. In [BL] Bellamy and Lysko showed that a compact and connected topological group has ample diagonal in $G \times G$ if and only if $G$ is locally connected. They asked if the product of a Knaster continuum $K$ and solenoid $S$ has the property that any subcontinuum of $K \times S$, that projects onto both coordinate spaces, has arbitrarily small connected open neighborhoods. Taking advantage of a result of Illanes [I] we answer their question in the affirmative (see [BPS]).

## References

[BL] Bellamy, D. P.; Lysko, J. M. Connected open neighborhoods of subcontinua of product continua with indecomposable factors Topology Proc., 44 (2014), 223-231.
[BPS] Boroński J.P.,Prier D.R., Smith M., Ample continua in Cartesian products of continua arXiv:1709.01885
[I] Illanes, A. Connected open neighborhoods in products Acta Math. Hungar. 148 (2016), 73-82
[PW] Prajs, J.; Whittington, K. Filament sets, aposyndesis, and the decomposition theorem of Jones
Trans. Amer. Math. Soc. 359 (2007), no. 12, 5991-6000.

A characterization of hereditarily contractible continua<br>Włodzimierz J. Charatonik (4:30-5:00 Wednesday)<br>Missouri University of Science and Technology<br>wjcharat@mst.edu

We will sketch a proof of a characterization of hereditarily contractible continua as pointwise smooth dendroids. The definition of a pointwise smooth dendroid and a proof that every hereditarily contractible continuum is a pointwise smooth dendroid are due to Stanislaw T. Czuba given in his work from the 1980's. The opposite implication requires a lot of preparation and use of Michael's selection Theorem.

## Janusz R. Prajs - an ample mathematician

Włodzimierz J. Charatonik (4:00-4:30 Thursday)
Missouri University of Science and Technology
wjcharat@mst.edu
We will recall ample discoveries in mathematics done by Janusz R. Prajs, pointing out that the inhomogeneity of his body of work adds spice to an already flavorful career.
$n$-ods, strong $n$-ods and weak $n$-ods
Alejandro Illanes
Instituto de Matematicas, UNAM
illanes@matem. unam.mx
Coauthors: Norberto Ordoñez
A continuum $X$ is: (a) an $n$-od if there exists a subcontinuum $A$ of $X$ such that $X \backslash A$ has at least $n$ components, (b) a strong $n$-od if there exists a subcontinuum $A$ of $X$ such that $X \backslash A$ is the union of $n$ nonempty pairwise disjoint subsets whose closures are pairwise disjoint, and (c) a weak $n$-od if there exist $n$ subcontinua $K_{1}, \ldots, K_{n}$ of $X$ such that the common intersection of all $K_{i}$ 's is nonempty and no $K_{i}$ is contained in the union of the others. In this talk we will discuss some relations among these concepts. In the case that $X$ is a finite graph, we also show how to see if $X$ is an $n$-od or a weak $n$-od.

## Continua that are Subcontinuum-like or Hereditarily Self-like

Matt Insall
Missouri University of Science and Technology
insall@mst.edu
Coauthors: Włodzimierz J. Charatonik
A (non-degenerate) continuum $X$ is hereditarily self-like provided that each of its non-degenerate subcontinua are $X$-like, while a (non-degenerate) continuum $X$ is subcontinuum-like iff for every non-degenerate subcontinuum $K$ of $X, X$ is $K$-like. This presentation reports on progress from our introduction of these notions last year, answering some of the problems that were open then. In particular, we construct a hereditarily self-like continuum that is not tree-like.

Markov set-valued functions and their inverse limits<br>James Kelly<br>Christopher Newport University<br>james.kelly@cnu.edu<br>Coauthors: Lori Alvin

We introduce the definition of a Markov set-valued function and show that the inverse limits of two similar Markov set-valued functions are homeomorphic. This generalizes results of S. Holte, I. Banič, M. Črepnjak, and T. Lunder. The definition we present differs from previous definitions of Markov interval functions in that we allow for points outside of the Markov partition to have non-degenerate images. Additionally, our definition focuses on the structure of the inverse of our function; we require that the inverse is a union of continuous mappings with specified restrictions on the domains, ranges, and points of intersection.

A Surprising Example for Topological Entropy<br>Judy Kennedy<br>Lamar University<br>kennedy9905@gmail.com<br>Coauthors: Goran Erceg

We define topological entropy for closed subsets of the unit square. Then we give an example of a closed subset of the square with 0 entropy, but if any other point of the square is added, the new set has infinite entropy.

An upper semicontinuous function $f$ whose graph is homeomorphic to the inverse limit of closed unit intervals with $f$ as a bonding function
Bostjan Lemez
University of Maribor
bostjan.lemez@gmail.com
We construct a nontrivial family of upper semicontinuous functions $f:[0,1] \rightarrow 2^{[0,1]}$ with the property that the graph of $f$ is homeomorphic to the inverse limit of the inverse sequence of closed unit intervals $[0,1]$ with $f$ as the bonding function. As a special case, we use this construction to produce the Gehman dendrite as a graph of such function.

## Homeomorphisms of the pseudo-arc

Wayne Lewis
Texas Tech University
wayne.lewis@ttu.edu
We present a few known results and a larger number of open questions about the topological group of homeomorphisms of the pseudo-arc

## On Jones' and Prajs' decomposition theorems

Sergio Macias
Institute of Mathematics, National University of Mexico
sergiom@matem. unam.mx
A continuum is a compact connected metric space. A continuum $X$ is homogeneous provided that for each pair of points $x_{1}$ and $x_{2}$ of $X$, there exists a homeomorphism $h: X \rightarrow X$ such that $h\left(x_{1}\right)=x_{2}$. We show:

Theorem. Given a homogeneous continuum $X$, let $\mathcal{G}=\left\{\mathcal{T}_{X}(\{x\}) \mid x \in X\right\}$ be Jones' decomposition, let $X_{J}=X / \mathcal{G}$ and let $q_{J}: X \rightarrow X_{J}$ be the quotient map. Let $\mathcal{Q}=\left\{Q_{x} \mid x \in X\right\}$ be Prajs' decompositon, let $X_{P}=X / \mathcal{Q}$ and let $q_{P}: X \rightarrow X_{P}$ be the quotient map. For $X_{J}$, let $\mathfrak{Q}_{J}=\left\{\mathcal{Q}_{\zeta} \mid \zeta \in X_{J}\right\}$ be Prajs' decomposition, let $X_{J P}=X_{J} / \mathfrak{Q}_{J}$ and let $q_{J P}: X_{J} \rightarrow X_{J P}$ be the quotient map. Then $X_{J P}$ is homeomrophic to $X_{P}$.

# A fixed point theorem for conical shells 

Marcus Marsh
Calif St Univ, Sacramento, Emeritus Professor
mmarsh@csus.edu
Coauthors: Andras Domokos
We prove a fixed point theorem for mappings $f$ defined on a conical shell $F$ in Euclidean $n$-space, where the image of $f$ need not be a subset of $F$, nor even a subset of the cone that contains $F$. Sufficiency for fixed points of $f$ is dependent only on the behavior of $f$ on the boundary of $F$, and is related to notions of compressing or extending $F$ as defined by Krasnoselskii. We also discuss possible extensions of our theorem to infinite dimensional Banach spaces.

## Symmetric Products and $n$-Fold Hyperspaces as Cones

Verónica Martínez de la Vega
Instituto de Matemáticas, UNAM
vmvm@matem.unam.mx
Coauthors: Alejandro Illanes, Daria Michalik
For a continuum $X$, let $F_{n}(X)$ be the hyperspace of all nonempty subsets of $X$ with at most $n$ points. We prove that if $X$ is a locally connected curve, then the following conditions are equivalent: (a) $X$ is a cone, (b) $F_{n}(X)$ is a cone for some $n \geq 2$, and (c) $F_{n}(X)$ is a cone for each $n \geq 2$. For a continuum $X$, let $C_{n}(X)$ be the hyperspace of nonempty closed subsets of $X$ with at most $n$ components. We prove that if $X$ is a fan and $n$ is different from 2 , then $C_{n}(X)$ is a cone if and only if $X$ is a cone.

## Locally connected generalized inverse limits

Faruq Mena
Missouri University of Science and Technology
famdn2@mst.edu
Coauthors: Włodzimierz J. Charatonik
We prove a theorem that under some conditions local connectedness is preserved under set-valued inverse limits. The theorem generalizes Capel's theorem that local connectedness is preserved under (single-valued) inverse limits with monotone bonding functions and its set-valued analogue by James Kelly (see J. Kelly, Monotone and weakly confluent set-valued functions and their inverse limits, Topology Appl. 228 (2017), 486-500). As a consequence we can characterize some set-valued inverse limits on intervals.

## Homogeneity degree of some Cartesian products

Daria Michalik
Cardinal Stefan Wyszyński University, Poland
d.michalik@uksw.edu.pl

Let $\mathcal{H}(X)$ denote the group of homeomorphisms of $X$ onto itself. By an orbit of $X$ we mean an orbit under the action of $\mathcal{H}(X)$ on $X$. We say that the homogeneity degree of $X$ is $n$ if $X$ has exactly $n$ orbits, in symbols $d_{H}(X)=n$.

Obviously, for every pair of topological spaces $X$ and $Y$,

$$
d_{H}(X \times Y) \leq d_{H}(X) \cdot d_{H}(Y)
$$

and the equality, in general, doesn't hold.
In my talk I shall determine the homogeneity degree of the Cartesian product $C \times M$ in terms of that of $C$ and $M$, for $C$ being a locally connected curve and $M$ being a manifold.

## Reversible properties of inverse limits with a single set valued function.

## Van Nall <br> University of Richmond <br> vnall@richmond.edu

If $f$ is an upper semi-continuous set valued function from a compact set $X$ into the closed subsets of $X$ and the inverse limit of $f$ is connected, then the inverse limit of $f^{-1}$ is connected. So we say being connected is a reversible property. We will explore other reversible properties but with restrictions on the bonding functions and the factor spaces since there do not appear to be any properties as broadly reversible as connectedness.

Commuting maps on triods, intervals, and related spaces that do or do not have coincidence and/or common fixed points.
Jeffrey Norden
Tennessee Tech University
jeff@math.tntech.edu
In early 80 's, as a new grad student, I read a preprint of the now famous Oversteegen-Rogers version of Bellamy's example, and decided to work on the following innocent looking question:

Do there exist a pair of maps on a simple triod which commute and are coincidence-point free?
(I.e, $f \circ g=g \circ f$ and $f \cap g=\emptyset$.)

Fortunately (for me) I eventually found a less innocent looking problem that I was able to solve and write a dissertation about. On the other hand, I never lost interest in the the triod problem. In this talk I'll survey what I have and (mostly) haven't been able to figure out about commutativity questions in the past 35 or so years.

Dendroids with low homogeneity degree.<br>Yaziel Pacheco Juárez<br>Facultad de Ciencias Exactas, Universidad Juárez del Estado de Durango.<br>yazi28@hotmail.com

The homogeneity degree of a topological space $X$ is the number of orbits of the action of the homeomorphism group of $X$ on $X$. A dendroid is an arcwise connected and hereditarily unicoherent continuum, and a fan is a dendroid with exactly one ramification point. In this talk we will discuss fans with homogeneity degree four, and smooth dendroids with homogeneity degree less than or equal to three.

Menger curve as a set-valued inverse limit<br>Şahika Şahan<br>Missouri University of Science and Technology<br>ssxx4@mst.edu<br>Coauthors: Włodzimierz J. Charatonik

We show that the Menger Curve can be represented as a set-valued inverse limit of intervals $[0,1]$ with a single bonding function. The graph of the function is homeomorphic to the Sierpinski Carpet. This answers a question by M. Hiraki and H. Kato.

## Generalized Inverse Limits that Banish their Graphs

## Scott Varagona

University of Montevallo
svaragona@montevallo.edu
Suppose $f:[0,1] \rightarrow 2^{[0,1]}$ is a set-valued surjective function, and suppose that $K$, the inverse limit with the single bonding function $f$, is a continuum. Then let us say the inverse limit $K$ banishes the graph of $f$ if $K$ contains no copy of the graph of $f$. Moreover, the inverse limit $K$ completely banishes the graph of $f$ if $K$ contains no non-degenerate subcontinuum of the graph of $f$. We will construct some generalized inverse limits that banish (or completely banish) their graphs, and discuss the implications of this phenomenon to the theory of inverse limits.

## On hyperspaces $C(p, X)$

Hugo Villanueva
Universidad Autónoma de Chiapas
hvillam@gmail.com
Given a metric continuum $X$, let $C(X)$ be the hyperspace of subcontinua of $X$. For each $p \in X$, let $C(p, X)$ be denote the hyperspace of all subcontinua of $X$ containing $p$, and let $\mathcal{K}(X)$ be the family of all hyperspaces $C(p, X)$. In this talk, we present some results concerning hyperspaces $C(p, X)$ and $\mathcal{K}(X)$.

## Dynamical Systems

Endpoints of inverse limits for a family of set-valued functions<br>Lori Alvin<br>Bradley University<br>lalvin@bradley.edu<br>Coauthors: James Kelly

Given a set-valued upper semi-continuous function whose inverse is the union of mappings, J. Kelly was able to characterize the collection of endpoints of the inverse limit assuming the following definition of an endpoint: $p$ is an endpoint of a continuum $X$ if for any two subcontinua $H, K \subseteq X$ which both contain $p$, either $H \subseteq K$ or $K \subseteq H$. It remained an open question as to whether this characterization would hold using other definitions of an endpoint. We provide an example where the characterization does not hold using Lelek's definition: $p$ is an endpoint of a continuum $X$ if $p$ is an endpoint of any arc in $X$ containing $p$. We then extend the example to study the collection of endpoints for a family of set-valued functions obtained by attaching an arc to the critical point of a symmetric tent map.

## Models of some spaces of complex polynomials of arbitrary degree

Alexander Blokh
$U A B$
ablokh@math.uab.edu
Coauthors: L. Oversteegen (UAB), V. Timorin (Higher School of Economics, Moscow, Russia)
Consider the family of all polynomials P of degree d with connected Julia set and such that (1) all their periodic points are repelling, and (b) there exists a fixed point at which all invariant external rays of P land. Using the original ideas of Thurston and relying upon our recent results we construct a model of this family of polynomials.

## The Denjoy-Rees construction on the pseudo-circle

Jan P. Boroński
AGH University of Science and TechnologyEIT4Innocations University of Ostrava
jBoroński@wms.mat.agh.edu.pl
Coauthors: Judy Kennedy, Xiaochuan Liu, and Piotr Oprocha
In 1981 Rees constructed a minimal homeomorphism of the $n$-torus with positive topological entropy, by enriching the dynamics of an irrational rotation $[R]$. Her construction was generalized and further developed by Béguin, Crovisier, and Le Roux [BCL]. We apply their results to exhibit minimal noninvertible maps on the pseudo-circle. This resolves a conjecture from $[\mathrm{BKS}],[\mathrm{S}]$ and adds to the recent results on minimal noninvertible maps obtained in [BCO].

## References

[BCL] F. Béguin, S. Crovisier and F. Le Roux. Construction of curious minimal uniquely ergodic homeomorphisms on manifolds: the Denjoy-Rees technique. Ann. Sci. cole Norm. Sup. (4), 40(2):251-308, 2007
[BCO] Boroński J.P.; Clark A.; Oprocha P., A compact minimal space $Y$ such that its square $Y x Y$ is not minimal, arXiv:1612.09179
[BKS] Bruin, H.; Kolyada, S.; Snoha, L’. Minimal nonhomogeneous continua. Colloq. Math. 95, 123-132 (2003)
[R] M. Rees, A minimal positive entropy homeomorphism of the 2-torus, J. London Math. Soc. 23 (1981) 537-550.
[S] Snoha, L'. On some problems in discrete dynamics (cycles, chaos, topological entropy, minimality), D.Sc. Thesis, Academy of Sciences of Czech Republic, Prague, 2005.

## Dynamical systems on $\mathbb{N}^{*}$ and their quotients

Will Brian
UNC Charlotte
wbrian.math@gmail.com
A "mod-finite permutation" of $\mathbb{N}$ is a bijection from one co-finite subset of $\mathbb{N}$ to another. If $p$ is a mod-finite permutation of $\mathbb{N}$, then it induces a self-homeomorphism $p^{*}$ on the space $\mathbb{N}^{*}$ of non-principal ultrafilters on $\mathbb{N}$. In this talk, I will try to convince you that the dynamical systems of the form ( $\mathbb{N}^{*}, p^{*}$ ) are important. Then I will present a recent theorem that characterizes exactly when a dynamical system $(X, f)$ is a quotient of a dynamical system of the form $\left(\mathbb{N}^{*}, p^{*}\right)$.

```
Matching for translated }\beta\mathrm{ -transformations.
Henk Bruin
University of Vienna
henk.bruin@univie.ac.at
Coauthors: Charlene Kalle (University of Leiden) Carlo Carminati (University of Pisa)
```

This talk is about the parameter space of a family of shifted $\beta$-transformations $T$ in regard to the property of matching. This means that at some iterate $T^{n}(0)=T^{n}(1)$, and this has a bearing on the invariant density of $T$. The prevalence of matching is proved under specific number-theoretic conditions: the slope $\beta$ is a (specifically quadratic) Pisot number.

Lakes of Wada rotational attractor<br>Jernej Cinc<br>Nipissing University<br>jernejc@nipissingu.ca<br>Coauthors: Jan P. Boroński

A topological attractor is called rotational attractor, if its external prime ends rotation number is nonzero. In this talk I will give a construction of a rotational attractor of the 2 -sphere that is Lakes of Wada continuum (i.e. the complement of the continuum has three connected components and the continuum is the boundary of each of them separately).

Volumes of Bounded Remainder Sets for Rotations on the Adelic Torus<br>Joanna Furno<br>University of Houston<br>jfurno@math.uh.edu<br>Coauthors: Alan Haynes, Henna Koivusalo

In joint work with Alan Haynes and Henna Koivusalo, we determine the volumes of bounded remainder sets for irrational rotations on the Adelic torus. The first part uses Fourier analysis to restrict the possible volumes of bounded remainder sets. The second part uses a cut-and-project construction to show that these possible volumes are realized. This cut-and-project construction on the Adelic torus is an adaptation of the construction for irrational rotations on the torus, developed by Alan Haynes, Michael Kelly, and Henna Koivusalo.

## Characterising the sets of periodic points for solenoidal automorphisms.

Sharan Gopal
Assistant Professor, BITS-Pilani, Hyderabad campus, India.
sharanraghu@gmail.com
Characterisation of the sets of periodic points of a family of dynamical systems is a well studied problem in the literature. Here, we consider this problem for the family of automorphisms on a solenoid. By definition, a solenoid is a compact connected finite dimensional abelian group. Equivalently, a topological group $\Sigma$ will be a solenoid if its Pontryagin dual is a subgroup of $\mathbb{Q}^{n}$ and also contains $\mathbb{Z}^{n}$ as a subgroup for some positive integer $n$. When the dual is equal to $\mathbb{Z}^{n}$, the solenoid is actually an $n$-dimensional torus.

The sets of periodic points of automorphisms on a 2-dimensional torus was published in J. Math. Anal. Appl. in 2010. In a recent paper, that appeared in Topology Proceedings (Gopal, Raja; Vol. 50 (2017), 49-57), we considered the problem for general solenoids. First, the characterization is done for higher dimensional toral automorphisms and then follows the one-dimensional solenoids. The main reason behind dealing with the one-dimensional solenoids separately was the availability of a neat description of subgroups of $\mathbb{Q}$. This helped us in giving a nice form to one-dimensional solenoids and thus characterizing the sets of periodic points of automorphisms on it. However, the general problem is still incomplete. In this talk, I will show all the above results giving a brief idea of some proofs and then discuss some ideas that probably will help in solving the general problem. As a part of this, an another approach to this problem by considering a one-dimensional solenoid as the inverse limit of an inverse system of circles will also be discussed.

## 2-manifolds and inverse limits of set-valued functions on intervals

Sina Greenwood
University of Auckland
sina@math.auckland.ac.nz
Coauthors: Rolf Suabedissen
Suppose for each $n \in \mathbb{N}, f_{n}:[0,1] \rightarrow 2^{[0,1]}$ is a function whose graph $\Gamma\left(f_{n}\right)=\left\{(x, y) \in[0,1]^{2}: y \in f_{n}(x)\right\}$ is closed in $[0,1]^{2}$ (here $2^{[0,1]}$ is the space of non-empty closed subsets of $[0,1]$ ). We show that the generalized inverse limit $\lim \left(f_{n}\right)=\left\{\left(x_{n}\right) \in[0,1]^{\mathbb{N}}: \forall n \in \mathbb{N}, x_{n} \in f_{n}\left(x_{n+1}\right)\right\}$ of such a sequence of functions cannot be an arbitrary continuum, answering a long-standing open problem in the study of generalized inverse limits. In particular we show that if such an inverse limit is a 2 -manifold then it is a torus and hence it is impossible to obtain a sphere.

Markov-like set-valued functions and their inverse limits<br>Hayato Imamura<br>Waseda University<br>hayato-imamura@asagi.waseda.jp

The same pattern of Markov maps on intervals was introduced by S. Holte (2002) and she showed that any two inverse limits with Markov bonding maps with the same pattern are homeomorphic. I. Banič and T. Lunder (2013) extended the notation of Markov map from continuous maps to set-valued functions, called generalized Markov interval functions, and applied the theory of generalized inverse limits with set-valued functions. In this talk, we introduce Markov-like functions as a generalization of generalized Markov interval functions and show that any two generalized inverse limits with Markov-like bonding functions having the same pattern are homeomorphic. Consequently, we can give a generalization of S. Holte, I. Banič, and T. Lunder. Recently we can have a generalization to Markov-like functions on finite graphs.

## Silent circulation of polio

James Keesling
University of Florida
kees@ufl.edu
Coauthors: Celeste Vallejo, Jim Koopman, Burt Singer
We present a stochastic dynamical system for the silent circulation of polio. Simulations of the system indicate that live polio virus can persist in a small isolated community for longer than was thought. This has implications for the eradication of the disease.

## Arboreal Cantor actions

Olga Lukina
University of Illinois at Chicago
lukina@uic.edu
The asymptotic discriminant is an invariant of actions of discrete groups on Cantor sets, recently introduced by the speaker in a joint work with Hurder. The asymptotic discriminant arises as a sequence of surjective group homomorphisms of certain profinite groups, associated to the action.

An arboreal representation of the absolute Galois group of a field is a profinite group, acting on the boundary of a spherically homogeneous rooted tree. In this talk, we show how one can compute the asymptotic discriminant for such representations. We give examples of arboreal representations with stable and wild asymptotic discriminant.

Weak mixing in general semiflows implies multi-sensitivity, but not thick sensitivity<br>Alica Miller<br>University of Louisville<br>alica.miller@louisville.edu

Research on relations between the mixing properties and sensitivity in semiflows has a long history. Let us mention some recent papers. In 2004 it was proved by L. He, X. Yan and L. Wang that weak mixing implies sensitivity in the context of Ergodic Theory. In 2006 it was proved by S. Lardjane that strong mixing implies sensitivity for topological semiflows. In 2007 T. S. Moothathu introduced the notion of multi-sensitivity for semiflows and mentioned that: (1) every weakly mixing compact cascade is multi-sensitive, and (2) every multi-sensitive compact cascade is thickly sensitive. (Note that in compact cascades every transition map is surjective.) In 2017 it was proved by T. Wang, J. Yin and Q. Yan that any semiflow on a compact metric space (with an arbitrary acting topological monoid) whose every transition map is surjective is thickly sensitive, thus generalizing Moothathu's comments. In this talk we will consider what happens if we do not have the assumptions of compactness of the phase space and surjectivity of the transition maps. We will show that the semiflow is still multi-sensitive, but that, however, it does not have to be thickly sensitive. We will thus generalize Moothathu's first comment to the case of general semiflows and show that his second comment does not hold in general semiflows.

## Continuous-wise distal homeomorphisms

C.A. Morales

Federal University of Rio de Janeiro, Brazil
morales@impa.br
Coauthors: D. Carrasco-Olivera.
We incorporate the distal homeomorphisms into the continuum theory through the notion of continuumwise distal homeomorphism. The cw-distal homeomorphisms constitute a class much larger than that of distal homeomorphisms. We study several properties of cw-distal homeomorphisms. Some results concerning distal homeomorphisms will be generalized to the case of cw-distal homeomorphisms. Notions of cw-distality for measures are also introduced and discussed.

## Exact maps of the pseudo-arc

Christopher Mouron
Rhodes College
mouronc@rhodes.edu
A pseudo-arc is an hereditarily indecomposable chainable continuum. A map $f: X \longrightarrow X$ is exact if for every nonempty open set $U \subset X$, there exists an $n$ such that $f^{n}(U)=X$. In this talk I will construct an exact map on the pseudo-arc. Then I will show that every exact map of the pseudo-arc that can be realized with commuting maps on the bonding maps of the inverse limit construction of the pseudo-arc must have infinite entropy.

Mixing properties in expanding Lorenz maps<br>Piotr Oprocha<br>AGH University, Poland<br>oprocha@agh.edu.pl<br>Coauthors: P. Potorski and P. Raith

In this talk we provide sufficient conditions when expanding Lorenz map is topologically mixing and fully characterize this property for $\mathrm{T}(\mathrm{x})=\beta \mathrm{x}+\alpha(\bmod 1)$ within some range of parameters $\alpha, \beta$. Furthermore, relations between renormalizability and Lorenz map being locally eventually onto are considered, and some gaps in classical results on the dynamics of Lorenz maps are corrected.

## Perfect subspaces of quadratic laminations

## Lex Oversteegen

$U A B$
overstee@uab.edu
Coauthors: A. Blokh and V. Timorin
The combinatorial Mandelbrot set is a continuum in the plane, whose boundary is defined as the quotient space of the unit circle by an explicit equivalence relation. This equivalence relation was described by Douady and, separately, by Thurston who used quadratic invariant geolaminations as a major tool. We showed earlier that the combinatorial Mandelbrot set can be interpreted as a quotient of the space of all limit quadratic invariant geolaminations with the Hausdorff distance topology. In this paper, we describe two similar quotients. In the first case, the identifications are the same but the space is smaller than that used for the Mandelbrot set. The resulting quotient space is obtained from the Mandelbrot set by "unpinching" the transitions between adjacent hyperbolic components. In the second case we identify renormalizable geolaminations that can be "unrenormalised" to the same hyperbolic geolamination while no two non-renormalizable geolaminations are identified.

## Periodic orbits in a chaotic attractor introduced by Clark Robison

Michael Sullivan
Southern Illinois University - Carbondale
mikesullivan@math.siu.edu
Coauthors: Ghazwan Alhashimi - University of Diyala, Iraq
We show that the periodic orbits in a chaotic attractor introduced by Clark Robinson are fibered knots.

## Zero-temperature measures, entropy and rotation sets

Christian Wolf
Department of Mathematics, The City College of New York, NY, NY, 10031
cwolf@ccny.cuny.edu
Coauthors: Michael Burr and Yun Yang
Zero-temperature measures are limits of equilibrium states when the temperature goes to zero. They play an important role in statistical physics. In this talk we consider subshifts of finite type and discuss a topological classification of locally constant potentials via their zero-temperature measures. Our approach is to analyze the relationship between the distribution of the zero-temperature measures and the boundary of higher dimensional generalized rotation sets. If time permits we also discuss computability results for the entropy of zero-temperature measures. The material presented in this talk combines joint works with Yun Yang and Michael Burr.

## Topological transitivity and representability of surfaces flows

Tomoo Yokoyama
Kyoto University of Education, Japan
tomoo@kyokyo-u.ac.jp
In this talk, we present a necessary and sufficient condition for the existence of dense orbits of continuous flows on compact connected surfaces, which is a generalization of a necessary and sufficient condition on area-preserving flows obtained by H. Marzougui and G. Soler López. Moreover, we consider what class of flows on compact surfaces can be characterized by finite labeled graphs. We show that a class of surface flows, up to topological conjugacy, which contains both the set of Morse Smale flows and the set of area-preserving flows with finite singular points can be characterized. In other words, we construct a complete invariant for surface flows of "finite type". In fact, although the set of topological equivalent classes of minimal flows on a torus is uncountable, we enumerate the set of topological equivalent classes of flows with non-degenerate singular points and with at most finitely many limit cycles but without non-closed recurrent orbits on a compact surface using finite labelled graphs.

# Geometric Group Theory 

Group trisections and smooth 4-manifolds<br>Aaron Abrams<br>Washington and Lee University<br>abramsa@wlu.edu<br>Coauthors: David Gay, Rob Kirby

The theory of 4-manifold trisections pioneered by Gay and Kirby offers a new and exciting approach to the study of smooth 4-dimensional topology. We will discuss a group-theoretic interpretation of trisections which in principle allows the transport of problems (e.g. the 4-dimensional Poincar conjecture) from smooth topology to group theory.

Embedding Right-Angled Artin Groups into Brin-Thompson Groups<br>Jim Belk<br>Bard College<br>belk@bard.edu<br>Coauthors: Collin Bleak, Francesco Matucci

The Brin-Thompson groups $n V$ are a family of higher-dimensional generalizations of Thompson's group $V$. We show that any right-angled Artin group can be embedded into $n V$ for sufficiently large $n$. It follow that many other groups can also be embedded into Brin-Thompson groups, and indeed the subgroup structure of these groups is much richer than the subgroup structure for Thompson's group $V$.

## The topology of representation varieties

Maxime Bergeron
University of Chicago
mbergeron@math.uchicago.edu
Let H be a finitely generated group, let G be a complex reductive algebraic group (e.g. a special linear group) and let K be a maximal compact subgroup of G (e.g. a special unitary group). I will discuss exceptional classes of groups H for which there is a deformation retraction of $\operatorname{Hom}(\mathrm{H}, \mathrm{G})$ onto $\operatorname{Hom}(\mathrm{H}, \mathrm{K})$, thereby allowing us to obtain otherwise inaccessible topological invariants of these representation spaces.

# Weakly aperiodic SFTs on lamplighter groups. 

David Bruce Cohen<br>University of Chicago<br>davidbrucecohen@gmail.com

A subshift of finite type (SFT) is a symbolic dynamical system defined by a finite collection of "local rules". For instance, for any natural number k and any group G equipped with a finite generating set S , the set of all valid k-colorings of the corresponding Cayley graph of G (colorings of the Cayley graph in which no two adjacent vertices have the same color) forms an SFT. It is clear that any SFT X over a group G carries a G-action, and X is said to be weakly aperiodic if it is nonempty and has no finite G-orbits. When $G=Z$, there are no weakly aperiodic SFTs over $G$, but when $G=Z^{2}$ such SFTs do exist, as was shown by Berger. Carroll and Penland conjectured that a group with no weakly aperiodic SFT must be virtually cyclic. We will discuss some known obstructions to a group G being a counterexample to this conjecture (meaning that G is not virtually cyclic, but still admits a weakly aperiodic SFT), and explain why lamplighter groups were the most natural candidate. Time permitting, we will briefly discuss our proof that a lamplighter group cannot actually be a counterexample.

## Subgroup distortion in hyperbolic groups.

Pallavi Dani
Louisiana State University
pdani@math.lsu.edu
Coauthors: Tim Riley
The distortion function of a subgroup measures the extent to which the intrinsic word metric of the subgroup differs from the metric induced by the ambient group. Olshanskii showed that there are almost no restrictions on which functions arise as distortion functions of subgroups of finitely presented groups. This prompts one to ask what happens if one forces the ambient group to be particularly nice, say, for example, to be hyperbolic. I will survey which functions are known to be distortion functions of subgroups of hyperbolic groups. I will then describe joint work with Tim Riley which adds to this list: we construct free subgroups of hyperbolic groups with distortion functions $2^{n^{p / q}}$, for all integers $p>q>0$.

Isomorphisms and abstract commensurations of big mapping class groups<br>Spencer Dowdall<br>Vanderbilt University<br>spencer.dowdall@vanderbilt.edu<br>Coauthors: Juliette Bavard and Kasra Rafi

It is a classic result of Ivanov that the mapping class group of a finite-type surface is equal to its own automorphism group. Relatedly (aside from low-complexity exceptions), it is well-known that nonhomeomorphic surfaces cannot have isomorphic mapping class groups. In the setting of "big mapping class groups", that is of infinite-type surfaces, these considerations are complicated by the fact that their sheer enormity and variety of behavior prevents group elements from having canonical descriptions in terms of normal forms. This talk will present work with Juliette Bavard and Kasra Rafi overcoming these difficulties and extending the above results to big mapping class groups. In particular, we show that any isomorphism between big mapping class groups is induced by a homeomorphism of the surface and that each big mapping class group is equal to its abstract commensurator.

## Hierarchies of Relatively Hyperbolic Non-Positively Curved Cube Complexes Eduard Einstein <br> Cornell University <br> ee256@cornell.edu

A non-positively curved (NPC) cube complex is a combinatorial complex constructed by gluing Euclidean cubes along faces in a way that satisfies a combinatorial local non-positive curvature condition. A hierarchy is an inductive method of decomposing the fundamental group of a cube complex. Cube complexes and hierarchies of cube complexes have been studied extensively by Wise and feature prominently in Agol's proof of the Virtual Haken Conjecture for hyperbolic 3-manifolds. In this talk, I will give an overview of the geometry of cube complexes, explain how to construct a hierarchy for a NPC cube complex, and discuss applications of cube complex hierarchies to hyperbolic and relatively hyperbolic groups.

## Simple closed curves with controlled intersections

Jonah Gaster
McGill University
jbgaster@gmail.com
Coauthors: Tarik Aougab, Ian Biringer
Farb and Leininger asked: How many distinct (isotopy classes of) simple closed curves on an orientable surface of Euler characteristic $\chi$ may pairwise intersect at most $k$ times? Przytycki has shown that this number grows at most as a polynomial in $|\chi|$ of degree $k^{2}+k+1$.

We present improved bounds. The most interesting case is that of $k=1$, in which case the size of a 'maximal 1-system' grows no faster than $|\chi|^{3} /(\log |\chi|)^{2}$. Following Przytycki, the proof uses the hyperbolic geometry of surfaces essentially. In particular, we make use of bounds for the maximum size of a collection of curves of length at most $L$ on a hyperbolic surface homeomorphic to $S$ that are independent of the hyperbolic structure. This is joint work with Tarik Aougab and Ian Biringer.

## Counting conjugacy classes in $\operatorname{Out}\left(F_{N}\right)$

Michael Hull
University of Florida
mbhull@ufl.edu
Coauthors: Ilya Kapovich
We show that if a f.g. group $G$ has a non-elementary WPD action on a hyperbolic metric space $X$, then the number of conjugacy classes of loxodromic elements of $G$ coming from a ball of radius $R$ in the Cayley graph of $G$ grows exponentially in $R$. As an application we prove that for $N \geq 3$ the number of distinct $O u t\left(F_{N}\right)$-conjugacy classes of fully irreducibles $\phi$ from an $R$-ball in the Cayley graph of $O u t\left(F_{N}\right)$ with $\log \lambda(\phi)$ on the order of $R$ grows exponentially in $R$.

A uniform McCarthy-type theorem for linearly growing outer automorphisms of a free group<br>Edgar A. Bering IV<br>Temple University<br>edgar.bering@temple.edu

In his proof of the Tits alternative for the mapping class group of a surface, McCarthy also proved that given any two mapping classes $\sigma$ and $\tau$, there exists an integer $N$ such that the group generated by $\left\langle\sigma^{N}, \tau^{N}\right\rangle$ is either free of rank two or abelian. In the setting of $\operatorname{Out}\left(F_{r}\right)$, whether or not such a statement is true remains open, though there are many partial results. Later work in the mapping class group setting due to Hamidi-Tehrani showed that for Dehn twists the power N is uniform, which Mangahas used to prove that the mapping class groups have uniform-uniform exponential growth. I will present an Out $\left(F_{r}\right)$ analog of Hamidi-Tehrani's result.

## The first Betti number of the level 4 braid group

## Kevin Kordek

Georgia Institute of Technology
kevin.kordek@math.gatech.edu
Coauthors: Dan Margalit
It is generally a difficult problem to compute the Betti numbers of a given finite-index subgroup of an infinite group, even when the ambient group is well-understood. In this talk I will describe recent joint work with Dan Margalit on the rational cohomology of the level 4 braid group, which is the kernel of the mod 4 reduction of the integral Burau representation. The main result of our work is an explicit formula for the first Betti number. I will conclude with a few applications to the structure of some closely related groups.

## Groups of Type $F P_{2}$

Robert Kropholler
Tufts University
robert.kropholler@tufts.edu
Coauthors: Ian Leary and Ignat Soroko
I will discuss various constructions of groups of type $F P_{2}$ which are not finitely presented. I will start with the work of Bestvina and Brady namely, how they applied a version of morse theory to find the first examples of such groups. I will then move onto more recent work of Leary giving a technique for constructing uncountably many such groups. Finally, I will look at further work of Leary, Soroko and myself showing that there are uncountably many such groups up to quasi-isometry.

## Homology of finite covers of surfaces and simple closed curves

Justin Malestein
University of Oklahoma
justinmalestein@gmail.com
Coauthors: Andrew Putman
In this talk, I will discuss examples of finite covers of punctured surfaces where the first rational homology is not spanned by lifts of simple closed curves. Additionally, I will discuss analogous results for primitive elements and the homology of finite index subgroups of a free group. A couple consequences of these results include a theorem that $\operatorname{Out}\left(F_{n}\right)$ modulo the group generated by kth powers of transvections often has infinite order elements. This is joint work with Andrew Putman.

## Quasi-geodesics in Out(Fn)

Yulan Qing
University of Toronto
yulan.qing@utoronto.ca
Coauthors: Kasra Rafi
We study the behavior of quasi-geodesics in Out(Fn). Given an element $\Phi \in \operatorname{Out}(\mathrm{Fn})$ there are several natural paths connecting the origin to $\Phi$ in $\operatorname{Out}(\mathrm{Fn})$, for example, a path associate to sequence of Staling folds and a path associated to standard geodesics in Outer space. We show that neither of these paths is, in general, a quasi-geodesic in Out(Fn). In fact, we construct examples where any quasi-geodesic connecting $\Phi$ to the origin will have to back-track in some free factor of Fn.

# Boundaries of CAT(0) spaces with Isolated Flats Property 

Kim Ruane
Tufts University
kim.ruane@tufts.edu
Coauthors: Chris Hruska
Let G be a one-ended group acting geometrically on a $\mathrm{CAT}(0)$ space with the isolated flats property. Such a group is hyperbolic relative to the class of flat stabilizers, thus we can consider the maximal peripheral splitting of G. If this splitting satisfies a certain finite index condition, then we can describethe boundary as a "tree of metric compacta" in the sense of Swiatkowski. We use this to show that the boundary of such a group is locally connected. I will discuss this theorem and its applications in my talk.

## Surface bundles, monodromy, and arithmetic groups

Nick Salter
University of Chicago
salter@math.harvard.edu
Coauthors: Bena Tshishiku
The study of monodromy groups is an age-old problem at the juncture of topology, algebraic geometry, and the theory of algebraic groups. Some beautiful and poorly-understood monodromy groups have an intimate connection with the mapping class group. In this talk, I will describe an ongoing project with Bena Tshishiku studying some examples of monodromy groups arising from surface subgroups of the mapping class group (or depending on ones taste, arising as complete algebraic curves inside moduli space). The main result is that these monodromy groups are "as large as number-theoretic constraints allow them to be'.

## Obstructions of Riemannian smoothings of locally CAT(0) manifolds <br> Bakul Sathaye <br> The Ohio State University <br> sathaye.2@osu.edu

In this talk I will focus on obstructions in dimension $=4$ to Riemannian smoothings of locally CAT(0) manifolds. I will discuss how the obstruction given by Davis-Januszkiewicz-Lafont can be extended to construct more examples of locally $\operatorname{CAT}(0)$ 4-manifolds that do not support Riemannian metric with nonpositive sectional curvature. The universal covers of these manifolds satisfy the isolated flats condition and contain a collection of 2-dimensional flats with the property that their boundaries at infinity form non-trivial links in the boundary 3 -sphere.

# Separating Nekrashevych groups via finiteness properties 

Rachel Skipper
Binghamton University
skipper@math.binghamton.edu
Coauthors: Stefan Witzel Matthew C.B. Zaremsky
A group is of finiteness type $F_{n}$ if it admits a classifying space with compact $n$-skeleton. We discuss some recent results about finiteness properties of Nekrashevych groups, a class of groups whose building blocks are self-similar groups and Higman-Thompson groups. Since these groups are often virtually simple and since finiteness properties are a quasi-isometry invariant, we use these results to build new examples of non-quasi-isometric simple groups.

Stable commutator length in right-angled Artin groups<br>Ignat Soroko<br>University of Oklahoma<br>ignat.soroko@gmail.com<br>Coauthors: Max Forester, Jing Tao (University of Oklahoma)

The stable commutator length ( scl ) of an element in a group is a remarkable numerical invariant, which has relevance in several areas of low-dimensional topology, bounded cohomology and dynamics. In general, scl is very hard to compute, but for many important classes of groups it has been shown that the spectrum of possible values of scl has a gap above zero. In particular, Culler showed that for an arbitrary element $g$ of a free group, $\operatorname{scl}(g)$ is at least $1 / 6$. By using geometry of hyperplanes in CAT(0) cubical complexes, we adapt Culler's approach to the case of right-angled Artin groups (RAAGs). As a result, we get for arbitrary element $g$ of any RAAG the estimate: $\operatorname{scl}(g) \geq 1 / 6 k$, where $k$ is the chromatic number of the defining graph of the RAAG. In addition, for two-dimensional RAAGs, we obtain the lower bound $\operatorname{scl}(g) \geq 1 / 20$.

## Fibrations of 3-manifolds and nowhere continuous functions

Balazs Strenner
Georgia Tech
bstrenner7@gatech.edu
We start with a 3-manifold fibering over the circle and investigate how the pseudo-Anosov monodromies change as we vary the fibration. Fried proved that the stretch factor of the monodromies varies continuously (when normalized in the appropriate sense). In sharp contrast, we show that another numerical invariant, the asymptotic translation length in the arc complex, does not vary continuously.

# Semiduality in group cohomology 

Daniel Studenmund
University of Notre Dame
dstudenm@nd.edu
Coauthors: Kevin Wortman
A duality group has a pairing exhibiting isomorphisms between its homology and cohomology groups. Examples include solvable Baumslag-Solitar groups and arithmetic groups over number fields, by work of Borel and Serre. Many naturally occurring groups fail to be duality groups, but are morally very close. In this talk we make this precise with the notion of a semiduality group, and sketch a proof that certain arithmetic groups in positive characteristic are semiduality groups. This talk covers work joint with Kevin Wortman.

A Family of Locally Solvable Subgroups<br>Amanda Taylor<br>Alfred University<br>tayloral@alfred.edu

In this talk, we discuss an uncountable family of distinct elementary amenable subgroups of Thompson's Group F. The groups are limits of finitely generated wreath products, and their isomorphism types are determined by countable ordered sets related to their generators.

Approximating simple locally compact groups by their dense subgroups<br>Phillip Wesolek<br>Binghamton University, SUNY<br>pwesolek@binghamton.edu<br>Coauthors: P.-E. Caprace and C.D. Reid

Simple locally compact groups appear throughout mathematics; examples include many locally compact groups acting on regular trees and the simple algebraic groups over local fields. In this talk, we explore the relationship between simple locally compact groups and their dense subgroups. We will see that dense subgroups have a restricted structure and give information on the structure of the simple group. Additionally, dense subgroups give new examples of simple locally compact groups.

## Geometric Topology

## Gromov-Hausdorff hyperspace of nonnegatively curved 2-spheres <br> Igor Belegradek <br> Georgia Tech <br> ib@math.gatech.edu

By Alexandrov realization theorem any nonnegatively curved 2-sphere is isometric to the boundary of a convex body in the Euclidean 3-space. Up to congruence the space of such convex bodies is homeomorphic to the Gromov-Hausdorff metric space of nonnegatively curved 2 -spheres. I will discuss topological properties of this metric space.

Decomposition theorems for asymptotic property $C$ and property $A$<br>Greg Bell<br>UNC Greensboro<br>gcbell@uncg.edu<br>Coauthors: Andrzej Nagórko

We combine aspects of the notions of finite decomposition complexity and asymptotic property C into a notion that we call finite APC-decomposition complexity. Any space with finite decomposition complexity has finite APC-decomposition complexity and any space with asymptotic property C has finite APC-decomposition complexity. Moreover, finite APC-decomposition complexity implies property A for metric spaces. We also show that finite APC-decomposition complexity is preserved by direct products of groups and spaces, amalgamated products of groups, and group extensions, among other constructions.

Cosmic Censorship of Smooth Structures on Spacetimes<br>Vladimir Chernov<br>Dartmouth College<br>vladimir.chernov@dartmouth.edu<br>Coauthors: Stefan Nemirovski

It is observed that on many 4-manifolds there is a unique smooth structure underlying a globally hyperbolic Lorentz metric. For instance, every contractible smooth 4-manifold admitting a globally hyperbolic Lorentz metric is diffeomorphic to the standard $\mathbb{R}^{4}$. Similarly, a smooth 4-manifold homeomorphic to the product of a closed oriented 3-manifold N and $\mathbb{R}$ and admitting a globally hyperbolic Lorentz metric is in fact diffeomorphic to $N \times \mathbb{R}$. Thus one may speak of a censorship imposed by the global hyperbolicty assumption on the possible smooth structures on (3+1)-dimensional spacetimes.

# Topological complexity of surfaces 

Daniel C. Cohen
Louisiana State University
cohen@math.lsu.edu
Topological complexity is a numerical homotopy-type invariant introduced by M. Farber about 15 years ago, motivated by the motion planning problem from robotics. For a given space, this invariant provides a measure of the complexity of navigation in the space. Computing this invariant is sometimes easy, sometimes hard. I will attempt to illustrate this with surfaces.

## Decomposition Complexity Growth

Trevor Davila
University of Florida
trevordavila@ufl.edu
We define a quasi-isometry invariant called decomposition complexity growth which generalizes both finite decomposition complexity and asymptotic dimension growth. We show that subexponential decomposition complexity growth implies Property A, and that finite decomposition complexity and subexponential dimension growth both imply subexponential decomposition growth. We also show that certain group extensions satisfy subexponential decomposition growth.

## Coarse coherence of metric spaces and groups and its permanence properties

Boris Goldfarb
University at Albany, SUNY
bgoldfarb@albany.edu
Coauthors: Jonathan Grossman
We introduce properties of metric spaces and, specifically, finitely generated groups with word metrics which we call "coarse coherence" and "coarse regular coherence". They are geometric counterparts of the classical notion of coherence in homological algebra and the regular coherence property of groups defined and studied by Waldhausen. The new properties make sense in the general context of coarse metric geometry and are coarse invariants. In particular, they are quasi-isometry invariants of spaces and groups. They are in fact a weakening of Waldhausen's regular coherence but can be used as effectively in K-theory computations. We show that coarse regular coherence implies weak regular coherence defined by Carlsson and Goldfarb, yet all groups known to be weakly regular coherent are also coarsely regular coherent. This is a large class of groups containing all groups with straight finite decomposition complexity defined by Dranishnikov and Zarichnyi. The new framework allows us to prove structural results by developing permanence properties for coarse coherence.

# Compactifications of manifolds with boundary 

Shijie Gu
University of Wisconsin Milwaukee
shijiegu@uwm.edu
In 1976, Chapman-Siebenmann provided criteria for a Hilbert cube manifold $X$ to admit a $\mathcal{Z}$ compactification. However, the question of (whether) the extension of their characterization can be extended to manifolds still remains open: If $M^{n}$ is a finite dimensional manifold and $M^{n} \times Q$ is $\mathcal{Z}$ compactifiable, is $M^{n}$ itself $\mathcal{Z}$-compactifiable? In this talk, the complete characterizations of completable manifolds and pseudo-collarable manifolds will be given, respectively. As two applications, the former one implies a best possible stabilization theorem: $M^{n} \times Q(n \geq 4)$ is $\mathcal{Z}$-compactifiable iff $M^{n} \times[0,1]$ is $\mathcal{Z}$-compactifiable. Applying the latter characterization together with knot theory and group theory, we prove that there exist $\mathcal{Z}$-compactifiable manifolds with boundary which are not pseudo-collarable. In addition, the "building block" is an open contractible 3-manifold which embeds in no compact 3-manifolds.

## Completions and Z-compactifications of Manifolds

Craig Guilbault
University of Wisconsin-Milwaukee
craigg@uwm.edu
Coauthors: Shijie Gu
This talk is about "nice" compactifications of manifolds. The simplest of these compactifications is the addition of a boundary to an open manifold (or to a manifold with compact boundary). That was the topic of Siebenmann's famous 1965 dissertation. When $M^{m}$ has noncompact boundary, one seeks a compactification $\widehat{M}^{m}$ that "completes" $\partial M^{m}$. That is a more delicate problem. Siebenmann addressed a very special case in his dissertation and O'Brien extended that work to cases where $M^{m}$ and $\partial M^{m}$ are 1-ended. We will present a full characterization, thereby completing an unfinished chapter in the study of noncompact manifolds. As an application of this work, we obtain some new results about Z-compactifications of manifolds.

## Calculating 5-fold coverings: Not in our lifetimes

Greg Kuperberg
UC Davis
greg@math.ucdavis.edu
Coauthors: Eric Samperton
Eric Samperton and I have recently shown that, given a fixed, finite simple non-abelian group G and a 3-manifold M regarded as computational input, the number of non-trivial homomorphisms from the fundamental group of M to G is almost parsimoniously \#P-hard. In the time allotted, I will discuss basically what this result means. Among other things, it means that obtaining any non-trivial information about the connected 5 -sheeted coverings of M is computationally intractable. If M is a reasonably large, generic example, then even though its 5 -sheeted coverings are computable in principle, humanity well probably never know anything about them.

Jay Leach<br>Florida State University<br>jleach@math.fsu.edu

Surfaces detected by the character variety of manifolds with symmetries

The components of character varieties for hyperbolic 3-manifolds can detect essential surfaces. I will talk on how some components of the character variety in manifolds with symmetries always detect surfaces that respect those symmetries. Then I will look at an family of 2-bridge knots that illustrate this property.

## On a metric of J. Nagata

Atish J. Mitra
Montana Tech of the University of Montana
atish.mitra@gmail.com
For positive integers $k \geq 2$, J. Nagata considered a metric $N_{k}$ on a set $X$ with the following property: for every $x, y_{1}, \cdots, y_{k} \in X$ there exist indices $i \neq j$ such that $d\left(y_{i}, y_{j}\right) \leq d\left(x, y_{i}\right)$, and he showed that $X$ admits such a metric $N_{n+2}$ iff $\operatorname{dim}(X) \leq n$. In this talk we explore some curious properties of this metric and some connections with both topology and coarse geometry.

## KKM type theorems, homotopy and cobordism classes of maps for spheres

Oleg R Musin
University of Texas Rio Grande Valley
oleg.musin@utrgv.edu
In this talk will be shown how using homotopy classes of maps for spheres to obtain generalizations of the classic KKM (Knaster-Kuratowski-Mazurkiewicz) and Sperner lemmas. We also show that for $m>n$ the set of cobordism classes of maps from $m$-sphere to $n$-sphere is trivial. The determination of the cobordism homotopy groups of spheres admits applications to the covers for spheres.

## Distortion of surfaces in 3manifolds

Hoang Nguyen
University of Wisconsin-Milwaukee
nguyen36@uwm.edu
In the 3manifold theory, a great deal of interest has focused on the study of immersed surfaces in 3 manifolds in last decades. One reason is that studying immersed surfaces will help us to understand the structures of 3manifolds. For instance, cubulation is used in the work of Wise and Agol to resolve the Virtuallly Haken conjecture on the hyperbolic manifolds. Wise observed that the following problem is important in the study of of cubulations of 3manifold groups: Determine the distortion of surface subgroups in 3 manifold groups. The answer to this problem has been answered by BonahonThurston in the hyperbolic case. In this talk, I will give a solution to this problem in the non-geometric 3 -manifold case.

## Growth series of CAT(0) cubical complexes

Boris Okun
University of Wisconsin Milwaukee
okun@uwm.edu
Coauthors: Rick Scott (Santa Clara University)
Let $X$ be a CAT(0) cubical complex. The growth series of $X$ at $x$ is $g_{x}(t)=\sum_{y \in \operatorname{Vert(X)}} t^{d(x, y)}$, where $d(x, y)$ denotes $\ell_{1}$-distance between $x$ and $y$. If $X$ is cocompact, then $g_{x}(t)$ is a rational function of $t$. In the case of when $X$ is the Davis complex of a right-angled Coxeter group it is well-known that $g_{x}(t)=1 / f_{L}(-t /(1+t))$, where $f_{L}$ denotes the $f$-polynomial of the link $L$ of a vertex of $X$. We obtain a similar formula for general cocompact $X$. We also obtain a simple relation between the growth series of individual orbits and the $f$-polynomials of various links. In particular, we get a simple proof of reciprocity of these series $\left(g_{x}(t)= \pm g_{x}\left(t^{-1}\right)\right)$ for Eulerian $X$.

## Coassembly for representation spaces

Daniel Ramras
Indiana University-Purdue University Indianapolis
dramras@iupui.edu
The deformation K-theory of a space X is the K-theory spectrum of the category of finite-dimensional unitary representations of the fundamental group of X. This is a ring spectrum, and in fact an algebra over the connective K-theory spectrum ku. I will give a hands-on description of the universal coassembly map linking deformation K-theory and topological K-theory, along with some geometric applications.

# Regular Finite Decomposition Complexity 

David Rosenthal
St. John's University
rosenthd@stjohns.edu
Coauthors: Daniel Kasprowski and Andrew Nicas
We introduce the notion of "regular finite decomposition complexity" of a metric family. This generalizes Gromov's finite asymptotic dimension and is motivated by the concept of finite decomposition complexity (FDC) due to Guentner, Tessera and Yu. Regular finite decomposition complexity implies FDC and has all the permanence properties that are known for FDC, as well as a new one. We show that for a collection containing all metric families with finite asymptotic dimension all other permanence properties follow from Fibering Permanence.

## On the Lusternik-Schnirelmann category and Topological complexity of the connected sum of manifolds and free products of groups.

Rustam Sadykov
Kansas State University
sadykov@ksu.edu
Coauthors: Alexander Dranishnikov
We give various estimates for and calculations of the invariants in the title. In particular, for orientable closed connected manifolds we show that the LS-category of a connected sum is the maximum of the LS-categories of the summands, and give bounds for the topological complexity of a connected sum. The LS-category of a discrete group was identified by Eilenberg and Ganea with the cohomology dimension of a group. Under certain conditions we calculate the topological complexity of the free product of groups.

## The topological complexity of moving robots on graphs <br> Steven Scheirer <br> Lehigh University <br> sts413@lehigh.edu

The topological complexity of a path-connected space $X$, denoted by $T C(X)$, is an integer which can be thought of as the minimum number of "continuous rules" required to describe how to move between any two points of $X$. We will consider the case in which $X$ is the unordered discrete configuration space of $n$ points on a graph $\Gamma$, which is denoted by $U D^{n}(\Gamma)$. This space can be interpreted as the space of configurations of $n$ robots which move along a system of one-dimensional tracks. We will discuss methods to determine $T C\left(U D^{n}(\Gamma)\right)$ for the case in which $\Gamma$ is a tree.

# Alexander-Briggs Presentations of Knot Groups 

Daniel S. Silver
University of South Alabama
silver@southalabama.edu
Coauthors: J. Scott Carter, Susan G. Williams
The fundamental group $G$ of the exterior of a knot the knot group is usually described by a Wirtinger presentation of a plane diagram. Here generators of $G$ correspond to arcs of the diagram while relations are read from the crossings. Less common but also well known is the Dehn presentation of $G$ with generators (resp. relators) corresponding to regions (resp. crossings). We introduce a third type of knot group presentation inspired by the 1926/27 paper of J.W. Alexander and G.B. Briggs. The AlexanderBriggs presentation of $G$ has generators corresponding to crossings and relations corresponding to regions of the diagram.

## On strongly quasiconvex subgroups

Hung Cong Tran
The University of Georgia
hung.tran@uga.edu
We introduce the concept of strongly quasiconvex subgroups of an arbitrary finitely generated group. Strong quasiconvexity generalizes quasiconvexity in hyperbolic groups and is preserved under quasiisometry. We prove that strongly quasiconvex subgroups have many properties analogous to those of quasiconvex subgroups of hyperbolic groups. We study strong quasiconvexity and stability in relatively hyperbolic groups, two dimensional right-angled Coxeter groups, and right-angled Artin groups. We note that the result on right-angled Artin groups strengthens the work of Koberda-Mangahas-Taylor on characterizing purely loxodromic subgroups of right-angled Artin groups.

## Spaces of embeddings and diffeomorphisms of discs

Victor Turchin
Kansas State University
turchin@math.ksu.edu
There are several approaches to deloop such spaces. One goes back to Burghelea and uses the smoothing theory. The other one comes from the Goodwillie-Weiss embedding calculus and uses the theory of operads. I will survey the known results and if time permits tell what the calculus approach says about the delooping of more general mapping spaces avoiding any type of multisingularity.

## Coarse hyperstructures with applications to groups

Nicol Zava
University of Udine
zava.nicolo@spes.uniud.it
Coauthors: Dikran Dikranjan, University of Udine; Igor Protasov, Kiev State University of Taras Shevchenko; Ksenia Protasova, Kiev State University of Taras Shevchenko

Let $(X, \mathcal{U})$ be a uniform space. Then the Hausdorff-Bourbaki hyperspace is a well-known classical topological object, namely, it is the uniform space $\left(\mathcal{P}(X), \mathcal{U}^{*}\right)$, where $\mathcal{U}^{*}=\left\{U^{*} \subseteq \mathcal{P}(X) \times \mathcal{P}(X) \mid U \in \mathcal{U}\right\}$ and, for every $U \in \mathcal{U}$, a pair $(A, B) \in U^{*}$ if and only if $A \subseteq U[B]$ and $B \subseteq U[A]$. Extending an idea of Protasov and Protasova, we show that, if $(X, \mathcal{E})$ is a coarse space, then also $\left(X, \mathcal{E}^{*}\right)$ is a coarse space, called coarse hyperspace. By comparing the two situations, we show some similar results and we justify the choice of these authors to take into account only the family of all bounded non-empty subsets as a support of the coarse hyperspace. Moreover, we focus on some special subspaces of coarse hyperspaces of groups. More precisely, if $G$ is a group, we endow it with the coarse structure $\mathcal{E}_{G}$ induced by the ideal of all finite subsets. We consider the coarse subspace $\mathcal{L}(G)$ of $\mathcal{E}_{G}^{*}$, whose support is the subgroup lattice of $G$. We give some interesting properties of those coarse spaces and results that compare them with other coarse structures on subgroup lattices. This investigation also bring to attention some purely algebraic properties of groups: for example, two subgroups of a group $G$ belong to the same connected component of $\mathcal{L}(G)$ if and only if they are commensurable. If two groups $G, H$ are isomorphic, then the spaces $\mathcal{L}(G)$ and $\mathcal{L}(H)$ are obviously asymorphic, although the converse need not be true in general. Particular emphasis is given to investigating sufficient conditions for which the opposite implication holds.

## Set-Theoretic Topology

## ON M-METRIC SPACES

Samer Assaf
American University of Kuwait
sassaf@auk.edu.kw
We make some observations concerning m-metric spaces and point out some discrepancies in the proofs found in the literature. To remedy this, we propose a new topological construction and prove that it is in fact a generalization of a partial metric space.

## $\kappa$-proximal spaces

Jocelyn Bell
Hobart and William Smith Colleges
bell@hws.edu
The proximal game is a two-player infinite game which takes place in a uniform space. We will introduce a modification of the proximal game and the class of $\kappa$-proximal spaces and will discuss properties of $\kappa$-proximal spaces.

An Extension of the Baire Property<br>Christopher Caruvana<br>University of North Texas<br>Christopher.Caruvana@unt.edu<br>Coauthors: Robert Kallman, University of North Texas

We will define for every Polish space $X$ a class of sets, the extended Baire property $E B P(X)$ sets, to work out many properties of $E B P(X)$ and to show their usefulness in analysis. For example, a proper generalization of the Pettis Theorem is provided in this context that furnishes a new automatic continuity result for Polish groups. The name extended Baire property sets is reasonable since $E B P(X)$ contains the Baire property sets and it is consistent with ZFC that the containment is proper. We will also consider a naturally defined ideal of sets related to $\operatorname{EBP}(X)$ and show that this ideal is, in general, strictly finer than the ideal generated by the meager sets and the universally null sets.

## pi-Base: A usable map of the forest

Steven Clontz
University of South Alabama
sclontz@southalabama.edu
Coauthors: James Dabbs
To paraphrase Mary Ellen Ruden in her review of Lynn Arthur Steen and J. Arthur Seebach, Jr.'s Counterexamples in Topology, "Topology is a dense forest of counterexamples, and a usable map of the forest is a fine thing." Inspired by this quote and the work done by Steen and Seebach, the pi-Base serves as a living database cataloging topological spaces, their properties, and the theorems that connect those properties. The presenter will give a brief history of the database, as well as ongoing plans for developing its content and software to support active researchers in general topology. The pi-Base may be accessed on any computer or smart device by visiting 〈http://pi-base.org〉.

## Applications of high dimensional Ellentuck spaces <br> Natasha Dobrinen <br> University of Denver <br> ndobrine@du.edu

We present an overview of some recent works by various authors, including Arias, Dobrinen, Girón, Hathaway, Mijares, and Zheng, which centrally rely on the Ramsey-theoretic structure inherent in high dimensional Ellentuck spaces.

The well-known Boolean algebra $\mathcal{P}(\omega) /$ Fin adds a Ramsey ultrafilter, and its properties with respect to Rudin-Keisler, Tukey, and $L(\mathbb{R})[\mathcal{U}]$, and preservation under certain forcings have been well-studied. The natural extension on $\omega \times \omega$ is the Boolean algebra $\mathcal{P}(\omega \times \omega) /$ Fin $\otimes$ Fin. By recursion, a large hierarchy of such Boolean algebras may be formed, each growing in complexity strength over previous ones. In the process of determining the exact Tukey structures below these forced ultrafilters, Dobrinen showed that each of these is forcing equivalent to a topological Ramsey space, which may be viewed as higher dimensional versions of the Ellentuck space. The Ramsey-theoretic content of these spaces have provided the structure necessary for several recent results involving new Banach spaces, preservation of these ultrafilters under Sacks forcging, as well as preservation of properties of $L(\mathbb{R})[\mathcal{U}]$.

## sigma-compact density

Alan Dow
UNC Charlotte
adow@uncc.edu
Coauthors: Istvan Juhasz
Arhangelskii and Stavrova introduced and studied generalized notions of tightness, including the notion of sigma-compact tightness. The major related open problem is whether sigma-compact tightness implied simply countable tightness in spaces besides compact spaces. This inspired Juhasz and van Mill to recently study other generalizations of sigma-compact tightness and led van Mill to formulate a problem asking if a compact space is necessarily separable if every dense subset has a dense sigma-compact subset. We answer this question affirmatively.

```
Spaces with a \mathbb{Q}\mathrm{ -diagonal}
Ziqin Feng
Auburn University
zzf0006@auburn.edu
```

Let $\mathcal{K}(M)$ be the collection of all compact subsets of a topological space $M$. Then $\mathcal{K}(M)$ is a directed set ordered by set inclusion. For any directed set $P$, a collection $\mathcal{C}$ of subsets of a space $X$ is $P$-directed if $\mathcal{C}$ can be represented as $\left\{C_{p}: p \in P\right\}$ such that $C_{p} \subseteq C_{p^{\prime}}$ whenever $p \leq p^{\prime}$. A space $X$ has an $M$-diagonal for some separable metric space $M$ if $X^{2} \backslash \Delta$ has a $\mathcal{K}(M)$-directed compact cover. We show that any compact space with a $\mathbb{Q}$-diagonal is metrizable, where $\mathbb{Q}$ is the space of rational numbers.

## Cardinal inequalities for topological spaces: new results and some open questions <br> Ivan S. Gotchev <br> Central Connecticut State University <br> gotchevi@ccsu.edu

In this talk recent results on cardinal inequalities for topological spaces will be presented and some old and new open questions will be discussed.

## Monotonical Monolithity, Point-countable Almost Subbase and $D$

Hongfeng Guo
Shandong University of Finance and Economics
guohongfeng17@gmail.com
Coauthors: Ziqin Feng
Tkachuk introduced monotonically monolithic spaces and proved that any monotonically monolithic space is hereditarily $D$. We discuss the finite union of monotonically monolithic spaces and show that any countably tight space which is a finite union of monotonically monolithic subspaces is $D$.

We will also discuss the relation between point networks and some $D$-relatives. We show that any space with an $\aleph_{0}$-Noetherian point network is thickly covered, hence $a D$ and linearly $D$.

# Super HS and HL Spaces 

Joan Hart
University of Wisconsin Oshkosh
hartj@uwosh.edu
Coauthors: Kenneth Kunen
We introduce super properties, including suHS and suHL, based on the standard separating sequence characterizations of HS (Hereditarily Separable) and HL (Hereditarily Lindelöf). Given a space $X$, let $\mathcal{U}$ range over arbitrary sequences $\left\langle\left(x_{\alpha}, U_{\alpha}\right): \alpha<\omega_{1}\right\rangle$, where each $U_{\alpha}$ is open and $x_{\alpha} \in U_{\alpha}$. Then $X$ is HS iff $X$ has no left separated sequence iff $\forall \mathcal{U} \exists \alpha<\beta\left[x_{\alpha} \in U_{\beta}\right]$. To define suHS we replace the pair $\alpha, \beta$ by $\aleph_{1}$ elements of $X$ : So $X$ is super HS (suHS) iff $\forall \mathcal{U} \exists I \in\left[\omega_{1}\right]^{\aleph_{1}} \forall \alpha, \beta \in I\left[\alpha<\beta\right.$ implies $\left.x_{\alpha} \in U_{\beta}\right]$. We define suHL and suHC likewise, where HC abbreviates Hereditarily CCC.

We note that suHS implies stHS, but suHC does not imply stHC, where, as usual, if P is a property of spaces, then $X$ is strongly $\mathrm{P}(\mathrm{stP})$ iff all finite powers of $X$ have P .

## Sets and mappings in $\beta S$ which are not Borel <br> Neil Hindman <br> Howard University <br> nhindman@aol.com <br> Coauthors: Dona Strauss

We extend theorems previously proved for $\mathbb{N}$ by showing that, if $S$ is a countably infinite right cancellative and weakly left cancellative discrete semigroup, then the following subsets of $\beta S$ are not Borel: the set of idempotents, the smallest ideal, any semiprincipal right ideal defined by an element of $S^{*}$, and $S^{*} S^{*}$. This has the imediate corollary that, if $S$ is any infinite right cancellative and weakly left cancellative semigroup, the set of idempotents in $\beta S$ is not Borel.

## Calibrating the Size of Complicated Quotients

Jared Holshouser
University of South Alabama
JaredHolshouser@southalabama.edu
Silver's theorem and the Harrington-Kechris Louveau theorem serve to calibrate the size of co-analytic and Borel quotients of Polish Spaces. Work of Shelah, Hjorth, and more recently Caicedo and Ketchersid extend these results, with some limitations. Shelah only extends Silver's theorem, but obtains an explicit bound for his quotients and Caicedo and Ketchersid extend both Silver's theorem and the HarringtonKechris Louveau theorem, but lose the explicit bound. We will adapt techniques from infinitary logic to create a form of light-face pointclasses running parallel to the classes of Suslin sets to get an extension of both Silver's theorem and the Harrington-Kechris Louveau theorem which provides explicit bounds for the quotients.

Introduction To Typed Topological Spaces<br>Wanjun Hu<br>1505 River Pointe<br>Wanjun.Hu@asurams.edu

Finite topological spaces are discrete when they are $T_{1}$, which renders them uninteresting. In this presentation, we will introduce the so-called typed topologies on finite sets, using which fine structures can be defined.

## Uncountable discrete subspaces and forcing

Akira Iwasa
University of South Carolina Beaufort
iwasa@uscb.edu
Suppose that a topological space $X$ has no uncountable discrete subspace. We discuss if $X$ can get an uncountable discrete subspace in forcing extensions. If $X$ is metrizable, then this is not possible. If $X$ is hereditarily separable and non-Lindelöf ( $S$-space), then this is possible.

## Functional tightness of infinite products

Mikolaj Krupski
University of Pittsburgh
m.krupski@pitt.edu

A classical theorem of Malykhin says that tightness behaves nicely under Cartesian products of compact spaces. In my talk, I will show a counterpart of Malykhin's theorem for functional tightness. In particular, it turns out that if there are no measurable cardinals then the functional tightness is preserved by arbitrarily large products of compacta. Our results answer a question posed by Okunev.

## All sufficiently regular sets of reals may be projective

Paul Larson
Miami University
larsonpb@miamioh.edu
Coauthors: Saharon Shelah
We will discuss some open problems about universally measurable sets of real numbers and outline a proof that, consistently, every universally measurable set of reals is the continuous image of a coanalytic set.

The proof generalizes to other regularity properties, including the corresponding universal version of the Baire property.

# Products of topological groups in which all closed subgroups are separable 

Arkady Leiderman
Ben-Gurion University of the Negev, Israel
arkady@math.bgu.ac.il
Coauthors: Mikhail G. Tkachenko
We say that a topological group $G$ is strongly separable (briefly, $S$-separable), if for any topological group $H$ such that every closed subgroup of $H$ is separable, the product $G \times H$ has the same property.

Theorem 1. Every separable compact group is $S$-separable.
The following problem is open.
Problem. 1) Is every separable locally compact group $S$-separable? 2) Does there exist a separable metrizable group which is not $S$-separable?

Let $\mathfrak{c}$ be the cardinality of the continuum.
Theorem 2. Assuming $2^{\omega_{1}}=\mathfrak{c}$, there exist:

- pseudocompact topological abelian groups $G$ and $H$ such that all closed subgroups of $G$ and $H$ are separable, but the product $G \times H$ contains a closed non-separable $\sigma$-compact subgroup;
- pseudocomplete locally convex vector spaces $K$ and $L$ such that all closed vector subspaces of $K$ and $L$ are separable, but the product $K \times L$ contains a closed non-separable $\sigma$-compact vector subspace.

The problem whether such pairs of spaces exist in ZFC is open.

## Pin homogeneity

David Milovich
Texas A $\mathcal{G} M$ International University
ultrafilter@gmail.com
This talk introduces a weak form of homogeneity. Call two points $a, b$ in a compact space $X$ pin equivalent if there exist a compact space $Y$, a quotient map $f: Y \rightarrow X$ that is invertible at $a$ and $b$, and a homeomorphism $g: Y \rightarrow Y$ such that $g\left(f^{-1}(a)\right)=f^{-1}(b)$. Call $X$ pin homogeneous if every two points in $X$ are pin equivalent.

Pin homogeneity is strictly weaker than homogeneity and strictly stronger than the property of all points having the same Tukey type. Assuming CH, infinite compact F-spaces are not pin homogeneous. On the other hand, every compact space has a pin homogeneous power.

If the above $f$ is required to be a homeomorphism instead of a mere quotient map, then pin homogeneity becomes homogeneity. This suggests a strategy for incremental progress towards solving the open problem of whether every compact space is a quotient of a homogeneous compact space: start with the positive solution to the analogous problem for pin homogeneous compacts and incrementally require more of $f$.

## Butterfly points in compacta

Peter Nyikos
University of South Carolina
nyikos@math.sc.edu
An elementary problem about ultrafilters is equivalent to a classic problem in set theoretic topology that has resisted a complete solution for over four decades.

Definition 1. A filter on a set $S$ is nowhere maximal if it does not trace an ultrafilter on any subset of $S$.

Problem 1. Is every free ultrafilter generated by two nowhere maximal filters?
The equivalent topological problem has to do with what are called butterfly points.
Definition 2. A point $p$ of a space $X$ is a butterfly point of $X$ if there exist two closed subsets $F_{0}$ and $F_{1}$ of $X$ whose intersection is $\{p\}$, and $p$ is nonisolated in the relative topology of both $F_{0}$ and $F_{1}$.

The following classic problem is equivalent to Problem 1:
Problem 1'. Is every point in the Stone-Čech remainder of every discrete space a buttefly point?
A closely related problem is whether the removal of any point in such a remainder gives a non-normal subspace. The answer to both questions is is Yes under GCH, but it is not known whether the usual (ZFC) axioms of set theory suffice to show it is Yes for even one infinite set, for either problem. And so, Problem 1 is also open for all infinite sets $S$.

It has been known since 1980 that ZFC is enough to show the existence of some butterfly points in the remainder of $\omega$ and hence in all these remainders. On the other hand, the following classic problem is open:

Problem 2. Does every infinite compact Hausdorff space have a butterfly point?
The problem is worded in this way because not every nonisolated point of every compact Hausdorff space is a butterfly point. A counterexample is the last point $\omega_{1}$ in $\omega_{1}+1$.
V. V. Fedorchuk found counterexamples for Problem 2 under the ZFC-independent axiom $\diamond$, but no ZFC counterexamples are known.

There are various "translations" of the concept of a butterfly point in analysis and algebra, in the spirit of the way Problem 1 is a "translation" of Problem 1'. Adequate and inadequate ways of translating Problem 2 will be outlined.

## On GO-spaces with a monotone closure-preserving operator

Strashimir G. Popvassilev
The City College and Medgar Evers College, CUNY (part time)
spopvassilev@ccny.cuny.edu
Coauthors: John E. (Ted) Porter, Murray State University
We generalize results by Bennett, Hart, Lutzer, and Peng, Li, by proving that the following conditions are equivalent for a GO-space $X$ whose underlying LOTS has a $\sigma$-closed-and-discrete dense subset: (1) $X$ has a monotone open locally-finite operator, (2) $X$ is monotonically metacompact, and (3) $X$ has a monotone (open or not) closure-preserving operator. Monotonically metacompact GO-spaces have a monotone open locally-finite operator. A GO-space with a $\sigma$-closed-discrete dense subset and a monotone closure-preserving operator is metrizable. A compact LOTS with a monotone open closure-preserving operator is metrizable.

## On the metrization of GO-spaces with the $N Z(\omega)$-property.

Ted Porter
Murray State University
jporter@murraystate.edu
In their study of $D$-spaces, Z. Feng and G. Gruenhage introduced the $N Z(\kappa)$-property. A topological space $X$ is said to satisfy $N Z(\kappa)$ for a cardinal $\kappa$ if for any collection $N=\left\{\left(x_{\lambda}, U_{\lambda}\right): \lambda \in \Lambda\right\}$ with $U_{\lambda}$ being an open neighborhood of $x_{\lambda}$ for each $\lambda \in \Lambda$, there exist a cover $\left\{\Lambda_{\alpha}: \alpha<\kappa\right\}$ of $\Lambda$ and open set $Z_{\lambda} \subset U_{\lambda}$ containing $x_{\lambda}$ for each $\lambda \in \Lambda$ such that for each $\alpha<\kappa$, if $\left\{\lambda_{n}: n \in \omega\right\} \subset \Lambda_{\alpha}$ is any sequence with $x_{\lambda_{n}} \notin \bigcup_{i<n} U_{\lambda_{i}}$ for each $n \in \omega$, then $\bigcap_{n \in \omega} Z_{\lambda_{n}}=\emptyset$. We will show GO-spaces with the $N Z(\omega)$-property and a $\sigma$-closed-discrete dense subsets are metrizable improving upon results of Peng and Li and Bennett, Hart, and Lutzer on monotonically metacompact spaces along with results on $\sigma$-NSR pair-bases that the speaker announced at last year's Spring Topology and Dynamical Systems Conference and Summer Topology Conference.

## Countably compact sequential groups of arbitrary orders.

Alexander Shibakov
Tennessee Tech University
ashibakov@tntech.edu
Coauthors: Dmitri Shakhmatov
We present a construction of a countably compact sequential group topology on a boolean group that has a prescribed sequential order. A number of open questions (that are finally appropriate to ask) will be mentioned. This is a joint work with Dima Shakhmatov.

## Extension of functions and metrics with variable domains <br> Ihor Stasyuk <br> Nipissing University <br> ihors@nipissingu.ca <br> Coauthors: T. Banakh, E.D.Tymchatyn and M. Zarichnyi

Let $(X, d)$ be a complete, bounded, metric space. For a nonempty, closed subset $A$ of $X$ denote by $C^{*}(A \times A)$ the set of all continuous, bounded, real-valued functions on $A \times A$. Denote by

$$
C=\cup\left\{C^{*}(A \times A) \mid A \text { is a nonempty closed subset of } X\right\}
$$

the set of all partial, continuous and bounded functions. We prove that there exists a linear, regular extension operator from $C$ endowed with the topology of convergence in the Hausdorff distance of graphs of partial functions to the space $C^{*}(X \times X)$ with the topology of uniform convergence on compact sets. The constructed extension operator preserves constant functions, pseudometrics, metrics and admissible metrics. For a fixed, nonempty, closed subset $A$ of $X$ the restricted extension operator from $C^{*}(A \times A)$ to $C^{*}(X \times X)$ is continuous with respect to the topologies of pointwise convergence, uniform convergence on compact sets and uniform convergence considered on both $C^{*}(A \times A)$ and $C^{*}(X \times X)$.

Ladder systems after forcing with a Suslin tree<br>Paul Szeptycki<br>York University<br>szeptyck@gmail.com<br>Coauthors: Cesar Corral

Uniformization and anti-uniformization properties of ladder systems in forcing in models obtained by forcing with a Suslin tree $S$ over a model of MA(S) are considered.

## Discrete selectivity in function spaces

Vladimir Tkachuk
Universidad Autonoma Metropolitana de Mexico
vova@xanum.uam.mx
Given a sequence $\mathbf{S}=\left\{U_{n}: n \in \omega\right\}$ of non-empty open subsets of a space $Y$, a set $\left\{x_{n}: n \in \omega\right\}$ is a selection of $\mathbf{S}$ if $x_{n} \in U_{n}$ for every $n \in \omega$. The space $Y$ is discretely selective if every sequence of non-empty open subsets of $Y$ has a closed discrete selection. We show that a space $X$ is uncountable if and only if $C_{p}(X)$ is discretely selective and study discrete selectivity in spaces $C_{p}(X,[0,1])$.

## The SIN Property in Homeomorphism Groups

Keith Whittington
University of the Pacific, Stockton, CA
kwhittin@pacific.edu
It is shown that if $G$ is a group of homeomorphisms of a compact space $X$, then under fairly general circumstances, $G$ is SIN if and only if $\bar{G}$ is compact. The case where $X$ is metric and $G$ is given the compact-open topology adds to the classical characterization of compactness of $\bar{G}$ given by $S$. Eilenberg. The work is then generalized using uniformities.

## Cofinally Polish spaces and domain representability

Lynne Yengulalp
University of Dayton
lyengulalp1@udayton.edu
Coauthors: Jila Niknejad and Vladimir Tkachuk
A space X is said to be cofinally Polish if for every continuous mapping f of X onto a separable metric space M, there is a Polish space P and continuous onto maps $g: X \rightarrow P$ and $h: P \rightarrow M$ such that $f=h \circ g$.

We show that if X has a countable network, then X is cofinally Polish if and only if it is domain representable.

