AFFABILITY OF LAMINATIONS DEFINED BY REPETITIVE PLANAR TILINGS

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Abstract

A *planar tiling* is a partition of $\mathbb{R}^2$ into *tiles*, which are polygons touching face-to-face obtained by translation from a finite set of *prototiles*. If we consider $\mathbb{T}(\mathcal{P})$ the set of tilings $\mathcal{T}$ constructed from a finite set of prototiles $\mathcal{P}$, it is possible to endow it with a natural topology, the *Gromov-Hausdorff topology* [2, 3], which turns it into a compact metrizable space laminated by the orbits $L_{\mathcal{T}}$ of the natural $\mathbb{R}^2$-action. If $\mathcal{T} \in \mathbb{T}(\mathcal{P})$ is a *repetitive* tiling (i.e. for any patch M, there exists a constant $R > 0$ such that any ball of radius R contains a translated copy of M), then the closure of its orbit $\mathbb{X} = \overline{L}_{\mathcal{T}}$ is a minimal closed subset of $\mathbb{T}(\mathcal{P})$, called the *continuous hull of* $\mathcal{T}$. If $\mathcal{T}$ is also *aperiodic* (i.e. $\mathcal{T}$ has no translation symmetries), then $\mathbb{X}$ is transversally modeled by a Cantor set. Dynamical and ergodic properties of these laminations are important for the theoretical study of *quasicrystals*.

Affable equivalence relations are orbit equivalent to inductive limits of finite equivalence relations on the Cantor set. This notion has been introduced by J. Renault [7] and T. Giordano, I.F. Puntnam and C.F. Skau [6]. One can think of affability as the topological version of hyperfiniteness. A transversally Cantor lamination will be said to be *affable* if the equivalence relation induced on any total transversal is affable.

In [4], T. Giordano, H. Matui, I. Putnam and C. Skau have proved that any free minimal $\mathbb{Z}^2$-action on the Cantor set is affable. In order to demonstrate it, they combine strong convexity arguments with an important result about extension of minimal affable equivalence relations, called Absorption Theorem, given in [5]. In [1], we show that, equivalently, the continuous hull of any repetitive and aperiodic planar tiling is affable. Our proof is based on a special inflation process, which is similar to that used to construct Robinson tilings. Though we still use the Absorption Theorem, it has the advantage that no convexity argument is needed. Here we want to present the main concepts referenced and illustrate this proof.

References

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