# Non-Existence of Non-Trivial Bi-Infinite Geodesics in Geometric Last Passage Percolation

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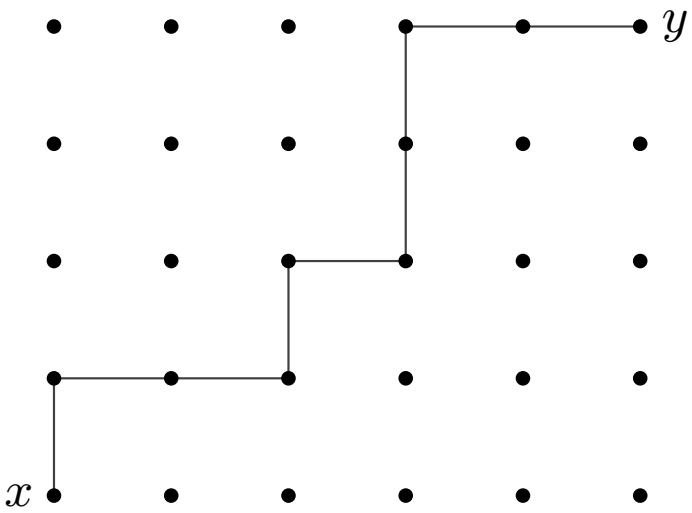
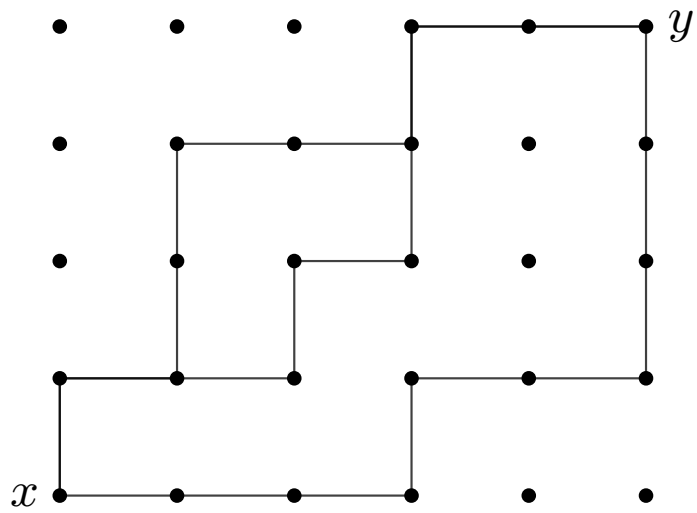
# Last Passage Percolation

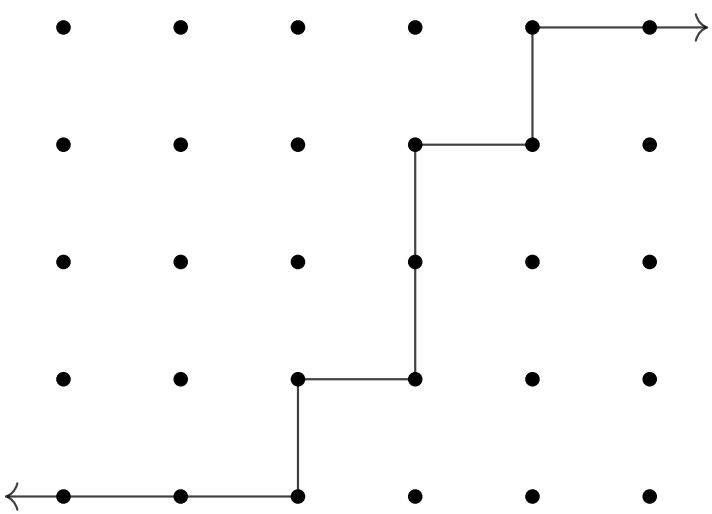
- i.i.d. **weights**: $\omega = (\omega_x)_{x \in \mathbb{Z}^2}$
- **last passage time**: $G_{x,y} = \max_{\pi \in \Pi_x^y} \sum_{v \in \pi} \omega_v$
- Maximizing paths are (finite) **geodesics**

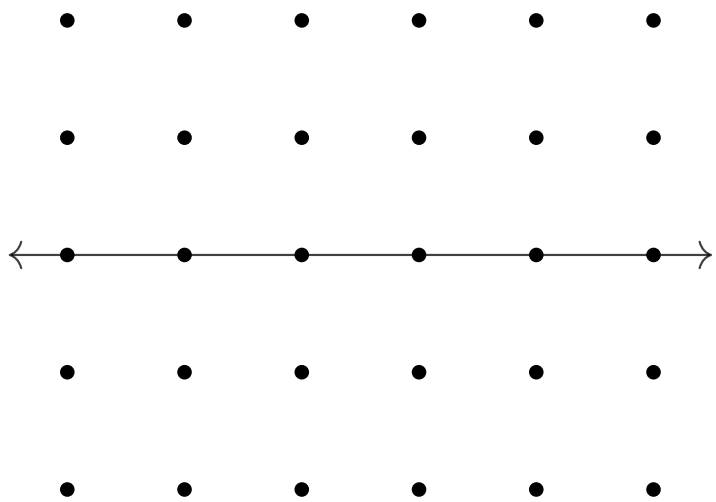


# Bi-Infinite Geodesics

- $\pi_{-\infty:\infty}$ is a **bi-infinite geodesic** if $\pi_{m:n}$ is a geodesic for all $m < n$
- Horizontal and vertical lines are **trivial** bi-infinite geodesics



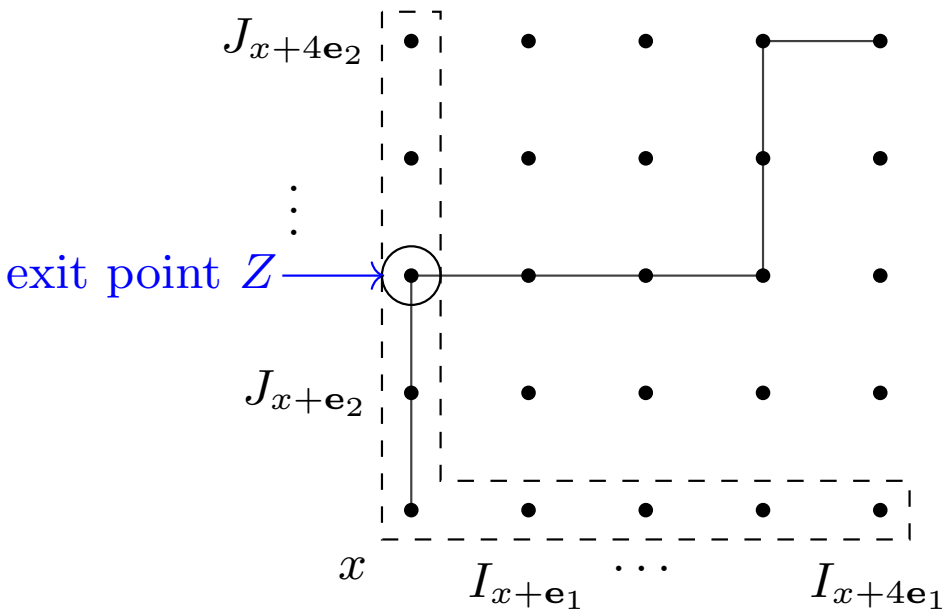


# Main Result

**Theorem**

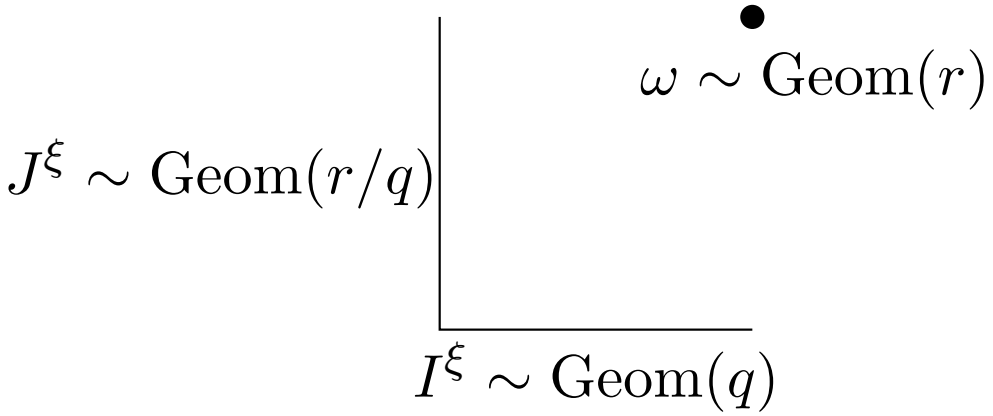
*Let $\omega_0 \sim \mathrm{Geom}(r)$ for some $r \in (0,1)$. With probability one, no nontrivial bi-infinite geodesics exist.*

# Models with Boundary



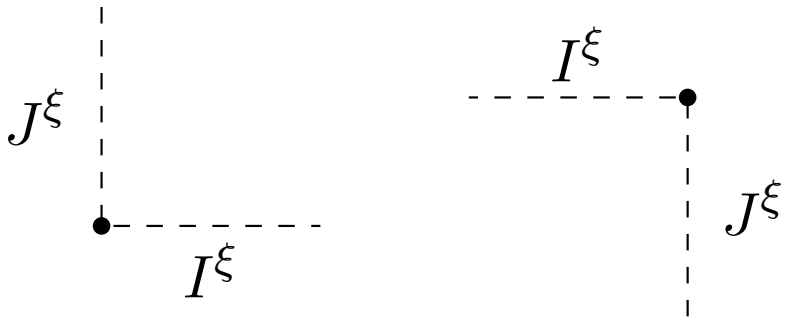

- Use recovering cocycle to get weights $I, J$
- Get additivity: $G^{\xi}_{x,y} + G^{\xi}_{y,z} = G^{\xi}_{x,z}$

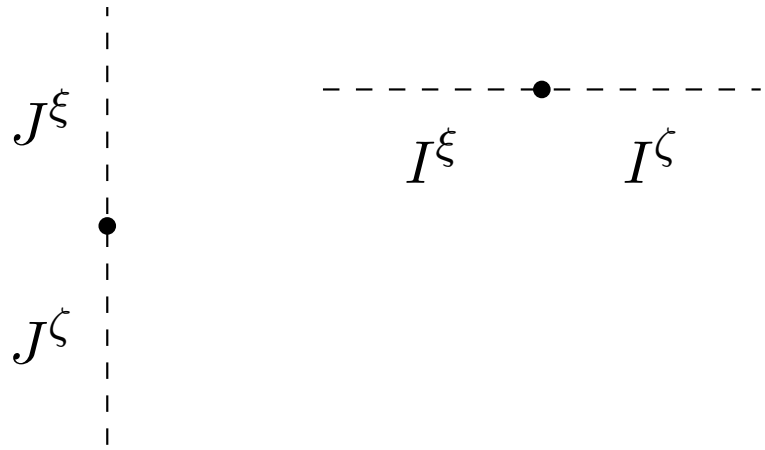
# Why Geometric?



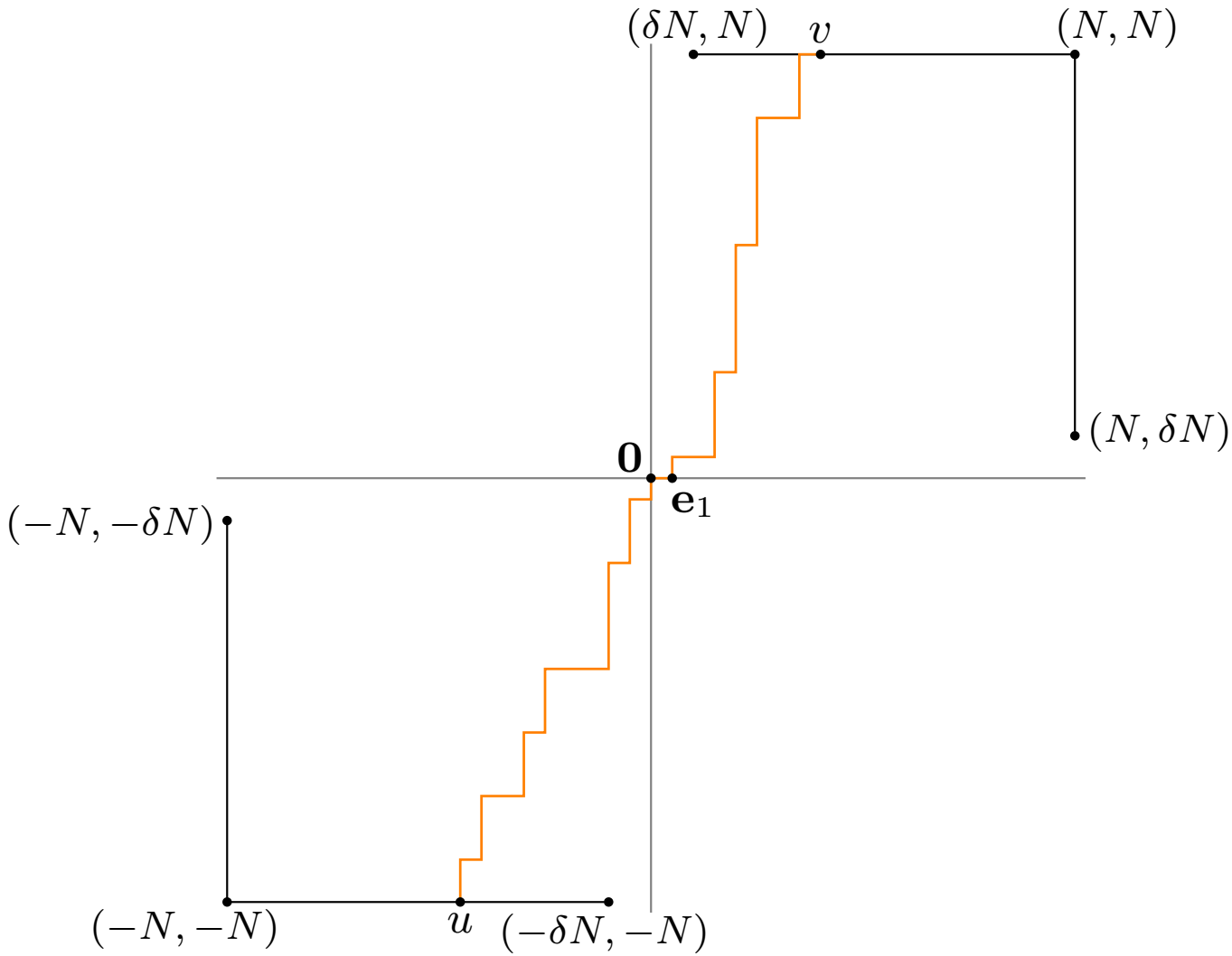

$$q = \frac{r + (r+1)\sqrt{r\xi_1\xi_2}}{\xi_1 + r\xi_2 + 2\sqrt{r\xi_1\xi_2}} \in (r, 1)$$

Independence Properties:

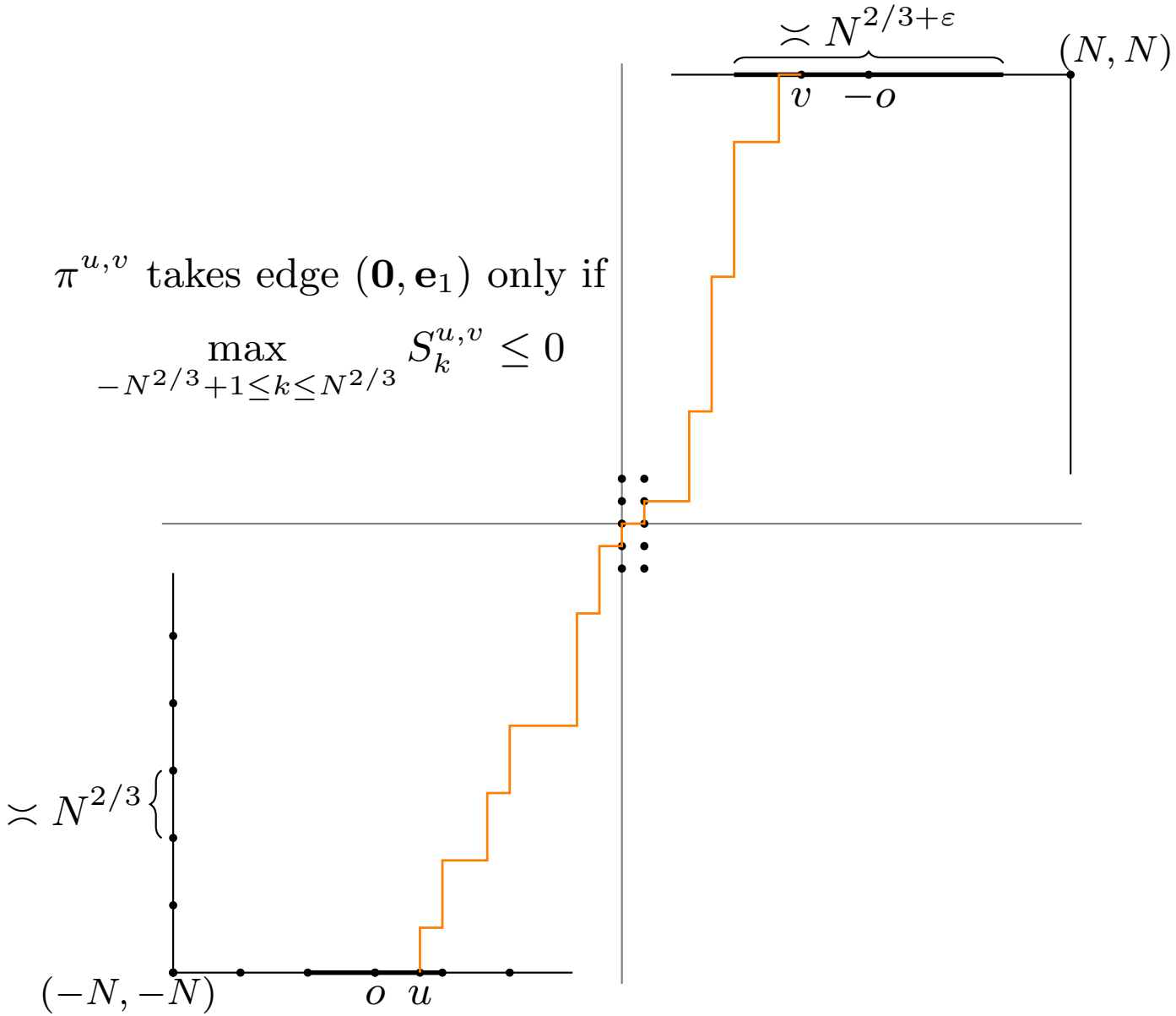


# No Bi-Infinite Geodesics Away from the Axes

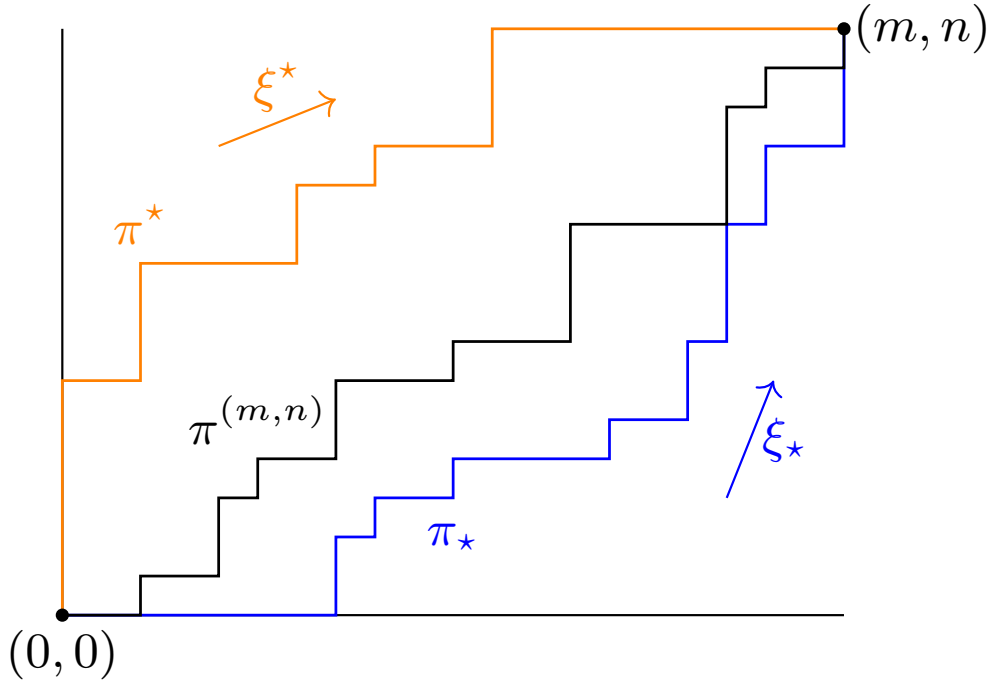


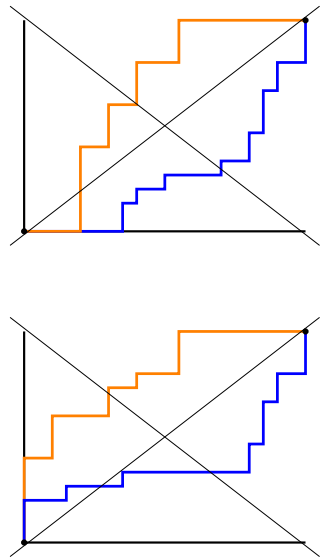
# Main Case


$\asymp N^{2/3+\varepsilon}$
$(N, N)$
$v$
$-o$
$\pi^{u,v}$ takes edge $(\mathbf{0}, \mathbf{e}_1)$ only if
$\max_{-N^{2/3}+1 \le k \le N^{2/3}} S_k^{u,v} \le 0$
$\asymp N^{2/3}$
$(-N, -N)$
$o$
$u$

Thank you!

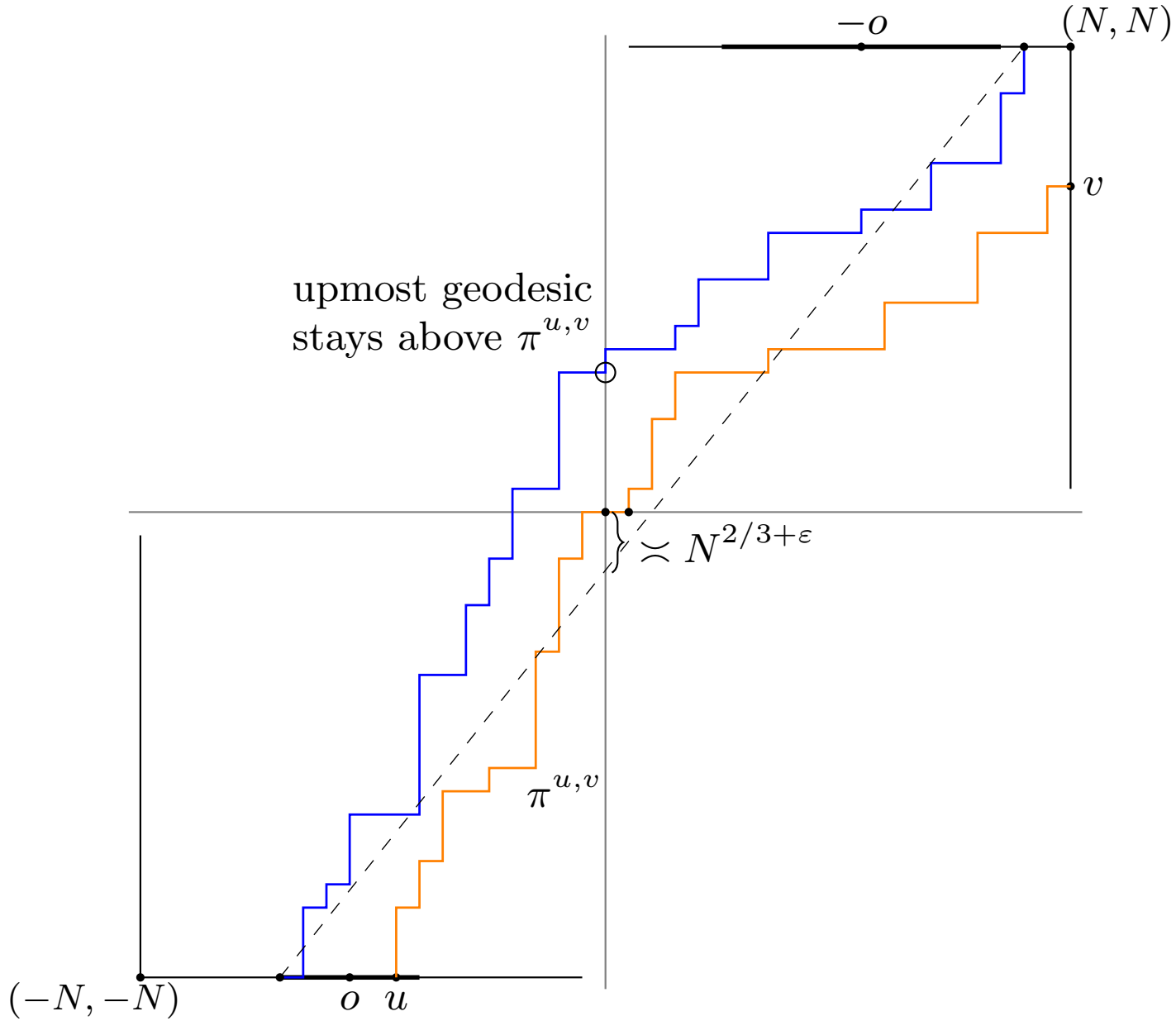
# Controlling Geodesics



On high probability event, $\pi^{m,n}$ is sandwiched between $\pi^\star, \pi_\star$

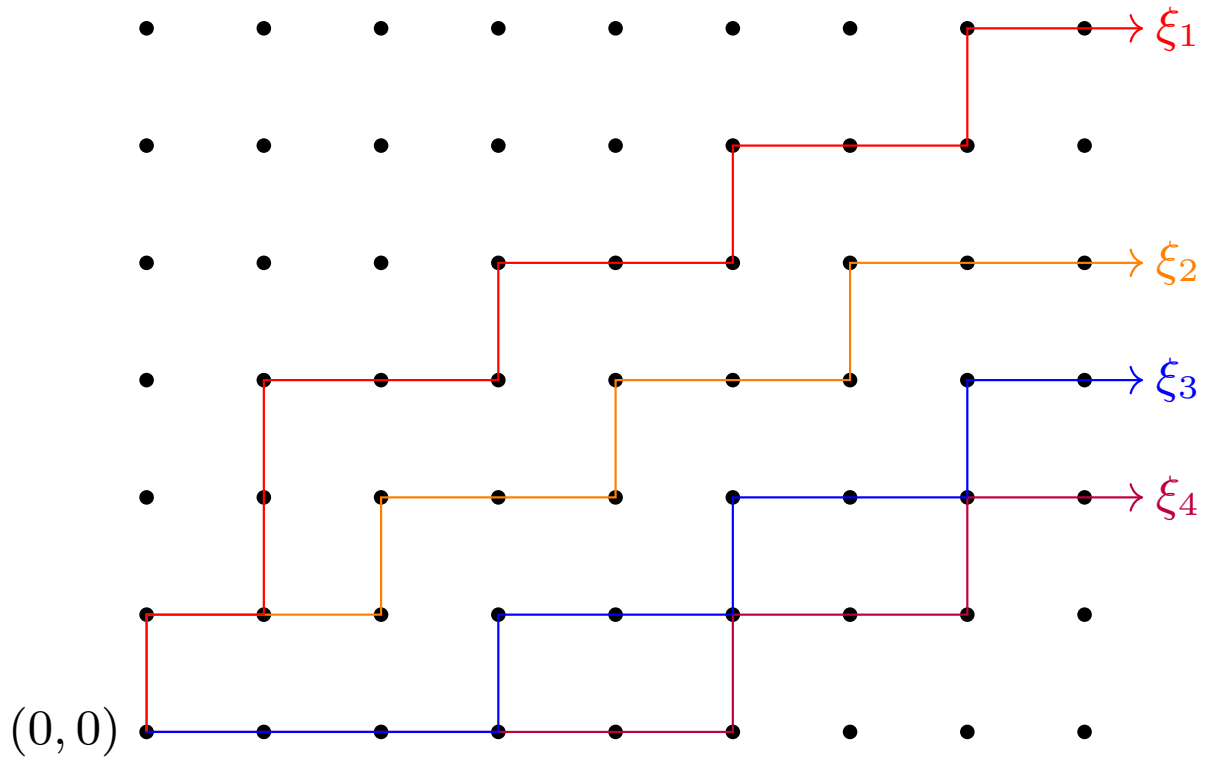
# Case 2


$-o$
$(N, N)$
$v$
upmost geodesic
stays above $\pi^{u,v}$
$\asymp N^{2/3+\varepsilon}$
$\pi^{u,v}$
$(-N, -N)$
$o$
$u$

# Axis-Directed Bi-infinite Geodesics

## Lemma

*For $x \in \mathbb{Z}^2_{\geq 0}$ and $\ell \in \{1, 2\}$, a.s. the only semi-infinite geodesic starting at $x$ satisfying $\varliminf_{k\to\infty} k^{-1} x_k \cdot \mathbf{e}_{3-\ell} = 0$ is the trivial geodesic $\{x + k\mathbf{e}_\ell\}_{k=0}^{\infty}$.*



# References


[1] R. Basu, C. Hoffman, and A. Sly, "Nonexistence of bigeodesics in integrable models of last passage percolation",, Nov. 2018, Preprint (arXiv 1811.04908). eprint: 1811.04908. [Online]. Available: https://arxiv.org/pdf/1811.04908.

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[3] E. Emrah, C. Janjigian, and T. Seppäläinen, "Right-tail moderate deviations in the exponential last-passage percolation", *arXiv e-prints*, arXiv:2004.04285, arXiv:2004.04285, Apr. 2020. arXiv: 2004.04285 [math.PR].

[4] O. Busani and T. Seppäläinen, "Non-existence of bi-infinite polymer Gibbs measures", *arXiv e-prints*, arXiv:2010.11279, arXiv:2010.11279, Oct. 2020. arXiv: 2010.11279 [math.PR].