Financial Mathematics

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 14. Exotic Options: I

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An exotic option, or nonstandard option, is simply an option with some contractual difference from standard options.

Exotic options solve particular business problems that an ordinary option do not.

They are often constructed by tweaking ordinary options in minor ways.

Questions

- How does the exotic payoff compare to that of a standard option?
- Can the exotic option be approximated by a portfolio of other options?
- Is the exotic option cheap or expensive relative to standard options?
- What is the rationale for the use of the exotic option?
- How easily can the exotic option be hedged?

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Chapter 14. Exotic Options: I

§ 14.1 Introduction

§ 14.2 Asian options

§ 14.3 Barrier options

§ 14.4 Compound options

§ 14.5 Gap options

§ 14.6 Exchange options

§ 14.7 Problems

The payoff of an Asian option is based on the average price over some period of time.

- It is less valuable than otherwise equivalent ordinary options.
- It is path-dependent.

Situations when Asian options are useful:

- When a business cares about the average exchange rate over time
- When a single price at a point in time might be subject to manipulation
- When price swings are frequent due to thin markets

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Eight possible Asian options:

{Call, Put} × {Arithmetic, Geometric} × {Average Price, Average Strike}

- Arithmetic Average: $A(T) = \frac{1}{N} \sum_{i=1}^{N} S_{ih}$.

  Geometric Average: $G(T) = \left(\prod_{i=1}^{N} S_{ih} \right)^{1/N}$.

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Eight possible Asian options:

$$\{\text{Call, Put}\} \times \{\text{Arithmetic, Geometric}\} \times \{\text{Average Price, Average Strike}\}$$

$$\begin{aligned}
\text{Arithmetic average price call} &= \max(0, A(T) - K) \\
\text{Arithmetic average price put} &= \max(0, K - A(T)) \\
\text{Arithmetic average strike call} &= \max(0, S_T - A(T)) \\
\text{Arithmetic average strike put} &= \max(0, A(T) - S_T)
\end{aligned}$$

Eight possible Asian options:

$$\{\text{Call, Put}\} \times \{\text{Arithmetic, Geometric}\} \times \{\text{Average Price, Average Strike}\}$$

$$\begin{aligned}
\text{Geometric average price call} &= \max(0, G(T) - K) \\
\text{Geometric average price put} &= \max(0, K - G(T)) \\
\text{Geometric average strike call} &= \max(0, S_T - G(T)) \\
\text{Geometric average strike put} &= \max(0, G(T) - S_T)
\end{aligned}$$

Comparing Asian options

Example 14.2-1 Reproduce the numbers in the following table:

TABLE 14.1 Premiums of at-the-money geometric average price and geometric average strike calls and puts, for different numbers of prices averaged, N. The case $N = 1$ for the average price options is equivalent to Black-Scholes values. Assumes $S = \$40$, $K = \$40$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0$, and $t = 1$.

	Average Price ($)		Average Strike ($)	
N	Call	Put	Call	Put
1	6.285	3.209	0.000	0.000
2	4.708	2.645	2.225	1.213
3	4.209	2.445	2.748	1.436
5	3.819	2.281	3.148	1.610
10	3.530	2.155	3.440	1.740
50	3.302	2.052	3.668	1.843
1000	3.248	2.027	3.722	1.868
∞	3.246	2.026	3.725	1.869

Solution. Bonus problem... □

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The barrier option has the payoff
that depends on whether over the option life the underlying price reaches a specified level.

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- Since barrier puts and calls never pay more than standard puts and calls, they are no more expensive than standard puts and calls
- Widely used in practice

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Types of Barrier Options

1. **Knock-out options**: Go out of existence

   down-and-out: if the asset price falls to reach the barrier

   up-and-out: if the asset price rises to reach the barrier

2. **Knock-in options**: Come into existence

   down-and-in: if the asset price falls to reach the barrier

   up-and-in: if the asset price rises to reach the barrier

3. **Rebate options**: make a fixed payment if the asset price reaches the barrier

   down rebates: if the asset price falls to reach the barrier

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$$\underbrace{\{\text{down}, \text{up}\}}_{\text{Knock}} \times \{\text{in}, \text{out}\} \times \{\text{call}, \text{put}\}$$

Knock-in option + Knock-out option = Normal option

Down-and-in call + Down-and-out call = Standard call
Down-and-in put + Down-and-out put = Standard put

Up-and-in call + Up-and-out call = Standard call
Up-and-in put + Up-and-out put = Standard put

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This section will be skipped.

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Problems: 14.1, 14.2, 14.3, 14.4, 14.5, 14.6.

Due Date: TBA