

# Financial Mathematics

MATH 5870/6870<sup>1</sup>  
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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 3. Insurance, Collars, and Other Strategies

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

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§ 3.1 Basic insurance strategies

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§ 3.4 Speculating on volatility

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Options can be

1. Used to insure long positions (floors)
2. Used to insure short positions (caps)
3. Written against asset positions (selling insurance)
  - Covered call writing
  - Covered put writing

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## Four positions

positions w.r.t. asset	put option	call option
long	purchased ( <b>floor</b> )	written
short	written	purchased ( <b>cap</b> )

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Buying insurance

**floor** = buying a **put** option

**cap** = buying a **call** option

Selling insurance

Covered **put** writing

Covered **call** writing

We will work under the following setup

S&S index

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month <b>call</b>	\$93.809
premium for 1000-strike 6-month <b>put</b>	\$74.201

## Insuring a long position

### – Floors

owning a home	owning a stock index
insuring the house	buying a put (floor)

Goal: to insure against a fall in the price of the underlying asset.

**Example 3.1-1** Under the following scenario, compute the combined profit of insuring a long position via **buying a put** for the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month <b>put</b>	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$900 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$900 - \$74.201 \times 1.02}_{\text{profit on put}} = -\$95.68.$$

□

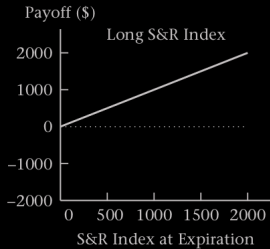
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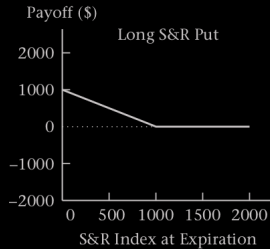
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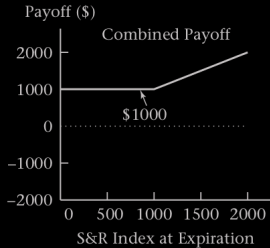




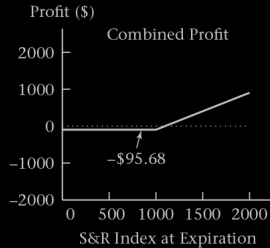
(a)



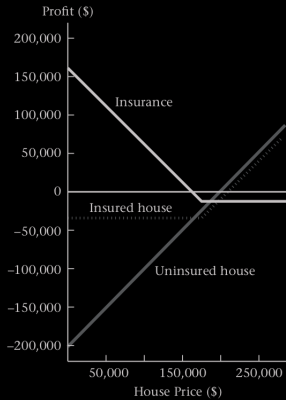
(b)



(c)



(d)





## Insuring a short position

### – Caps

If we have a short position in the S&R index, we experience a loss when the index rises.

We can insure a short position by purchasing a call option (cap) to protect against a higher price of repurchasing the index.

**Example 3.1-2** Under the following scenario, compute the combined profit for insuring a short position via **buying a call** of the following S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month <b>call</b>	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,000 \times 1.02}_{\text{future value of short S\&R index}} - \underbrace{\$93.809 \times 1.02}_{\text{FV of premium for call}} - \underbrace{\$1,000}_{\text{exercise the call option}} = -\$75.685.$$

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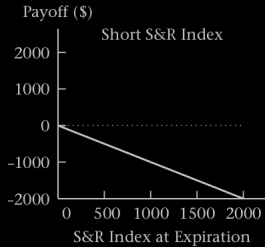
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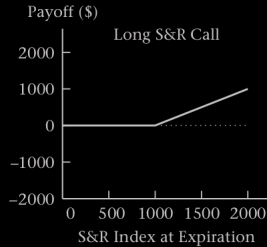
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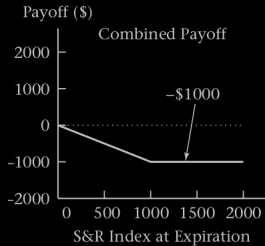
□



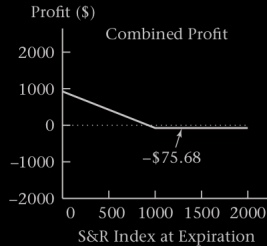
(a)



(b)



(c)



(d)

# Selling insurance

For every insurance buyer there must be an insurance seller

---

## Strategies used to sell insurance

- ▶ Covered writing (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
- ▶ Naked writing is writing an option when the writer does not have a position in the asset.

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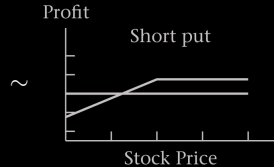
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- ▶ **Covered writing** (option overwriting or selling a covered call) is writing an option when there is a corresponding long position in the underlying asset.
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## Covered call writing

Long position of the asset + Sell a call option



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## Covered put writing

Short position of the asset + Sell a put option



## Covered call writing

Example 3.1-3 Under the following scenario, compute the combined profit for writing a **covered call** for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month <b>call</b>	\$93.809
index price at expiration	\$1,100

Solution.

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$



## Covered call writing

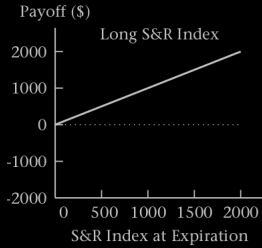
**Example 3.1-3** Under the following scenario, compute the combined profit for writing a **covered call** for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month <b>call</b>	\$93.809
index price at expiration	\$1,100

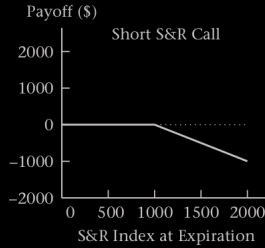
**Solution.**

$$\underbrace{\$1,100 - \$1,000 \times 1.02}_{\text{profit on S\&R index}} + \underbrace{\$1,000 - \$1,100 + \$93.809 \times 1.02}_{\text{profit on written call}} = \$75.68.$$

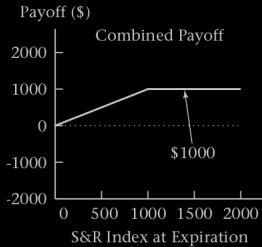




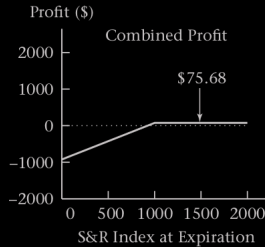
(a)



(b)



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## Covered put writing

**Example 3.1-4** Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

Solution.

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$



## Covered put writing

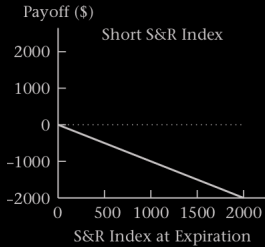
**Example 3.1-4** Under the following scenario, compute the combined profit for writing a covered put for S&R index.

index price today	\$1,000
6-month interest rate	2%
premium for 1000-strike 6-month put	\$74.201
index price at expiration	\$900

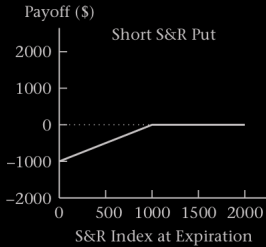
**Solution.**

$$\underbrace{\$1,000 \times 1.02 - \$900}_{\text{profit on selling S\&R index}} + \underbrace{\$900 - \$1,000 + \$74.201 \times 1.02}_{\text{profit on written put}} = \$95.685.$$

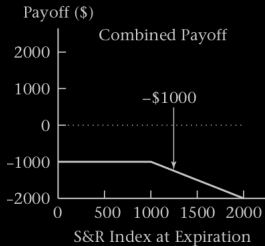




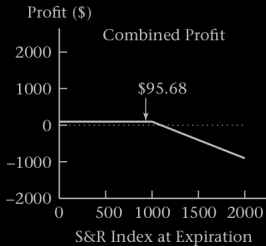
(a)



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It is possible to mimic a long forward position on an asset by  
buying a call + selling a put,  
with each option having the same strike price and expiration time.

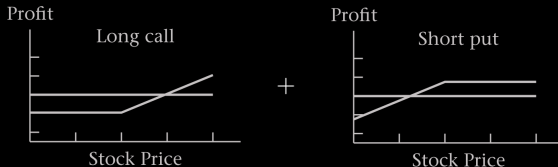
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A synthetic forward

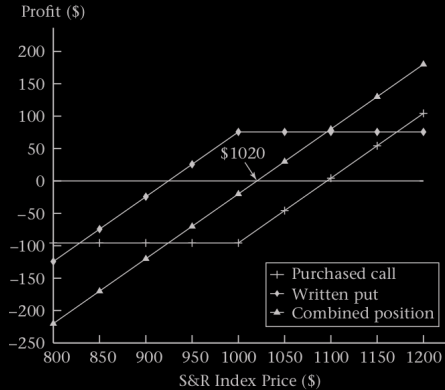
Example 3.2-1 Working with the S&R index. Suppose that

6-month interest rate	2%
premium for 1000-strike 6-month call	\$93.809
premium for 1000-strike 6-month put	\$74.201

Draw profit diagram for the combined position of a purchased call with a written put, namely,



Solution.



□

## A synthetic long forward contract

We pay the net option premium

We pay the strike price

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## The actual forward

We pay zero premium

We pay the forward price

## Basic Assumption

The net cost of buying the index using options  
must equal  
the net cost of buying the index using a forward contract.

**NO ARBITRAGE!**

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## The Put-Call parity equation

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$

- ▶  $K$  is the price
- ▶  $T$  is expiration date
- ▶  $\text{Call}(K, T)$  is the premium for call
- ▶  $\text{Put}(K, T)$  is the premium for put
- ▶  $F_{0,T}$  is the forward price at time  $T$  if you enter at time 0 into a long forward position
- ▶  $\text{PV}()$  is the present value function



## The Put-Call parity equation

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$

- ▶  $K$ : strike price
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- ▶  $\text{Call}(\cdot, \circ)$ : the premium for call.
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- ▶  $\text{PV}(\cdot)$ : the present value function.

**Example 3.2-2** Check Example 3.2-1 to see if the put-call parity equation is satisfied.

Solution. We need to check:

$$\$93.809 - \$74.201 \stackrel{?}{=} \text{PV}(\$1,000 \times 1.02 - \$1,000)$$

Clearly, LHS = \$19.61. On the other hand, the RHS is equal to

$$\begin{aligned} \text{PV}(\$1,000 \times 1.02 - \$1,000) &= \text{PV}(1,000 \times (1.02 - 1)) \\ &= \text{PV}(1,000 \times 0.02) \\ &= \frac{1,000 \times 0.02}{1.02} \\ &= \$19.61. \end{aligned}$$

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Hence, the put-call parity equation is satisfied. □

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$



$$\text{PV}(F_{0,T}) + \text{Put}(K, T) = \text{Call}(K, T) + \text{PV}(K)$$

---

Buying the index and buying the put

generate the same payoff as

buying the call and buying a zero-coupon bond (i.e. lending)  $\text{PV}(K)$

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$



$$\text{PV}(F_{0,T}) - \text{Call}(K, T) = \text{PV}(K) - \text{Put}(K, T)$$

---

Writing a covered call

has the same profit as

lending  $\text{PV}(K)$  and selling a put

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$

---

Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)		
Inuring a short position (caps)		
Covered call writing		
Covered put writing		

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$

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Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)	Index + Put	
Inuring a short position (caps)		
Covered call writing		
Covered put writing		

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$

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Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)	Index + Put	Bound + Call
Inuring a short position (caps)		
Covered call writing		
Covered put writing		



$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$


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Position	Meaning	equivalent to
Inuring a long position (floors)	Index + Put	Bound + Call
Inuring a short position (caps)	-Index + Call	-Bound + Put
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Revisit four positions in Section 3.1

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Inuring a long position (floors)	Index + Put	Bound + Call
Inuring a short position (caps)	-Index + Call	-Bound + Put
Covered call writing	Index - Call	Bound - Put
Covered put writing		

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$


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Revisit four positions in Section 3.1

Position	Meaning	equivalent to
Inuring a long position (floors)	Index + Put	Bound + Call
Inuring a short position (caps)	-Index + Call	-Bound + Put
Covered call writing	Index - Call	Bound - Put
Covered put writing	-Index - Put	

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$


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Covered put writing	-Index - Put	-Bound - Call

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§ 3.1 Basic insurance strategies

§ 3.2 Put-call parity

§ 3.3 Spreads and collars

§ 3.4 Speculating on volatility

§ 3.5 Problems

# Chapter 3. Insurance, Collars, and Other Strategies

§ 3.1 Basic insurance strategies

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It is always possible

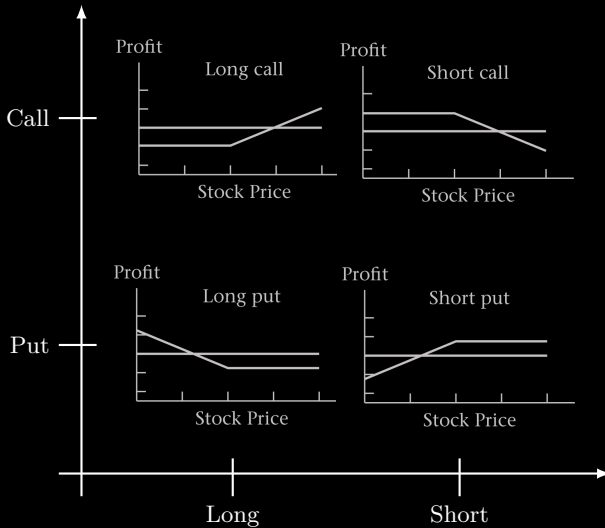
to

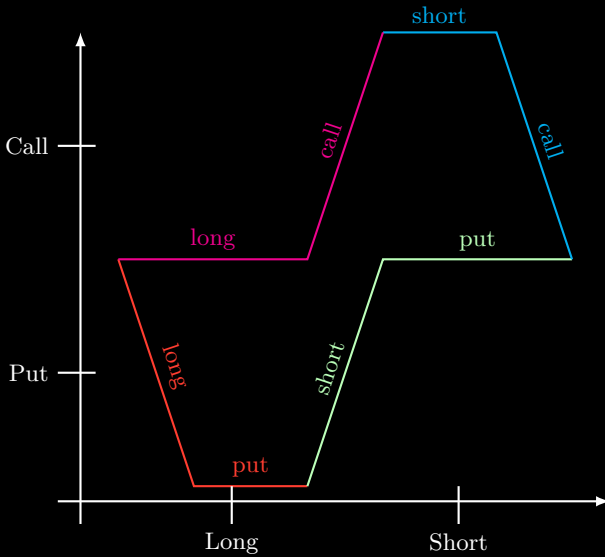
lower the cost of a position

by

reducing its payoff!

By combining two or more options, we find many well-known strategies.





An **option spread** is a position consisting of only calls or only puts, in which some options are purchased and some written.

- ▶ Bull and bear spreads
- ▶ Box spreads
- ▶ Ratio spreads
- ▶ Collars

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## Example for this section

### Black-Scholes option prices

Stock price = \$40

Volatility = 30%

Effective annual risk-free rate = 8.33%

Dividend yield = \$0

Expiration days = 91 days

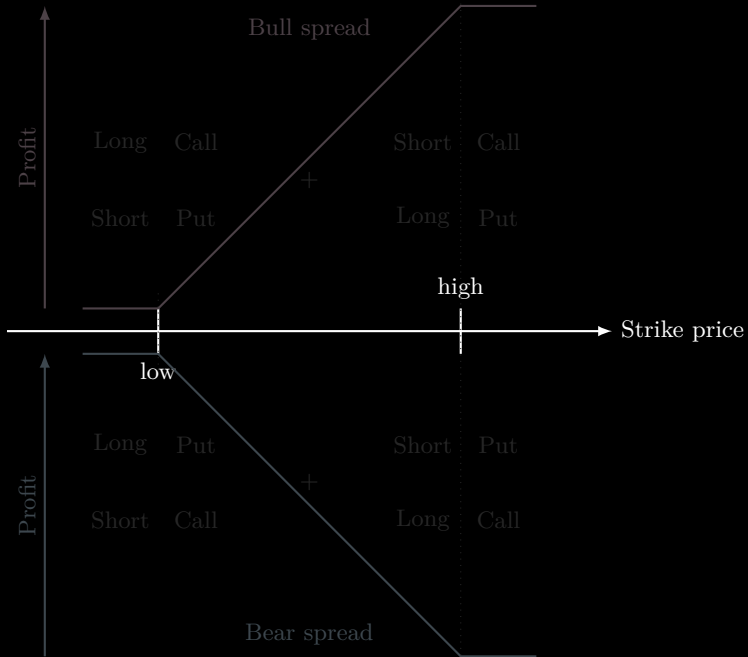
Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
45	0.97	5.08

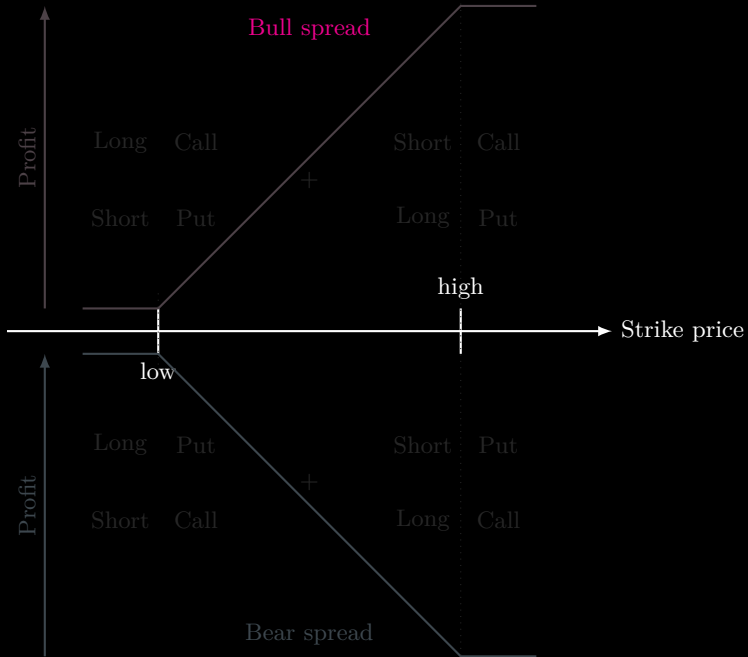


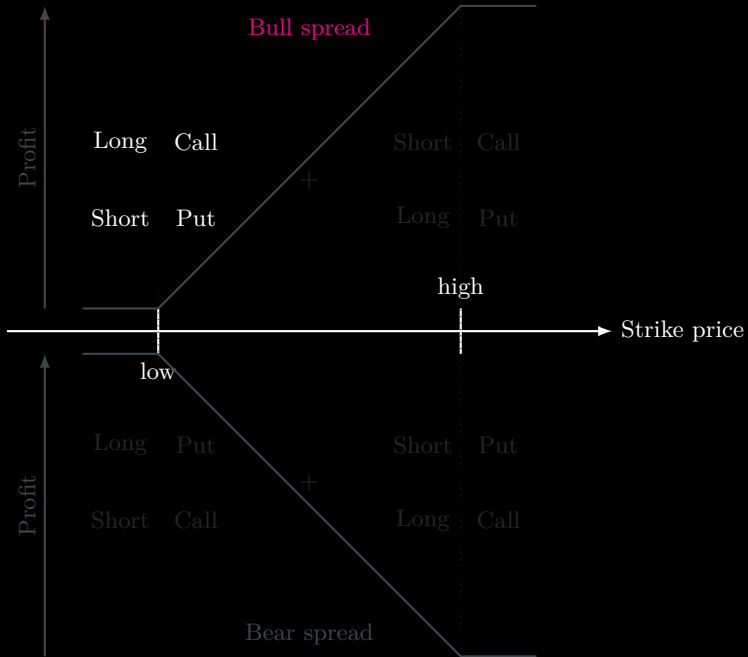
## Bull and bear spreads

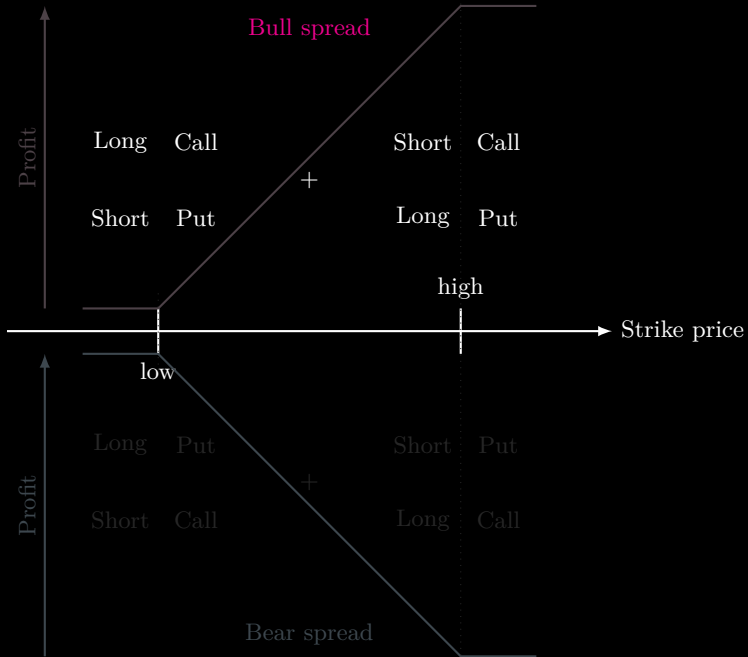
A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a **bull spread**. Bull spreads can also be constructed using puts.

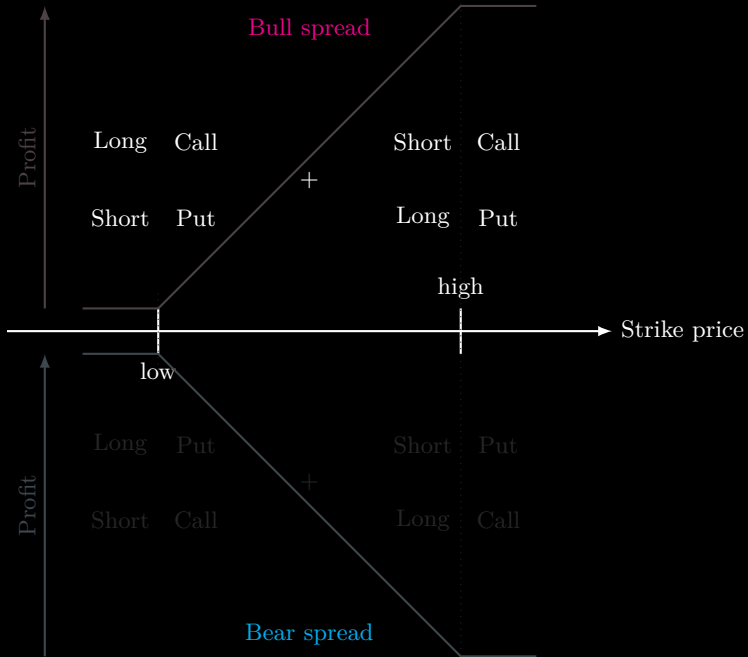
The opposite of a bull spread is a **bear spread**.

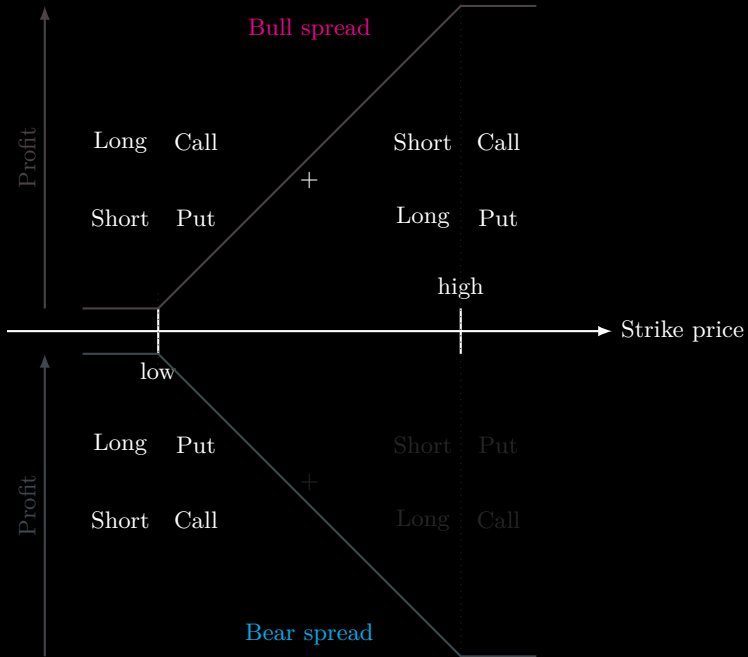


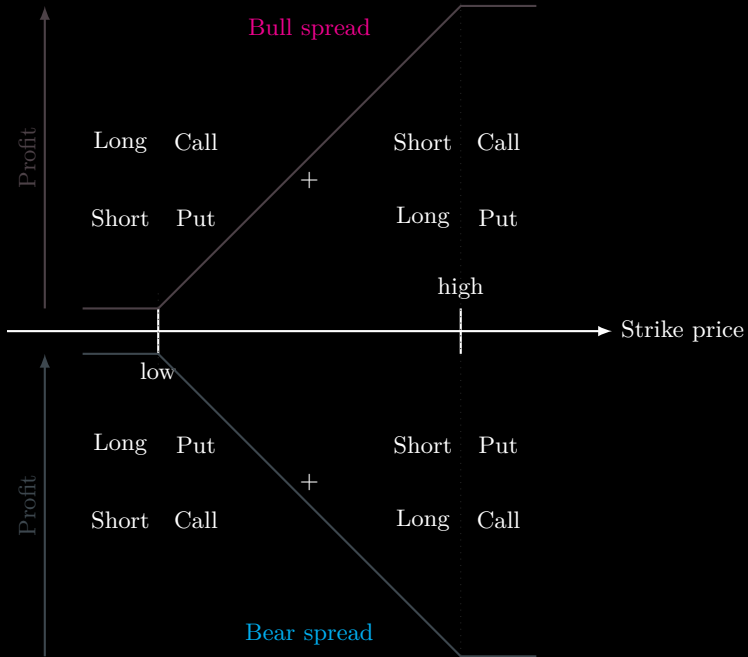




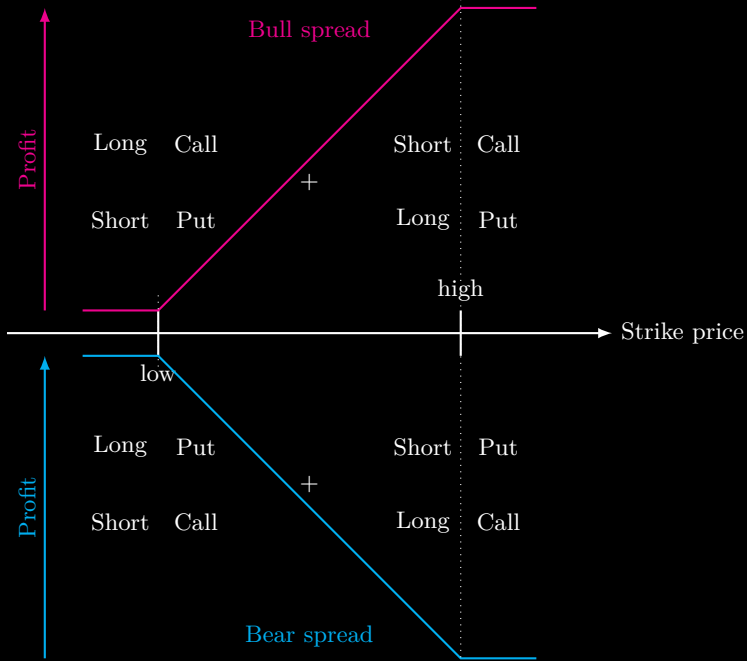












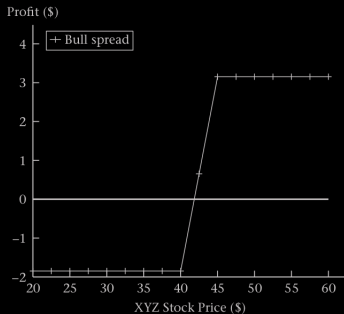
**Example 3.3-1** Draw profit diagram for a 40-45 bull spread, namely, buying a 40-strike call and selling a 45-strike call.

Solution.

We only need to determine the two levels.

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Solution(Continued).

(a) Suppose that the index price is \$ 30 at the expiration:

$$(\$2.78 - \$0.97) \times (1 + 0.0833)^{1/4} = \$1.85.$$

(b) Suppose that the index price is \$50 at the expiration:

$$(\$50 - \$40) - (\$40 - \$45) - \$1.85 = \$3.15.$$



Solution(Continued).

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$$(\$50 - \$40) - (\$40 - \$45) - \$1.85 = \$3.15.$$

□

## Box spreads

A **box spread** is accomplished by using options to create a **synthetic long forward** at one price and a **synthetic short forward** at a different price.

This strategy guarantees a cash flow in the future.

Hence, it is an option spread that is purely a means of borrowing or lending money. It is costly but has no stock price risk.

**Example 3.3-2** Suppose we simultaneously enter into the following two transactions:

1. Buy a 40-strike call and sell a 40-strike put.
2. Sell a 45-strike call and buy a 45-strike put.

Explain why there is no free lunch. Draw the profit diagram.

Solution. The profit is

$$5 + \underbrace{(1.99 - 2.78) \times (1.0833)^{1/4}}_{\text{Synthetic long forward}} + \underbrace{(0.97 - 5.08) \times (1.0833)^{1/4}}_{\text{Synthetic short forward}} = \$0.0099851.$$

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□

Buy a \$40-strike call

Sell a \$40-strike put


Synthetic long forward

Sell a \$45-strike call


Buy a \$45-strike put

Synthetic short forward

Bull spread

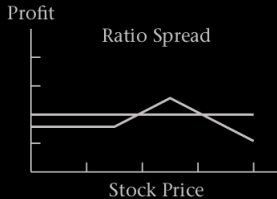


Bear spread



## Ratio spreads

A **ratio spread** is constructed by buying  $m$  options at one strike and selling  $n$  options at a different strike, with all options having the same type (call or put), same time to maturity, and same underlying asset.



**Example 3.3-3 (Problem 3.15)** Compute profit diagrams for the following ratio spreads:

- a Buy 950-strike call, sell two 1050-strike calls.
- b Buy two 950-strike calls, sell three 1050-strike calls.
- c Consider buying  $n$  950-strike calls and selling  $m$  1050-strike calls so that the premium of the position is zero. Considering your analysis in (a) and (b), what can you say about  $n/m$ ? What exact ratio gives you a zero premium?

Strike	Call	Put
\$950	\$120.405	\$51.777
1000	93.809	74.201
1020	84.470	84.470
1050	71.802	101.214
1107	51.873	137.167

Solution. ...



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Solution. ...



## Collars

A **collar** is the purchase of a put option and the sale of a call option with a higher strike price, with both options having the same underlying asset and having the same expiration date.

If the position is reversed, i.e., sale of a put and purchase of a call, the collar is written.

The **collar width** is the difference between the call and put strikes.

**Example 3.3-4** Draw the profit diagram for a purchased collar:  
selling a 45-strike call + buying a 40-strike put.

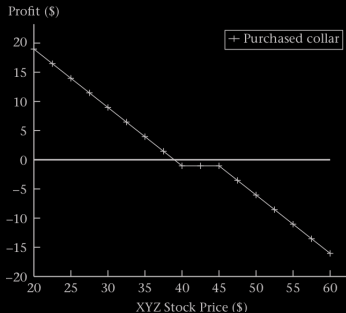
**Solution.** One can easily draw the profit graph. We only need to determine the level when the curve is flat. Hence, suppose the price is \$43. Then the profit is

$$(0.97 - 1.99) \times (1.083)^{1/4} = -\$1.0405.$$

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It is possible to find strike prices for the put and call such that the two premiums exactly offset one another. This position is called a **zero-cost collar**.

Example 3.3-5 Consider XYZ:

Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
41.72	1.99	—
45	0.97	5.08

where we need to use **Black-Scholes formula** to find out the strike price, which is **41.72**, when the put premium is **\$1.99**. This gives a zero-cost collar.

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## Directional positions

- ▶ Bull spread
  - ▶ Bear spread
  - ▶ Collars
  - ▶ Box spreads
- 

## Nondirectional positions

- ▶ Straddles
- ▶ Strangles
- ▶ Butterfly spread

Investors who do not care whether the stock goes up or down, but only **how much it moves**.

Investors are speculating on

**volatility**

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## Example for this section

### Black-Scholes option prices

Stock price = \$40

Volatility = 30%

Effective annual risk-free rate = 8.33%

Dividend yield = \$0

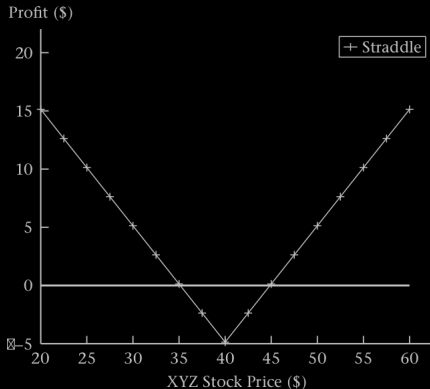
Expiration days = 91 days

Strike	Call	Put
35	6.13	0.44
40	2.78	1.99
45	0.97	5.08

# Straddles

**Straddle** is the strategy of buying a call and a put with the same strike price and time to expiration.

A straddle is a bet that **volatility will be high** relative to the market's assessment



**Example 3.4-1** Draw the profit graph for a \$40=strike straddle.

**Solution.** We only need to determine the tip of the graph:

$$-(2.78 + 1.99) \times (1 + 0.083)^{1/4} = -\$4.8660.$$

Hence,



□

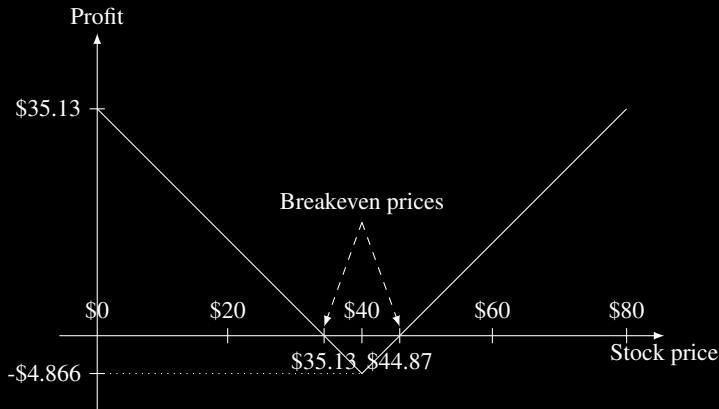


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Solution. We only need to determine the tip of the graph:

$$-(2.78 + 1.99) \times (1 + 0.083)^{1/4} = -\$4.8660.$$

Hence,



# Strangle

**Strangle** is the strategy of buying an out-of-the-money call and put with the same time to expiration.

A **strangle** can be used to reduce the high premium cost, associated with a **straddle**.

	Buying call at a strike price	Buying put at a strike price
<b>Straddle</b>	Same	Same
<b>Strangle</b>	High	Low

**Example 3.4-2** Draw profit diagram for 40-strike straddle and strangle composed of 35-strike put + 45-strike call.

**Solution.** We know the shape of the graph and need only to determine the level of the flat part. Hence, suppose the stock price is \$40. Then the profit is

$$-(0.44 + 0.97) \times (1 + 0.083)^{1/4} = -\$1.4384.$$

The breakeven prices are

$$45 + 1.4384 = \$46.4384 \quad \text{and} \quad 35 - 1.4384 = \$33.562.$$

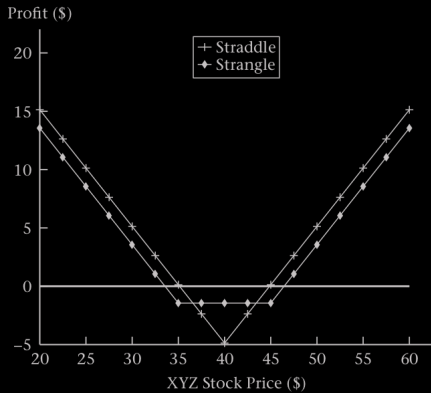
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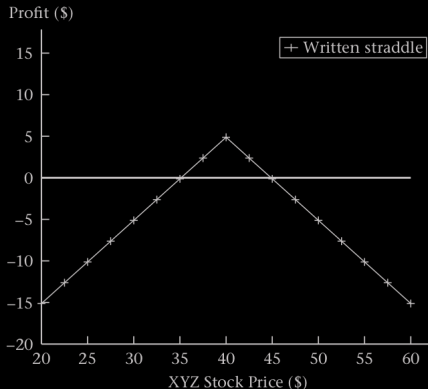
## Written straddles

**Written straddle** is the strategy of selling a call and put with the same strike price and time to maturity.

Unlike a purchased straddle, a written straddle is a bet that

volatility will be low

relative to the market's assessment.

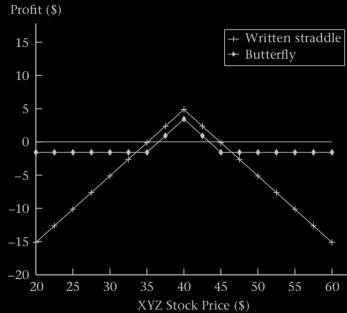
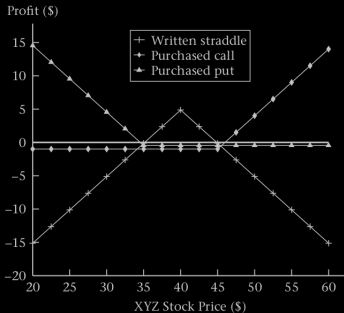


# Butterfly spreads

**Butterfly spreads** = Insured wrien straddle

= **Written straddle** + **purchased straggle**

A butterfly spread insures against large losses on a straddle.



**Example 3.4-3** Draw the profit graph for the butterfly spread:

Written \$40 **straddle** + purchased 35-45 **strangle**.

**Solution.** First notice that this spread corresponds:

Strike	Call	Put
35	6.13	0.44 (long)
40	2.78 (short)	1.99 (short)
45	0.97 (long)	5.08

We know the general shape of the profit graph and need only to determine the level when the graph is flat. For this, suppose that the stock price is \$  $x < 30$ . In this case, only both puts are in the money and the profit is

$$(2.78 + 1.99 - 0.44 - 0.97) \times (1 + 0.083)^{1/4} + (35 - x) + (x - 40) = -\$1.5724.$$



Example 3.4-3 Draw the profit graph for the butterfly spread:

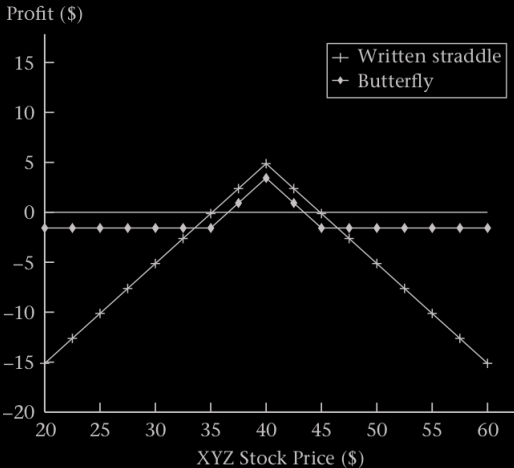
Written \$40 **straddle** + purchased 35-45 **straggle**.

**Solution.** First notice that this spread corresponds:

Strike	Call	Put
35	6.13	0.44 ( <b>long</b> )
40	2.78 ( <b>short</b> )	1.99 ( <b>short</b> )
45	0.97 ( <b>long</b> )	5.08

We know the general shape of the profit graph and need only to determine the level when the graph is flat. For this, suppose that the stock price is \$  $x < 30$ . In this case, only both puts are in the money and the profit is

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Problems: 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.11, 3.13, 3.14, 3.15, 3.17, 3.18.

Due Date: TBA