# Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 9. Parity and other option relationships

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§ 9.1 Put-call parity

§ 9.2 Generalized parity and exchange options

§ 9.3 Comparing options with respect to style, maturity, and strike

§ 9.4 Problems

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§ 9.4 Problems

How does one value the right to back away from a commitment?

**TABLE 9.1** IBM option prices, dollars per share, May 6, 2011. The closing price of IBM on that day was $168.89.

| | | Calls | | Puts | |
|---|---|---|---|---|---|
| **Strike** | **Expiration** | **Bid ($)** | **Ask ($)** | **Bid ($)** | **Ask ($)** |
| 160 | June | 10.05 | 10.15 | 1.16 | 1.20 |
| 165 | June | 6.15 | 6.25 | 2.26 | 2.31 |
| 170 | June | 3.20 | 3.30 | 4.25 | 4.35 |
| 175 | June | 1.38 | 1.43 | 7.40 | 7.55 |
| 160 | October | 14.10 | 14.20 | 5.70 | 5.80 |
| 165 | October | 10.85 | 11.00 | 7.45 | 7.60 |
| 170 | October | 8.10 | 8.20 | 9.70 | 9.85 |
| 175 | October | 5.80 | 5.90 | 12.40 | 12.55 |

*Source:* Chicago Board Options Exchange.

- What determines the difference between put and call prices at a given strike?
- How would the premiums change if these options were European rather than American?
- It appears that, for a given strike, the October options are more expensive than the June options. Is this necessarily true?
- Do call premiums always decrease as the strike price increases? Do put premiums always increase as the strike price increases?
- Both call and put premiums change by less than the change in the strike price. Does this always happen?

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# European options

$$C(K,T) - P(K,T) = \mathrm{PV}_{0,T}\left(F_{0,T} - K\right) = e^{-rT}\left(F_{0,T} - K\right)$$

Buying a call and selling a put
with the strike both equal to the forward price (i.e., $K = F_{0,T}$)
creates a synthetic forward contract
and hence must have a zero price.

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Parity generally fails for American options!

# European options

$$C(K,T) - P(K,T) = \mathrm{PV}_{0,T}\,(F_{0,T} - K)$$
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# Parity for stocks

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

Example 9.1-1 Suppose that the price of a non-dividend-paying stock is \$40, the continuously compounded interest rate is 8%, and options have 3 months to expiration. If a 40-strike European call sells for \$2.78, find the price for a 40-strike European put sells.

Solution. Let the price for put be $y$. Then

$$\$2.78 = y + \$40 - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$1.99.$$

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# Why is a call more expensive than a put?

When $S_0 = K$ and $\mathrm{Div} = 0$, then

$$C(K,T) - P(K,T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is
the time value of money.

## Why is a call more expensive than a put?

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When $S_0 = K$ and $\text{Div} = 0$, then

$$C(K,T) - P(K,T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is the time value of money.

# Why is a call more expensive than a put?

When $S_0 = K$ and $\text{Div} = 0$, then

$$C(K, T) - P(K, T) = K\left(1 - e^{-rT}\right)$$

The difference of a call and put is the time value of money.

Example 9.1-2 Make the same assumptions as in Example 9.1-1, except suppose that the stock pays a \$5 dividend just before expiration. If the price of the European call is \$0.74, what would be the price of the European put?

Solution. Let the price for put be $y$. Then

$$\$0.74 = y + \left(\$40 - \$5e^{-0.08\times 0.25}\right) - \$40e^{-0.08\times 0.25}$$

Hence,

$$y = \$4.85.$$

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# Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic stock

$$S_0 = C(K,T) - P(K,T) + \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

- Synthetic Treasury bill (T-bill)

$$\underbrace{S_0 - C(K,T) + P(K,T)}_{\text{a } \textbf{conversion}} = \mathrm{PV}_{0,T}(\mathrm{Div}) + e^{-rT}K$$

Motivation:

A hedged position that has no risk but requires investment.

T-bills are taxed differently than stocks.

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

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# Synthetic securities

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

---

- Synthetic options

$$C(K,T) = P(K,T) + (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) - e^{-rT}K$$

$$P(K,T) = C(K,T) - (S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})) + e^{-rT}K$$

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Generalize the parity to apply to the case where the strike asset is not necessarily cash but could be any other asset.

We will skip this section and
leave it for motivated students.

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# European versus American options

$$C_{\text{Amer}}(S, K, T) \geq C_{\text{Eur}}(S, K, T)$$

$$P_{\text{Amer}}(S, K, T) \geq P_{\text{Eur}}(S, K, T)$$

# Maximum and minimum option prices

$$S \geq C_{\mathrm{Amer}}(S, K, T) \geq C_{\mathrm{Eur}}(S, K, T) \geq \max\left(0, \mathrm{PV}_{0,T}(F_{0,T}) - \mathrm{PV}_{0,T}(K)\right)$$

$$K \geq P_{\mathrm{Amer}}(S, K, T) \geq P_{\mathrm{Eur}}(S, K, T) \geq \max\left(0, \mathrm{PV}(K) - \mathrm{PV}_{0,T}(F_{0,T})\right)$$

# Early exercise for American options

Calls on stocks with no dividend

No early exercise!

---

$$
\begin{aligned}
C_{\text{Ame}}(S_t, K, T-t) &\geq C_{\text{Eur}}(S_t, K, T-t) \\
&= \underbrace{S_t - K}_{\text{Exercise value}} + \underbrace{P_{Eur}(S_t, K, T-t)}_{\text{Insurance against } S_T < K} + \underbrace{K\left(1 - e^{-r(T-t)}\right)}_{\text{Time value of money on } K} \\
&\geq S_t - K
\end{aligned}
$$

Instead of $C(S_t, K, T-t) \geq S_t - K$ one can prove a stronger version:

$$C(S_t, K, T-t) \geq S_t - Ke^{-r(T-t)}$$

---

| Transaction | Time t | Expiration or Exercise, Time T | |
|---|---|---|---|
| | | $S_T < K$ | $S_T > K$ |
| Buy call | $-C$ | $0$ | $S_T - K$ |
| Short stock | $S_t$ | $-S_T$ | $-S_T$ |
| Lend $Ke^{-r(T-t)}$ | $-Ke^{-r(T-t)}$ | $K$ | $K$ |
| Total | $S_t - Ke^{-r(T-t)} - C$ | $K - S_T$ | $0$ |

# Early exercise for American options

Calls on stock with dividends

| Interest beats dividends?<br>$K - \mathrm{PV}_{t,T}(K) > \mathrm{PV}_{t,T}(\mathrm{Div})$ | Early exercise? |
|---|---|
| ✓ | ✗ |
| ✗ | possibly |

When dividends do make early exercise rational, one should exercise at the last moment before the ex-dividend date.

## Early exercise for puts
## (no dividend case)

In order to receive interest, one may exercise early
(think about the case when $S_T = 0$)

| No early exercise | Early exercise |
|---|---|
| $PV_{t,T}(K)$ | $K$ |

Early exercise for puts
(no dividend case)

No-exercise condition:

$$P\left(S_t, K, T-t\right) > K - S_t$$

$$\Updownarrow$$

$$C\left(S_t, K, T-t\right) > K - \mathrm{PV}_{t,T}(K)$$

---

$$P(S_t, K, T-t) = C(S_t, K, T-t) - S_t + \mathrm{PV}_{t,T}(K)$$

| | calls | puts |
|---|---|---|
| Receive | stock | cash |
| Motivation for early exercise | sufficient dividends | sufficient interest |

One can view interest as the dividend on cash.

Dividends are the sole reason to early-exercise an option.

# Time to expiration
# – the $K$ fixed

The longer the more expensive

- American call/put options
- European call option on stock with no dividend

$$C_{Eur} = C_{Ame}$$

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The longer, might be cheaper

- European call option on stock with dividend
- European put option

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# Time to expiration
# – $K_t = ke^{rt}$

**Theorem 9.3-1** When $K_t = e^{rt}K$, i.e., the strike grows at the interest rate, the premiums on European calls and puts on a non-dividend-paying stock increases with time to maturity.

*Proof.* We only prove the case for puts and leave the calls as exercise. Let $T > t$. In order to show that

$$P_{\text{Euro}}(S_T, K_T, T) > P_{\text{Euro}}(S_t, K_t, t),$$

it suffices to find an arbitrage when

$$P_{\text{Euro}}(S_T, K_T, T) \leq P_{\text{Euro}}(S_t, K_t, t).$$

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Proof (continued).

| | | Payoff at Time $T$ | | | |
|---|---|---|---|---|---|
| | | $S_T < K_T$ | | $S_T > K_T$ | |
| | | Payoff at Time $t$ | | | |
| **Transaction** | **Time 0** | $S_t < K_t$ | $S_t > K_t$ | $S_t < K_t$ | $S_t > K_t$ |
| Sell $P(t)$ | $P(t)$ | $S_T - K_T$ | 0 | $S_T - K_T$ | 0 |
| Buy $P(T)$ | $-P(T)$ | $K_T - S_T$ | $K_T - S_T$ | 0 | 0 |
| Total | $P(t) - P(T)$ | 0 | $K_T - S_T$ | $S_T - K_T$ | 0 |

For example, at time $t$, if $S_t < K_t$, one has to buy the stock. The payoff at time $t$ is

$$S_t - K_t = S_t - Ke^{rt}$$

One keeps this stock to time $T$, the stock price becomes $S_T$, and the future value of $Ke^{rt}$ that one spent at time $t$ becomes $Ke^{rt+r(T-t)} = Ke^{rT}$. Hence, the payoff of this strategy at time $T$ is

$$S_T - Ke^{rT} = S_T - K_T.$$

□

# Different strike prices

$$K_1 \leq K_2 \leq K_3$$

| Relation | Ideas in proof, arbitrage in |
|---|---|
| $C(K_1) \geq C(K_2)$ | a call bull spread |
| $P(K_1) \leq P(K_2)$ | a put bear spread |
| $C(K_1) - C(K_2) \leq K_2 - K_1$ | a call bear spread |
| $P(K_2) - P(K_1) \leq K_2 - K_1$ | a put bull spread |
| $\frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$ | an asymmetric butterfly spread |
| $\frac{P(K_2) - P(K_1)}{K_2 - K_1} \leq \frac{P(K_3) - P(K_2)}{K_3 - K_2}$ | an asymmetric butterfly spread |

Convexity revisited

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$

$$\Updownarrow$$

$$C(K_2) \geq \lambda C(K_1) + (1 - \lambda) C(K_3).$$

with

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1}$$

Example 9.3-1 Suppose that

| | | |
|---|---|---|
| Strike | 50 | 55 |
| Call Premium | 18 | 12 |

1. What no-arbitrage property is violated?
2. What spread position would you use to effect arbitrage?
3. Demonstrate that the spread position is an arbitrage.

Solution. Check Example 9.4 on p. 283. □

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Example 9.3-2 Suppose that

| Strike | 50 | 59 | 65 |
|---|---|---|---|
| Call premium | 14 | 8.9 | 5 |

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Solution. Check Example 9.5 on p. 284. □

Example 9.3-3 Suppose that

| | | | |
|---|---|---|---|
| Strike | 50 | 55 | 70 |
| Put premium | 4 | 8 | 16 |

1. What no-arbitrage property is violated?
2. What spread position would you use to effect arbitrage?
3. Demonstrate that the spread position is an arbitrage.

Solution. Check Example 9.6 on p. 284. □

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|---|---|---|---|
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§ 9.4 Problems

Problems: 9.1, 9.2, 9.3, 9.4, 9.8, 9.9, 9.10, 9.11, 9.15.

Due Date: TBA