

# Financial Mathematics

MATH 5870/6870<sup>1</sup>  
Fall 2021

Le Chen

lzc0090@auburn.edu

Last updated on  
September 28, 2021

Auburn University  
Auburn AL

---

<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 10. Binomial Option Pricing: Basic Concepts

# Chapter 10. Binomial Option Pricing: Basic Concepts

§ 10.1 A one-period Binomial tree

§ 10.2 Constructing a Binomial tree

§ 10.3 Two or more binomial periods

§ 10.4 Put options

§ 10.5 American options

§ 10.6 Options on other assets

§ 10.7 Problems

# Chapter 10. Binomial Option Pricing: Basic Concepts

§ 10.1 A one-period Binomial tree

§ 10.2 Constructing a Binomial tree

§ 10.3 Two or more binomial periods

§ 10.4 Put options

§ 10.5 American options

§ 10.6 Options on other assets

§ 10.7 Problems

$$u = e^{(r-\delta)h + \sigma\sqrt{h}}$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}}$$

- ▶  $r$ : continuously compounded annual interest rate.
- ▶  $\delta$ : continuously dividend yield.
- ▶  $\sigma$ : annual volatility.
- ▶  $h$ : the length of a binomial period in years.

# Continuously Compounded Returns

$$r_{t,t+h} = \ln(S_{t+h}/S_t)$$

$$S_{t+h} = S_t e^{r_{t,t+h}}$$

$$r_{t,t+nh} = \sum_{i=1}^n r_{t+(i-1)h,t+ih}$$

---

Go over 3 examples on p. 301

# Volatility

The **volatility** of an asset is the standard deviation of continuously compounded returns.

- ▶ A year is divided into  $n$  periods (say,  $n = 12$ ) of length  $h = 1/n$ .
- ▶ Let  $\sigma^2$  be the annual continuously compounded return.
- ▶ Assuming that the continuously compounded returns are independent and identically distributed
- ▶ We have

$$\sigma^2 = 12 \times \sigma_{\text{monthly}}^2$$

and

$$\sigma_h = \sigma \sqrt{h} \quad \text{or} \quad \sigma = \frac{\sigma_h}{\sqrt{h}}.$$

## Constructing $u$ and $d$

With no volatility

$$S_{t+h} = F_{t,t+h} = S_t e^{(r-\delta)h}$$

---

With volatility

$$uS_t = F_{t,t+h} e^{+\sigma\sqrt{h}}$$

$$dS_t = F_{t,t+h} e^{-\sigma\sqrt{h}}$$

⇓

$$u = e^{(r-\delta)h + \sigma\sqrt{h}}$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}}$$



# Estimating Historical Volatility

TABLE 10.1

Weekly prices and continuously compounded returns for the S&P 500 index and IBM, from 7/7/2010 to 9/8/2010.

Date	S&P 500		IBM	
	Price	$\ln(S_t/S_{t-1})$	Price	$\ln(S_t/S_{t-1})$
7/7/2010	1060.27		127	
7/14/2010	1095.17	0.03239	130.72	0.02887
7/21/2010	1069.59	-0.02363	125.27	-0.04259
7/28/2010	1106.13	0.03359	128.43	0.02491
8/4/2010	1127.24	0.01890	131.27	0.02187
8/11/2010	1089.47	-0.03408	129.83	-0.01103
8/18/2010	1094.16	0.00430	129.39	-0.00338
8/25/2010	1055.33	-0.03613	125.27	-0.03238
9/1/2010	1080.29	0.02338	125.77	0.00398
9/8/2010	1098.87	0.01705	126.08	0.00246
Standard deviation	0.02800		0.02486	
Standard deviation $\times \sqrt{52}$	0.20194		0.17926	

- ▶ Volatility computation should exclude dividend.
- ▶ But since dividends are small and infrequent; the standard deviation will be similar whether you exclude dividends or not when computing the standard deviation.

## One-period Example with a Forward Tree

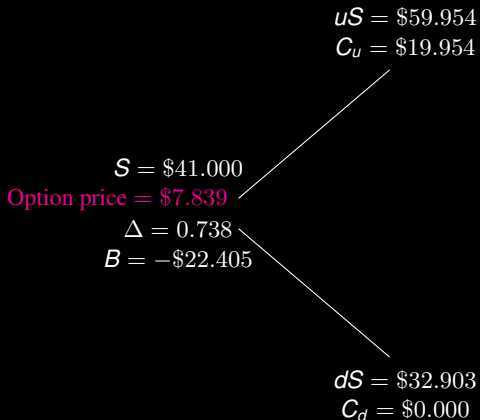
**Example 10.2-1** Consider a European call option on a stock, with a \$40 strike and 1 year to expiration. The stock does not pay dividends, and its current price is \$41. Suppose the volatility of the stock is 30%. The continuously compounded risk-free interest rate is 8%.

Use these inputs to calculate the followings:

1. the final stock prices  $uS$  and  $dS$
2. the final option values  $C_u$  and  $C_d$
3.  $\Delta$  and  $B$
4. the option price:  $\Delta S + B$ .

Solution. In summary:

$$S = 41, K = 40, r = 0.08, \delta = 0, \sigma = 0.30, h = 1.$$



□

## Questions

- ▶ How to handle more than one binomial period?
- ▶ How to price put options?
- ▶ How to price American options?