Financial Mathematics

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Chapter 11. Binomial Option Pricing: Selected Topics

Options may be rationally exercised prior to expiration

By exercising, the option holder

+ Receives the stock and thus receives dividends

− Pays the strike price prior to expiration (this has an interest cost)

− Loses the insurance implicit in the call against the possibility that the stock price will be less than the strike price at expiration

Example 11.1-1 For a call option, let $K = 100$, $r = 0.05$, $\delta = 0.05$, $\sigma = 0$ and the stock price today is $S = 200$. Shall we exercise the call?

Solution.

+ Receives the stock and thus receives dividends:

$$S \times \delta = 200 \times 0.05 = \$10.00.$$

− Pays the strike price prior to expiration (this has an interest cost)

$$K \times r = 100 \times 0.05 = \$5.00.$$

− Loses the insurance: $\$0$ because $\delta = 0$.

Hence, we need to early exercise! □

If volatility is zero, the value of insurance is zero. Then, it is optimal to defer exercise as long as interest savings on the strike exceed dividends lost

$$rK > \delta S$$

$$\Downarrow$$

It is optimal to exercise $\iff$ $S > \dfrac{rK}{\delta}$

E.g. If $r = \delta$, any in-the-money option should be exercised immediately.

If $r = 3\delta$, we exercise when the stock price is 3 times of the strike price.

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When volatility is positive, the implicit insurance has value that varies with time to expiration.

Early-exercise boundary
– American call

FIGURE 11.1

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American call option. In all cases, $K = \$100$, $r = 5\%$, and $\delta = 5\%$.

Exercise Boundary

$\sigma = 0.50$

$\sigma = 0.30$

$\sigma = 0.10$

Time in Years
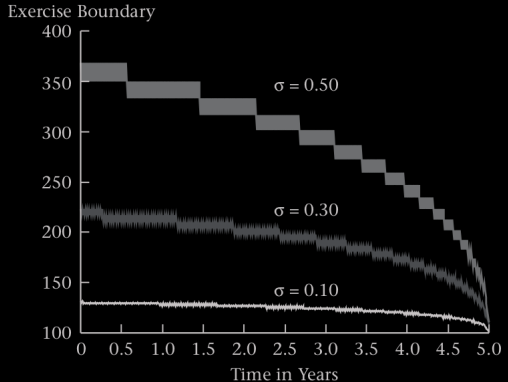
▶ Curve computed using 500 binomial steps.
▶ When $\sigma = 0$, the boundary should be $S = K = \$100$.
▶ The value of insurance diminishes in time.

Early-exercise boundary
– American put

FIGURE 11.2

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American put option. In all cases, $K = \$100$, $r = 5\%$, and $\delta = 5\%$.

Exercise Boundary

$\sigma = 0.10$

$\sigma = 0.30$

$\sigma = 0.50$

Time in Years