Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 11. Binomial Option Pricing: Selected Topics

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Options may be rationally exercised prior to expiration

By exercising, the option holder

+ Receives the stock and thus receives dividends

– Pays the strike price prior to expiration (this has an interest cost)

– Loses the insurance implicit in the call against the possibility that the stock price will be less than the strike price at expiration

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Example 11.1-1 For a call option, let $K = 100$, $r = 0.05$, $\delta = 0.05$, $\sigma = 0$ and the stock price today is $S = 200$. Shall we exercise the call?

Solution.

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$$S \times \delta = 200 \times 0.05 = \$10.00.$$

− Pays the strike price prior to expiration (this has an interest cost)

$$K \times r = 100 \times 0.05 = \$5.00.$$

− Loses the insurance: \$0 because $\delta = 0$.

Hence, we need to early exercise! □

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If volatility is zero, the value of insurance is zero. Then, it is optimal to defer exercise as long as interest savings on the strike exceed dividends lost

$$rK > \delta S$$

$$\Downarrow$$

$$\text{It is optimal to exercise} \quad \Longleftrightarrow \quad S > \frac{rK}{\delta}$$

E.g. If $r = \delta$, any in-the-money option should be exercised immediately.

If $r = 3\delta$, we exercise when the stock price is 3 times of the strike price.

When volatility is positive, the implicit insurance has value that varies with time to expiration.

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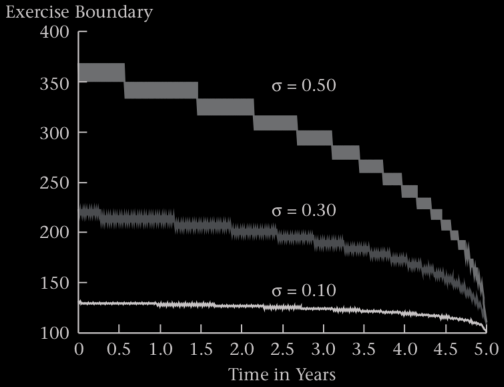
Early-exercise boundary – American call

FIGURE 11.1

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American call option. In all cases, $K = \$100$, $r = 5\%$, and $\delta = 5\%$.



- Curve computed using 500 binomial steps.
- When $\sigma = 0$, the boundary should be $S = K = \$100$.
- The value of insurance diminishes in time.

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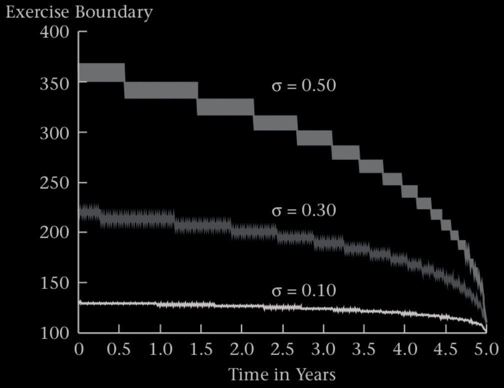

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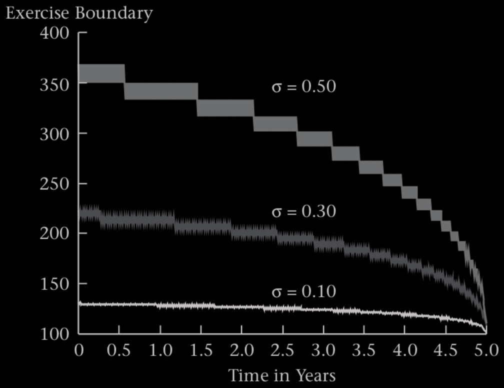

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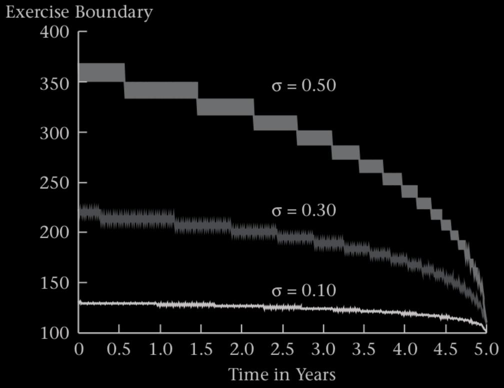

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Early-exercise boundary
– American put

FIGURE 11.2

Early-exercise boundaries for volatilities of 10%, 30%, and 50% for a 5-year American put option. In all cases, $K = \$100$, $r = 5\%$, and $\delta = 5\%$.