

Financial Mathematics

MATH 5870/6870¹
Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 13. Market-Making and Delta-Hedging

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§ 13.1 What do market-makers do?

§ 13.2 Market-maker risk

§ 13.3 Delta-Hedging

§ 13.4 The mathematics of Delta-hedging

§ 13.5 The Black-Scholes analysis

§ 13.6 Market-Making as insurance

§ 13.7 Problems

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§ 13.7 Problems

- ▶ Market-makers attempt to hedge the risk of their positions.
- ▶ Market-makers can control risk by Delta-hedging.
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TABLE 13.1

Price and Greek information for a call option with $S = \$40$, $K = \$40$, $\sigma = 0.30$, $r = 0.08$ (continuously compounded), $T - t = 91/365$, and $\delta = 0$.

| | Purchased | Written |
|------------|-----------|---------|
| Call price | 2.7804 | -2.7804 |
| Delta | 0.5824 | -0.5824 |
| Gamma | 0.0652 | -0.0652 |
| Theta | -0.0173 | 0.0173 |

Example 13.2-1 Under setting of the above table,

- ▶ compute call price, Delta, Gamma and Theta.
- ▶ If stock price increases to $S = 40.75$,
 - find the exact gain/loss of the market-maker.
 - find the approximate gain/loss of the market-maker via Δ .
- ▶ If stock price decreases to $S = 39.25$,
 - find the exact gain/loss of the market-maker.
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(Assume we liquidate the position at the same day)

Solution. Try codes/Section_13-2.nb



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