# Financial Mathematics

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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

# Chapter 13. Market-Making and Delta-Hedging

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From the previous section we see that

$$\text{Market-maker's profit} = -\left(\frac{1}{2}\sigma^2 S_t^2 \Gamma_t + \theta_t + r\left[\Delta_t S_t - C(S_t)\right]\right) h$$

If one believes that via one-standard deviation move, the market-maker's profit is approximately zero, we arrive at the Black-Scholes equation:

$$\boxed{\frac{1}{2}\sigma^2 S_t^2 \Gamma_t + \theta_t + r\Delta_t S_t = rC(S_t)}$$

Equivalently, this can be written as a standard PDE:

$$\mathcal{L}_{\mathrm{BS}} V(t, S) = 0$$

where $V(t, S)$ refers to option (call or put) price and

$$\mathcal{L}_{\mathrm{BS}} = \frac{\partial}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} + r\, S \frac{\partial}{\partial S} V(t, S) - r.$$

One still needs to put the correct boundary conditions.

- Under the following assumptions:

  Underlying asset and the option do not pay dividends

  Interest rate and volatility are constant

  The stock does not make large discrete moves

- The equation is valid only when early exercise is not optimal