Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 14. Exotic Options: I

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The payoff of an Asian option is based on the average price over some period of time.

- It is less valuable than otherwise equivalent ordinary options.
- It is path-dependent.

Situations when Asian options are useful:

- When a business cares about the average exchange rate over time
- When a single price at a point in time might be subject to manipulation
- When price swings are frequent due to thin markets

Eight possible Asian options:

{Call, Put} × {Arithmetic, Geometric} × {Average Price, Average Strike}

- Arithmetic Average: $A(T) = \frac{1}{N}\sum_{i=1}^{N} S_{ih}$.

  Geometric Average: $G(T) = \left(\prod_{i=1}^{N} S_{ih}\right)^{1/N}$.

Eight possible Asian options:

$$\{\text{Call}, \text{Put}\} \times \{\text{Arithmetic}, \text{Geometric}\} \times \{\text{Average Price}, \text{Average Strike}\}$$

$$\begin{aligned}
\text{Arithmetic average price call} &= \max(0, A(T) - K) \\
\text{Arithmetic average price put} &= \max(0, K - A(T)) \\
\text{Arithmetic average strike call} &= \max(0, S_T - A(T)) \\
\text{Arithmetic average strike put} &= \max(0, A(T) - S_T)
\end{aligned}$$

Eight possible Asian options:

$$\{\text{Call, Put}\} \times \{\text{Arithmetic, Geometric}\} \times \{\text{Average Price, Average Strike}\}$$

$$\begin{aligned}
\text{Geometric average price call} &= \max(0, G(T) - K) \\
\text{Geometric average price put} &= \max(0, K - G(T)) \\
\text{Geometric average strike call} &= \max(0, S_T - G(T)) \\
\text{Geometric average strike put} &= \max(0, G(T) - S_T)
\end{aligned}$$

Comparing Asian options

Example 14.2-1 Reproduce the numbers in the following table:

TABLE 14.1 Premiums of at-the-money geometric average price and geometric average strike calls and puts, for different numbers of prices averaged, N. The case $N = 1$ for the average price options is equivalent to Black-Scholes values. Assumes $S = \$40$, $K = \$40$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0$, and $t = 1$.

	Average Price ($)		Average Strike ($)	
N	Call	Put	Call	Put
1	6.285	3.209	0.000	0.000
2	4.708	2.645	2.225	1.213
3	4.209	2.445	2.748	1.436
5	3.819	2.281	3.148	1.610
10	3.530	2.155	3.440	1.740
50	3.302	2.052	3.668	1.843
1000	3.248	2.027	3.722	1.868
∞	3.246	2.026	3.725	1.869

Solution. Bonus problem... □