Financial Mathematics

MATH 5870/6870[1]
Fall 2021

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[1]Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 14. Exotic Options: I

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The payoff of an Asian option is based on the average price over some period of time.

- ► It is less valuable than otherwise equivalent ordinary options.
- ► It is path-dependent.

Situations when Asian options are useful:

- ► When a business cares about the average exchange rate over time
- ► When a single price at a point in time might be subject to manipulation
- ► When price swings are frequent due to thin markets

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Eight possible Asian options:

{Call, Put} × {Arithmetic, Geometric} × {Average Price, Average Strike}

▶ Arithmetic Average: $A(T) = \frac{1}{N} \sum_{i=1}^{N} S_{ih}$.

Geometric Average: $G(T) = \left(\prod_{i=1}^{N} S_{ih} \right)^{1/N}$.

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$$\text{Arithmetic average price call} = \max(0, A(T) - K)$$
$$\text{Arithmetic average price put} = \max(0, K - A(T))$$
$$\text{Arithmetic average strike call} = \max(0, S_T - A(T))$$
$$\text{Arithmetic average strike put} = \max(0, A(T) - S_T)$$

Eight possible Asian options:

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$$\text{Geometric average price call} = \max(0, G(T) - K)$$
$$\text{Geometric average price put } = \max(0, K - G(T))$$
$$\text{Geometric average strike call} = \max(0, S_T - G(T))$$
$$\text{Geometric average strike put } = \max(0, G(T) - S_T)$$

Comparing Asian options

Example 14.2-1 Reproduce the numbers in the following table:

TABLE 14.1 Premiums of at-the-money geometric average price and geometric average strike calls and puts, for different numbers of prices averaged, N. The case $N = 1$ for the average price options is equivalent to Black-Scholes values. Assumes $S = \$40$, $K = \$40$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0$, and $t = 1$.

| | Average Price ($) | | Average Strike ($) | |
N	Call	Put	Call	Put
1	6.285	3.209	0.000	0.000
2	4.708	2.645	2.225	1.213
3	4.209	2.445	2.748	1.436
5	3.819	2.281	3.148	1.610
10	3.530	2.155	3.440	1.740
50	3.302	2.052	3.668	1.843
1000	3.248	2.027	3.722	1.868
∞	3.246	2.026	3.725	1.869

Solution. Bonus problem... □