

# Financial Mathematics

MATH 5870/6870<sup>1</sup>  
Fall 2021

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Last updated on  
October 27, 2021

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<sup>1</sup>Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

## Chapter 14. Exotic Options: I

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§ 14.1 Introduction

§ 14.2 Asian options

§ 14.3 Barrier options

§ 14.4 Compound options

§ 14.5 Gap options

§ 14.6 Exchange options

§ 14.7 Problems

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- ▶ Path-dependent
- ▶ Since barrier puts and calls never pay more than standard puts and calls, they are no more expensive than standard puts and calls.
- ▶ Widely used in practice.

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# Types of Barrier Options

## 1. **Knock-out options:** Go out of existence

down-and-out: if the asset price falls to reach the barrier

up-and-out: if the asset price rises to reach the barrier

## 2. **Knock-in options:** Come into existence

down-and-in: if the asset price falls to reach the barrier

up-and-in: if the asset price rises to reach the barrier

## 3. **Rebate options:** make a fixed payment if the asset price reaches the barrier

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$$\underbrace{\{\text{down, up}\}}_{\text{Knock}} \times \{\text{in, out}\} \times \{\text{call, put}\}$$

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Knock-in option + Knock-out option = Normal option

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Down-and-in call + Down-and-out call = Standard call  
Down-and-in put + Down-and-out put = Standard put

Up-and-in call + Up-and-out call = Standard call  
Up-and-in put + Up-and-out put = Standard put