Financial Mathematics

MATH 5870/6870[1]
Fall 2021

Le Chen

lzc0090@auburn.edu

Last updated on

November 3, 2021

Auburn University
Auburn AL

[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 18. The Lognormal Distribution

Chapter 18. The Lognormal Distribution

§ 18.1 The normal distribution

§ 18.2 The lognormal distribution

§ 18.3 A lognormal model of stock prices

§ 18.4 Lognormal probability calculations

§ 18.5 Problems

- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0, T) \sim N$$

- Let $R(t,s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot,\cdot)$ has to satisfy the additivity property:

$$R(t_0,t_2) = R(t_0,t_1) + R(t_1,t_2)$$

- For time interval $[0,T]$ divided into n subintervals of equal length T/n, we have

$$R(0,T) = R(0,h) + R(h,2h) + \cdots + R((n-1)h,T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h,ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h,ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0,T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0,T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0,T) \sim N$$

- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0, T) \sim N$$

- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0, T) \sim N$$

- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0, T) \sim N$$

- Let $R(t, s)$ be the continuously compounded return from time t to a later time s.
- For $t_0 < t_1 < t_2$, $R(\cdot, \cdot)$ has to satisfy the additivity property:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

- For time interval $[0, T]$ divided into n subintervals of equal length T/n, we have

$$R(0, T) = R(0, h) + R(h, 2h) + \cdots + R((n-1)h, T)$$

Assume that

$$\mathbb{E}\left(R((i-1)h, ih)\right) = \alpha_h \quad \text{and} \quad \mathrm{Var}\left(R((i-1)h, ih)\right) = \sigma_h^2$$

Then

$$\mathbb{E}\left(R(0, T)\right) = n\alpha_h \quad \text{and} \quad \mathrm{Var}\left(R(0, T)\right) = n\sigma_h^2$$

- By central limit limit theorem, as $n \to \infty$, one can assume that

$$R(0, T) \sim N$$

$$\ln(S_t/S_0) \sim N\left([\alpha - \delta - 0.5\sigma^2]t, \sigma^2 t\right)$$

$$\ln(S_t/S_0) = [\alpha - \delta - 0.5\sigma^2]t + \sigma\sqrt{t}\,Z$$

$$S_t = S_0 e^{[\alpha-\delta-0.5\sigma^2]t} e^{\sigma\sqrt{t}\,Z}$$

$$\mathbb{E}[S_t] = S_0 e^{[\alpha-\delta]t} \quad \text{and} \quad \text{Median stock price} = e^{[\alpha-\delta-0.5\sigma^2]t}$$

$$\text{One standard deviation} \begin{cases} \text{move up} = e^{[\alpha-\delta-0.5\sigma^2]t+\sigma\sqrt{t}\times 1} \\ \text{move down} = e^{[\alpha-\delta-0.5\sigma^2]t-\sigma\sqrt{t}\times 1} \end{cases}$$

Go over examples 18.4 and 18.5 on textbook on p. 555.