Financial Mathematics

MATH 5870/6870[1]
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[1] Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 19. Monte Carlo Valuation

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$$V(S_0, 0) = \frac{1}{n} e^{-rT} \sum_{n=1}^{n} V\left(S_T^i, T\right)$$

where

- $S_T^1, \cdots, S_T^n$ are n randomly drawn time-T stock prices.
- For European Call:

$$V(S_T^i, T) = \max\left(0, S_T^i - K\right)$$

Similarly one finds the expression for European put.

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Example 19.4-1 Carry out the Monte Carlo valuation of the European call under the setting of the following table:

TABLE 19.2 Results of Monte Carlo valuation of European call with $S = \$40$, $K = \$40$, $\sigma = 30\%$, $r = 8\%$, $t = 91$ days, and $\delta = 0$. The Black-Scholes price is \$2.78. Each trial uses 500 random draws.

Trial	Computed Price (\$)
1	2.98
2	2.75
3	2.63
4	2.75
5	2.91
Average	2.804

Solution. Check

codes/Table_19-2.py

□

Example 19.4-2 Carry out the Monte Carlo valuation of the Asian call under the setting of the following table:

TABLE 19.3

Prices of arithmetic average-price Asian options estimated using Monte Carlo and exact prices of geometric average price options. Assumes option has 3 months to expiration and average is computed using equal intervals over the period. Each price is computed using 10,000 trials, assuming $S = \$40$, $K = \$40$, $\sigma = 30\%$, $r = 8\%$, $T = 0.25$, and $\delta = 0$. In each row, the same random numbers were used to compute both the geometric and arithmetic average price options. σ_n is the standard deviation of the estimated arithmetic option prices, divided by $\sqrt{10{,}000}$.

Number of	Monte Carlo Prices ($)		Exact	
Averages	Arithmetic	Geometric	Geometric Price ($)	σ_n
1	2.79	2.79	2.78	0.0408
3	2.03	1.99	1.94	0.0291
5	1.78	1.74	1.77	0.0259
10	1.70	1.66	1.65	0.0241
20	1.66	1.61	1.59	0.0231
40	1.63	1.58	1.56	0.0226

Solution. Check

codes/Table_19-3.py

□