

Financial Mathematics

MATH 5870/6870¹
Fall 2021

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¹Based on Robert L. McDonald's *Derivatives Markets*, 3rd Ed, Pearson, 2013.

Chapter 2. An Introduction to Forwards and Options

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§ 2.1 Forward contracts

§ 2.2 Call options

§ 2.3 Put options

§ 2.4 Options are insurance

§ 2.5 Summary of forward and option positions

§ 2.6 Problems

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Can one modify the forward contract so that the buyer can walk away from the deal at expiration?

Definition 2.2-1 A **call option** is a contract where the buyer has the right to buy, but not the obligation to buy.

Example 2.2-1 S&R index: Buyers' perspective

- ▶ Today: call buyer acquires the right to pay \$1,020 in six months for the index, but is not obligated to do so
 - ▶ In six months at contract expiration:
 - if the spot price is \$1,100, call buyers payoff = $\$1,100 - \$1,020 = \$80$
 - if the spot price is \$900, call buyer walks away, buyers payoff = \$0.
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Example 2.2-2 S&R index: Sellers' perspective

- ▶ Today: call seller is obligated to sell the index for \$1,020 in six months, if asked to do so
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- ▶ **Strike (or exercise) price:** the amount paid by the option buyer for the asset if he/she decides to exercise.
- ▶ **Exercise:** the act of paying the strike price to buy the asset.
- ▶ **Expiration:** the date by which the option must be exercised or become worthless.
- ▶ **Exercise style:** specifies when the option can be exercised.

Style	can be exercised
European	only at expiration date
American	at any time before expiration
Bermudan	during specified periods

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Payoff of purchased call = $\max(0, \text{spot price at expiration} - \text{strike price})$

Profit of purchased call = **payoff** of purchased call
– future value of option premium

Payoff of written call = $-\max(0, \text{spot price at expiration} - \text{strike price})$

Profit of written call = **payoff** of written call
+ future value of option premium

Example 2.2-3 S&R Index 6-month European call option

Strike price = \$1,000,

Premium = \$93.81,

6-month risk-free rate = 2%.

Compute both payoff and profit of the **purchased** call option if the index value in six months **\$1,100** (resp. **\$900**).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned}\text{Payoff} &= \max(0, \$1,100 - \$1,000) \\ &= \$100\end{aligned}$$

$$\begin{aligned}\text{Profit} &= \$100 - \$93.81 \times 1.02 \\ &= \$4.32.\end{aligned}$$

If index value in six months = \$900,

$$\begin{aligned}\text{Payoff} &= \max(0, \$900 - \$1,000) \\ &= \$0\end{aligned}$$

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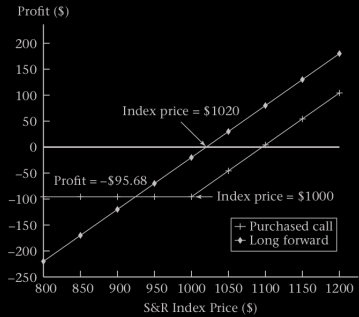
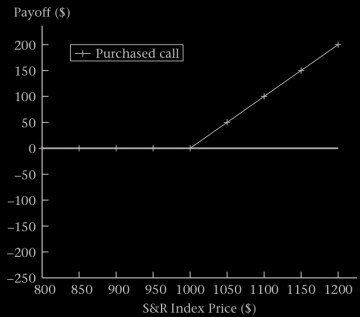
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Example 2.2-4 S&R Index 6-month European call option

Strike price = \$1,000,

Premium = \$93.81,

6-month risk-free rate = 2%.

Compute both payoff and profit of the **written** call option if the index value in six months **\$1,100** (resp. **\$900**).

Solution.

If index value in six months = \$1,100,

$$\begin{aligned}\text{Payoff} &= -\max(0, \$1,100 - \$1,000) \\ &= -\$100\end{aligned}$$

$$\begin{aligned}\text{Profit} &= -\$100 + \$93.81 \times 1.02 \\ &= -\$4.32.\end{aligned}$$

If index value in six months = \$900,

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